

Hydrostatics :-

Hydrostatic Pressure is that branch of science which relating to fluids at rest or to the pressures they exert or transmit Hydrostatic pressure.

Fluid :-

- ⇒ Fluid is a substance that continuously deforms (flow) under an applied shear stress.
- ⇒ Fluids are a subset of the phase of matter and include liquids, gases, plasma and to some extent, plastic solids.
- ⇒ Fluid is a substance which is capable of following
- ⇒ Conform the shape of the containing vessel.
- ⇒ Deform continuously under application of small shear force.
- ⇒ Both liquid & gas comes under fluid categories.

Types of Fluid :-

There are basically two types of fluid

(i) Ideal Fluid

(ii) Real Fluid

* Mass Density :-

=> The mass density of fluid ρ , is the mass of the fluid per unit volume.

=> In SI unit is kg/m^3

=> In CGS unit is gm/cm^3

* Weight Density (γ) :-

=> Weight Density is define as the ratio between weight per unit volume.

=> SI unit is N/m^3 / N/m^3

=> CGS unit is dyne/cm^3 or dyne/cc

* Specific Gravity (G)

=> The Specific Gravity of any fluid is define as the ratio of the density of that fluid to the density of the standard fluid at a given temperature.

$$G = \frac{\text{Weight density of the Substance}}{\text{Weight density of standard fluid at a given temperature.}}$$

Standard of fluid taken as water at 4°C for water

$$\gamma_w = 9.8 \text{ N/m}^3 \text{ or } 1 \text{ gm/cc}$$

$$G = \frac{\rho_s}{\rho_w} \text{ or } \frac{\gamma_s}{\gamma_w}$$

$$\rho_w = 1000 \text{ kg/m}^3 \text{ or } 1 \text{ gm/cc}$$

$$\gamma_w = \frac{1000 \times 9.81}{1000} = 9.81 \text{ kN/m}^3$$

Unit Weight of R.C.C. =

$$\rho_s = 2500 \text{ kg/m}^3$$

$$\gamma_w = 25 \text{ kN/m}^3$$

Specific gravity of R.C.C (G)

$$= \frac{2500}{1000} = 2.5$$

=> Specific gravity has no unit.

Specific gravity of Mercury is 13.6.

Specific Volume :-

Specific volume of a fluid is defined as the volume of a fluid occupied by a unit mass or volume per unit mass of a fluid is called specific volume.

$$\text{Specific volume :- } \frac{\text{Volume of fluid}}{\text{Mass of fluid}}$$

$$= \frac{\text{m}^3}{\text{kg}} \text{ or } \frac{\text{cm}^3}{\text{gm}}$$

Specific volume is reciprocal of mass density.

* Viscosity :-

Viscosity is defined as the property of fluid by virtue of which there is a resistance to flow of liquid layers one over another. is called viscosity.

Mathematically :-

$$\tau = \mu \frac{du}{dy}$$

Where,

$\tau \Rightarrow$ Shear stress

$\frac{du}{dy} \rightarrow$ is called velocity gradient.

$\mu \rightarrow$ Proportionality constant called coefficient of dynamic viscosity.

$u \rightarrow$ Velocity of flow of liquid layers.

$y \rightarrow$ The distance of the layer from a solid boundary.

Newton's Law of viscosity :-

It states that the shear stress developed in a fluid layer is directly proportional to the rate of change of velocity with respect to distance.

(i) Newtonian fluid (obeys Newton's law)

(ii) Non-Newtonian fluid (doesn't obey Newton's law)

Mathematically :-

$$\tau = \mu \frac{du}{dy}$$

Unit of Dynamic viscosity :-

$$\tau = \mu \frac{du}{dy}$$

$$\mu = \frac{\tau}{\frac{du}{dy}} = \frac{\text{Shear stress}}{\frac{\text{Change of velocity}}{\text{Change of distance}}} = \frac{\frac{\text{Force/Area}}{\frac{\text{Length}}{\text{Time}} \times \frac{1}{\text{Length}}}}{\frac{1}{\text{Time}}} = \frac{\text{Force} \times \text{Time}}{(\text{Length})^2}$$

MKS System $\mu = \frac{\text{kgf}}{\frac{\text{m}^2}{\frac{\text{m}}{\text{sec}}}} = \frac{\text{kgf sec}}{\text{m}^2}$

$$\boxed{\mu = \frac{\text{kgf sec}}{\text{m}^2}}$$

CGS System $= \frac{\text{gm} \cdot \text{sec}}{\text{cm}^2} = \frac{\text{dyne} \cdot \text{sec}}{\text{cm}^2}$

$$\boxed{\text{SI} = \frac{\text{N sec}}{\text{m}^2}}$$

$1 \text{ kgf} = 9.81 \text{ N}$

$$1 \frac{\text{dyne sec}}{\text{cm}^2} = 1 \text{ poise}$$

$$= \frac{1 \text{ kgf sec}}{\text{m}^2} = \text{poise}$$

$$= \frac{9.81 \text{ N sec}}{\text{m}^2}$$

$$= \frac{9.81 \times 10^5}{(10^2)^2 \text{ cm}^2}$$

$$= 9.81 \times 10^1 \frac{\text{dyne sec}}{\text{cm}^2}$$

$$= 98.1 \text{ Poise}$$

$$\boxed{\frac{1 \text{ kg f sec}}{\text{m}^2} = 98.1 \text{ Poise}}$$

$$1 \frac{\text{N sec}}{\text{m}^2}$$

$$= \frac{10^5 \text{ sec}}{(10^2)^2}$$

$$= 10 \frac{\text{dyne sec}}{\text{cm}^2}$$

$$= 10 \text{ poise}$$

$$\boxed{\frac{1 \text{ N sec}}{\text{m}^2} = 10 \text{ poise}}$$

$$\boxed{1 \text{ Poise} = \frac{1}{10} \frac{\text{m sec}}{\text{m}^2}}$$

Kinematic viscosity :- $\nu (\text{nu})$

it is the ratio between dynamic viscosity and the density of a fluid.

$$\nu = \frac{\mu}{\rho} \quad \frac{\frac{\text{N/sec}}{\text{m}^2}}{\frac{\text{kg}}{\text{m}^3}}$$

$$= \frac{\text{kg} \cdot \text{m} \times \text{sec}}{\text{sec}^2 \times \text{m}^2}$$

$$\text{kg/m}^2$$

$$= \frac{\text{kg} \cdot \text{m} \times \text{sec}}{\text{sec}^2 \times \text{m}^2} \times \frac{\text{m}^2}{\text{kg}}$$

$$= \frac{\text{m}^2}{\text{sec}}$$

Units :-

MKS / SI .

MKS .

$$\nu = \frac{\text{m}^2}{\text{sec}}$$

$$\text{CGS} = \frac{\text{cm}^2}{\text{sec}}$$

$$\nu = \frac{\text{cm}^2}{\text{sec}}$$

$$1 \text{ Stoke} = \frac{1 \text{ cm}^2}{\text{sec}}$$

$$= 10^{-4} \text{ m}^2/\text{sec}$$

variation of viscosity with temperature

For Liquid, viscosity is inversely proportional to temperature

$$\text{Viscosity} \propto \frac{1}{\text{temperature}}$$

For gas, viscosity is directly proportional to temperature

viscosity $\propto$ temperature

- (1) Calculate the Specific weight, density and Specific Gravity of 1 liter of a liquid is weight 7 N.

given:-

Volume of Liquid = 1 l.

$$= 10^{-3} \text{ m}^3$$

Wt of the Liquid = 7 N

$$\begin{aligned}\text{Specific weight} &= \frac{7 \text{ N}}{10^{-3}} \\ &= 7000 \text{ N/m}^3\end{aligned}$$

$$\begin{aligned}\text{Density} = \rho &= \frac{M}{V} = \frac{\gamma}{G} \\ &= \frac{7000}{9.81}\end{aligned}$$

Specific gravity (G)

$$= \frac{S_s}{S_w}$$

$$= \frac{713.5}{1000}$$

Ans = 0.713

2 Calculate Density, Specific weight and weight of 1l of Petrol of Specific gravity 0.7.

Given:-

$$G = 0.7$$

$$\text{volume} = 1\text{ l}$$

$$= 10^{-3} \text{ m}^3$$

Density =

$$G = \frac{S_s}{S_w}$$

$$S_s = G S_w$$

$$= 0.7 \times 1000$$
$$= 700$$

specific weight :-

$$\gamma = \rho g$$

$$= 700 \times 9.81$$

$$= 6867 \text{ N/m}^3$$

$$wt = \frac{6867}{1000}$$

$$= 6.867 \text{ kg}$$

3

The velocity Profile of a fluid is given by

$$u = \frac{2}{3} y - y^2$$

Where u = velocity of the fluid in m/sec at a distance y m. above the plate. Calculate the shear stress develop at the surface of the plate and at a distance 0.15 m above the plate. take the dynamic viscosity of a fluid as 8.63 poise.

$$\frac{du}{dy} = \frac{d}{dy} \left(\frac{2}{3} y - y^2 \right)$$
$$= \frac{2}{3} - 2y$$

τ = Shear stress at the Surface of the plate

$$y=0 \quad \left[\frac{du}{dy} \right]_y = 0$$

$$= 8.63 \left(\frac{2}{3} - 2 \times 0 \right)$$

$$= 8.63 \times \frac{2}{3}$$

$$= 5.75 \frac{\text{dyne}}{\text{cm}^2}$$

$$\tau_y = 0.15$$

$$= 8.63 \left(\frac{2}{3} - 2 \times 0.0015 \right)$$

$$= 5.72 \text{ dyne/cm}^2$$

Assignment:-

Q Two horizontal plates are placed 1.25 cm apart the space between them being filled with oil of viscosity 14 poises. Calculate the shear stress in oil if upper plate is moved with a velocity of 2.5 m/s.

Given:-

$$\begin{aligned} \text{Plate distance (dy)} &= 1.25 \text{ cm} \\ &= 0.0125 \text{ m.} \end{aligned}$$

$$\text{viscosity } (\mu) = 14 \text{ poise}$$

$$= \frac{14}{10} \frac{\text{N sec}}{\text{m}^2}$$

$$\text{velocity} = 2.5 \text{ m/sec}$$

(a) - Surface tension

Surface tension is defined as the tensile force acting on the surface of a liquid which tends to increase the surface area of the liquid. It is a property of all liquids and is caused by the cohesive forces between the molecules of the liquid.

The magnitude of this force per unit length of the surface is denoted by τ . It is denoted by $\tau = \frac{dF}{dl}$.
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Units of surface tension

$$\tau = \mu \frac{du}{dy}$$

$$\frac{\text{kg}}{\text{m}} = \frac{\text{kg}}{\text{m}}$$

$$\frac{\text{kg}}{\text{m}} = \frac{\text{kg}}{\text{m}}$$

$$= \frac{14}{10} \times \frac{2.5}{0.0125}$$

$$= 280 \text{ N/m}^2$$

Surface Tension :- (σ)

$\Rightarrow$ Surface tension is defined as the tensile force acting on the surface of a liquid in contact with a gas or on the surface between two immiscible liquids such that the contact surface behaves like a membrane under tension.

$\Rightarrow$ The magnitude of this force per unit length of the free surface will have the same value as the surface energy per unit area.

$\Rightarrow$ It is denoted by greek letter sigma (σ).

Units of Surface tension :-

$$\text{MKS Units} = \frac{\text{kgf}}{\text{m}}$$

$$\text{SI Units} = \text{N/m}$$

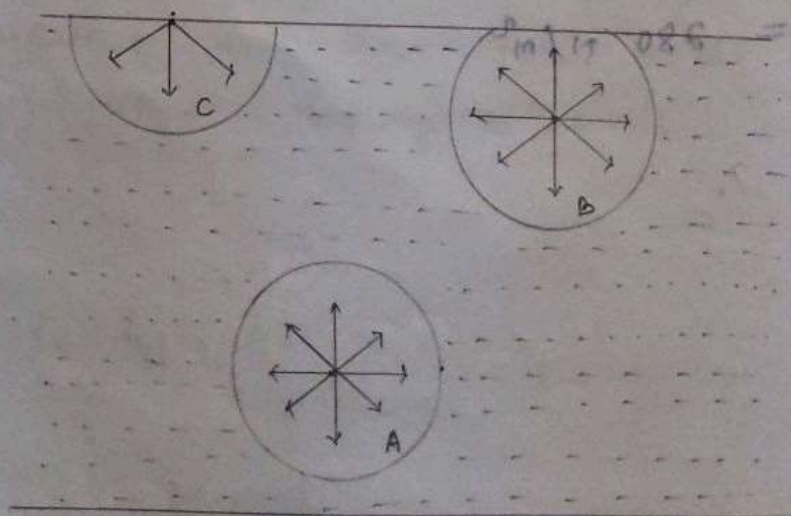
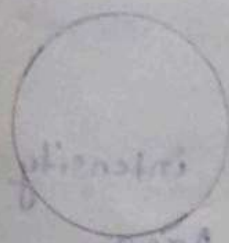
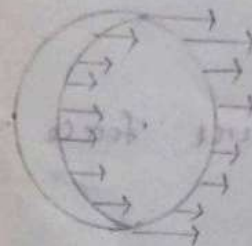


Fig Surface Tension

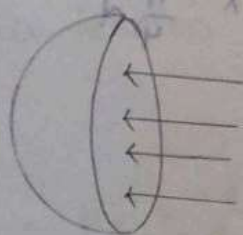
* Surface Tension on Liquid Droplet :-



Droplet



Surface tension



Consider a small spherical droplet of a liquid of radius r , on the entire surface of the droplet, the tensile force into the surface tension will be acting.

$$\frac{2\pi r}{d} = 1$$

Let σ = Surface tension of liquid

P = Pressure intensity inside the droplet

d = Diameter of droplet.

Let the droplet is cut into two halves

The tensile force due to surface tension acting around the circumference of the cut portion.

$$T = 6 \times \text{Circumference}$$

$$= 6 \times \pi d \quad \text{--- (i)}$$

Pressure force = Pressure intensity inside the droplet $\times$ Area

$$= P \times \frac{\pi}{4} d^2 \quad \text{--- (ii)}$$

A equilibrium

From equation (i) & (ii)

$$6 \times \pi d = P \times \frac{\pi}{4} d^2$$

$$\Rightarrow \boxed{P = \frac{4\sigma}{d}}$$

* Surface Tension in Bubble :- (outside)

$$\text{Surface tension developed} = 2 \times \sigma \times \pi d$$

$$\text{Inside Pressure} = P$$

$$\text{Inside pressure force} = P \times \frac{\pi}{4} \times d^2$$

At equilibrium

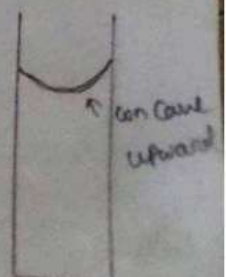
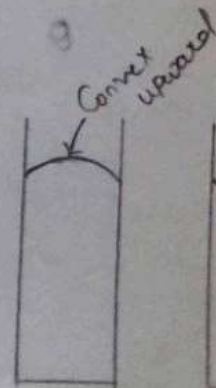
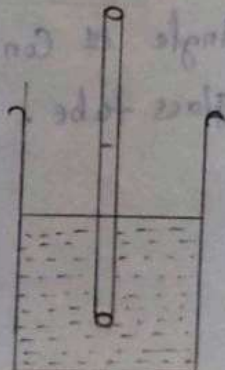
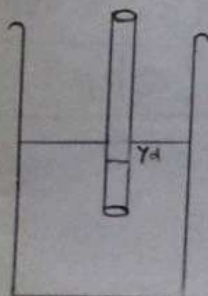
$$\text{Surface tension developed} = \text{Inside pressure force}$$

$$\Rightarrow 2 \times \sigma \times \pi d = P \times \frac{\pi}{4} d^2$$

$$\Rightarrow \boxed{P = \frac{8\sigma}{d}}$$

* Capillarity :-

Capillarity is a property of fluid by virtue of which it rises or dips when a pipe is inserted inside the fluid.



⇒ Capillarity can be measured in two ways

- (i) Capillary Rise
- (ii) Capillary depression

→ Rising of fluid inside the tube.

→ Lowering of the fluid inside the tube.

⇒ It is measured in mm, cm, m, etc.

Mathematically

$$h = \frac{4 \sigma \cos \theta}{\rho g d}$$

$$\frac{\sigma}{\rho g d} = h$$

Where,

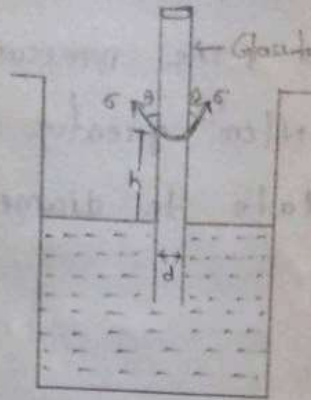
σ → Surface tension

ρ → density

g → acceleration due to gravity

d → diameter of pipe

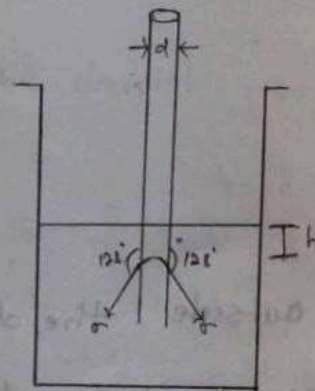
θ → Angle of Contact between liquid and glass tube.



For water, $\theta = 0^\circ$

$$so = \boxed{h = \frac{4\sigma}{\rho g d}}$$

$\Rightarrow$ For mercury $\theta = 128^\circ$



$$\boxed{h = \frac{4\sigma \times \cos 128^\circ}{\rho g d}}$$

10 The Surface tension of water in contact with air at 20°C is 0.0725 N/m . The pressure inside the droplet of water is 0.02 N/cm^2 greater than the outside pressure. Calculate the diameter of the droplet of water.

Ans:-

$$\sigma = 0.0725 \text{ N/m}$$

$$P = 0.02 \text{ N/cm}^2$$

$$= 0.02 \times 10^4 \text{ N/m}^2$$

$$P = \frac{4\sigma}{d} \quad d = \frac{4\sigma}{P}$$

$$d = \frac{4 \times 0.0725}{0.02 \times 10^4}$$

$$= 1.45 \times 10^{-3} \text{ m}$$

$$= 1.45 \text{ mm}$$

Q.2

The pressure outside the droplet of water of diameter 0.04 mm is 10.32 N/cm^2 (atmospheric pressure). Calculate the pressure within the droplet if surface tension is 0.0725 N/m of water.

Given

$$d = 0.04 \text{ mm} = 0.04 \times 10^{-3} \text{ m}$$

$$P = 10.32 \text{ N/cm}^2$$

$$= 10.32 \times 10^3$$

$$\sigma = 0.0725 \text{ N/m}$$

$$P = \frac{4\sigma}{d}$$

$$= \frac{4 \times 0.0725}{0.04 \times 10^{-3}}$$

$$= 0.725 \text{ N/cm}^2$$

Pressure inside the droplet = $0.0725 + \text{outside pressure}$

$$= 0.725 + 10.32$$

$$= 11.045 \text{ N/cm}^2$$

Q- Calculate the capillary rise in a glass tube of 2.5 mm diameter when immersed vertically in (a) water (b) mercury

Take $\sigma = 0.0725 \text{ N/m}$ For water &
 0.52 N/m For mercury in contact with air.

The Specific gravity of mercury is 13.6 & the angle of contact is 130° .

Given:-

$$d = 2.5 \text{ mm} \\ = 2.5 \times 10^{-3} \text{ m.}$$

$$\sigma \text{ For water} = 0.0725$$

$$\rho = 1000 \text{ kg/m}^3$$

$$\theta = 0$$

For water

$$h = \frac{4 \times \sigma \times \cos \theta}{\rho g d}$$

$$= \frac{4 \times 0.0725 \times \cos(0)}{1000 \times 9.81 \times 2.5 \times 10^{-3}}$$

$$= 0.01182 \text{ m.}$$

$$= 11.82 \text{ mm.}$$

For mercury

$$\sigma = 0.52$$

$$\rho = 13.6$$

$$\theta = 130^\circ$$

$$d = 2.5 \text{ m.}$$

$$F_s = \rho_w \times g$$

$$= 1000 \times 13.6$$

$$h = \frac{4\sigma \cos\theta}{\rho g d}$$

$$= \frac{4 \times 0.52 \times \cos 130^\circ}{1000 \times 13.6 \times 9.81 \times 2.5 \times 10^{-3}}$$

$$= -0.004008 \text{ m.}$$

$$= -4.008 \text{ mm}$$

Q Find out the minimum size of glass tube that can be used to measure water level, the capillary rise in the tube is to be restricted to 2mm. Consider the surface tension of water in contact with air as 0.073575 N/m .

Given:-

$$h = 2 \text{ mm}$$

$$= 2 \times 10^{-3}$$

$$\sigma = 0.073575$$

$$\theta = 0$$

$$h = \frac{4 \sigma \cos \theta}{\rho g d}$$

$$\Rightarrow 2 \times 10^{-3} = \frac{4 \times 0.073675 \times \cos(0)}{1000 \times 9.81 \times d}$$

$$\Rightarrow 2 \times 10^{-3} = \frac{0.2943}{9810 d}$$

$$\Rightarrow 19.62 d = 0.2943$$

$$\Rightarrow d = \frac{0.2943}{19.62}$$

$$\Rightarrow d = 0.015 \text{ m.}$$

$$d = 15 \text{ mm}$$

Find the minimum size of glass tube that can be used to measure water level, the capillary rise in the tube is to be restricted to 2 mm. Consider the surface tension of water in contact with air as 0.073675 N/m.

Fluid Pressure & Its Measurement

Fluid Pressure At a Point :-

Consider a small area dA in a large mass of fluid, then the force exerted by the surrounding fluid on the area dA will always be perpendicular to the surface of dA .

Let dF is the force acting on the area dA in normal direction then the intensity of pressure or pressure,

$$p = \frac{dF}{dA}$$

If the force is uniformly distributed over the area A then the pressure at any point is given by,

$$p = \frac{F}{A}$$

$$F = p \times A$$

Units of Pressure :-

In MKS $\rightarrow$ kgf/m^2 or kgf/cm^2

In SI $\rightarrow$ N/m^2 or N/mm^2

$$1 \text{ N/m}^2 = 1 \text{ Pascal} \\ = 1 \text{ Pa}$$

$$\left[\begin{array}{l} 1 \text{ kPa} = 10^3 \text{ Pa} \\ 1 \text{ MPa} = 10^6 \text{ Pa} \end{array} \right]$$

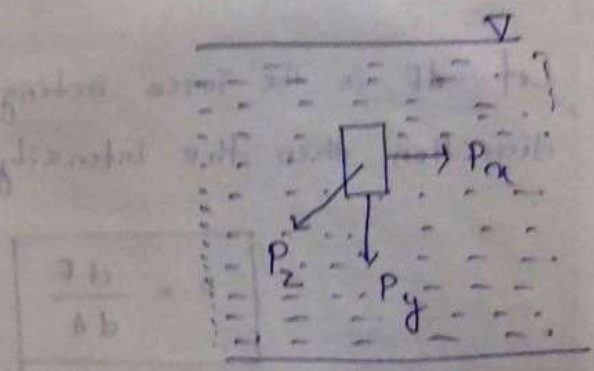
$$\left[\begin{array}{l} 1 \text{ bar} = 100 \text{ kPa} \\ \quad = 100 \times 10^3 \text{ Pa} \\ 1 \text{ bar} = 10^5 \text{ Pa} \end{array} \right]$$

* Pascal's Law :-

It states that, "the pressure or intensity of pressure at a point in a static fluid is equal in all direction".

mathematically,

$$P_x = P_y = P_z$$

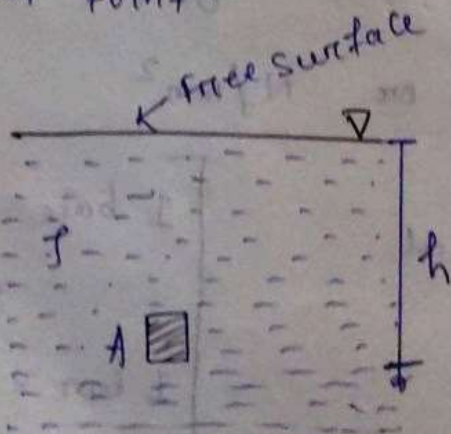


* Variation of pressure in a fluid at rest :-

The pressure at any point in a fluid at rest is obtained by hydrostatic law

* Hydrostatic Law :-

It states that, the rate of increase of pressure in a vertically downward direction must be equal to the specific weight or weight density of the fluid at that point.



Mathematically,

$$\boxed{P = \rho g h} \rightarrow \text{Hydrostatic Pressure}$$

$$\text{Hydrostatic force} = P \times A$$

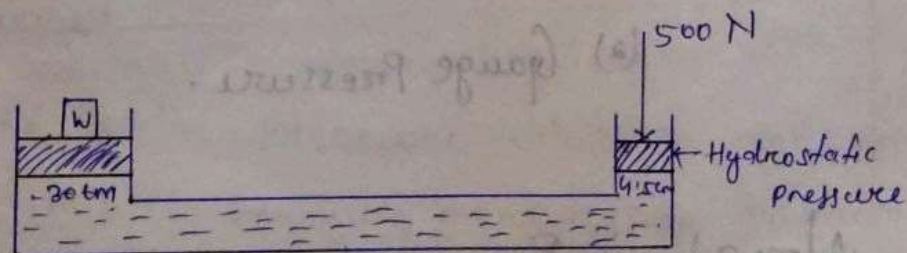
Where p is the pressure above the atmospheric pressure.

g = Acceleration due to gravity

h = Pressure head.

$$\boxed{h = \frac{P}{\rho g}}$$

Q- A hydraulic press as a RAM of 30 cm diameter and a plunger of 4.5 cm diameter find the weight lifted by the hydraulic press when the force applied at the plunger is 500 N.



$$d_1 = 30 \text{ cm}$$

$$d_2 = 4.5 \text{ cm}$$

$$F_2 = 500 \text{ N}$$

$$P = \frac{F}{A} = \frac{500}{\frac{\pi}{4} \times (0.045)^2}$$

$$= 314380.13 \text{ N/m}^2$$

$$W_f = P \times \text{Area of RAM}$$

$$= 314380.13 \times \frac{\pi}{4} \times (0.3)^2$$

$$= 22222.22 \text{ kN}$$

Types of Fluid Pressure :-

Two type of Pressure measurement Systems.

(1) Absolute Pressure / absolute zero / Complete vacuume Pressure

(2) Gauge Pressure.

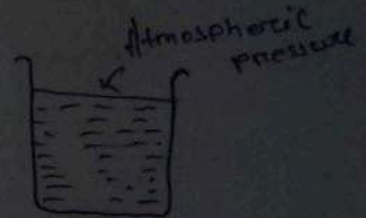
Atmospheric Pressure :-

The Pressure at the sea level at 15°C is 1.013×10^5
 or 10.13 N/cm^2 (SI System)

$$1.033 \text{ kgf/cm}^2 \text{ (MKS)}$$

Pressure head - 760 mm Hg

10.33 m of water.



Vacuum Pressure!—

It is the Pressure below atmospheric Pressure.

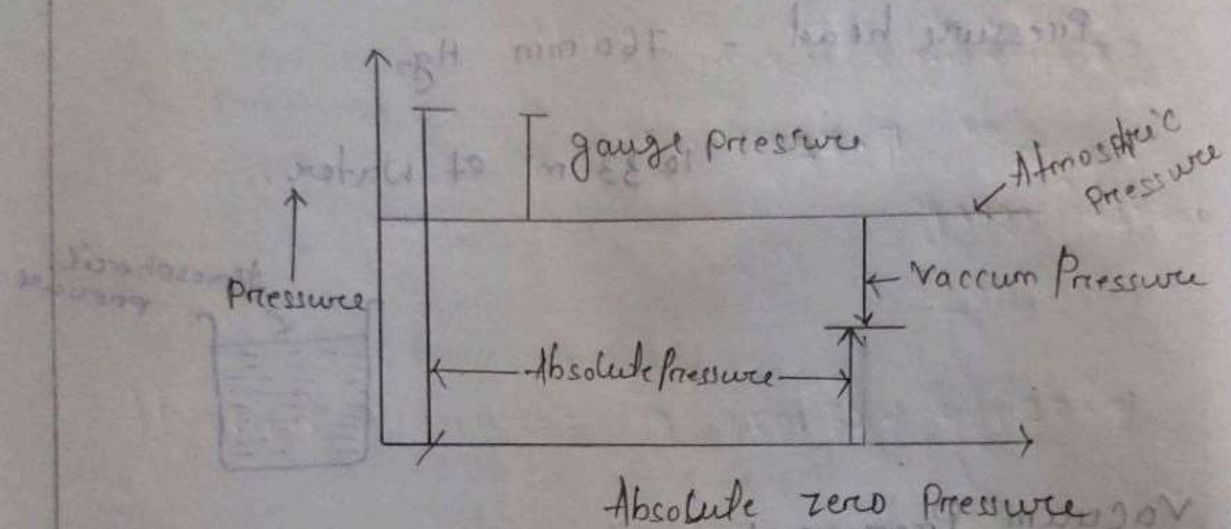
Absolute Pressure!—

It is defined as the Pressure which is measured with respect to absolute vacuum Pressure.

Gauge Pressure!—

It is defined as the Pressure which is measured with the help of pressure measuring instrument in which atmospheric pressure which is taken as datum.

The atmospheric pressure the scale is being mark as zero.



Mathematically;

$$\text{Absolute Pressure} = \text{Gauge Pressure} + \text{Atmospheric Pressure}$$

$$\text{Vacuum Pressure} = \text{Atmospheric Pressure} - \text{Absolute Pressure}$$

Q What are the gauge Pressure and absolute pressure at a Point 3 m. below the free Surface of liquid having density $1.53 \times 10^3 \text{ kg/m}^3$, if atmospheric pressure is equivalent to 750 mm of mercury. the Specific gravity of mercury is 13.6 and density of water 1000 kg/m^3 .

given data,

Depth of point from Free Surface of liquid = 3 m.

Density of liquid = $\rho = 1.53 \times 10^3 \text{ kg/m}^3$

Atmospheric pressure head = 750 mm Hg

Specific gravity (g) Hg = 13.6

Density of water = 1000 kg/m^3

Atmospheric pressure = $\rho g h$

$$= 13.6 \times 1000 \times 9.81 \times 0.75$$

$$= 100062 \text{ kg/m}^3$$

Pressure at 3m depth. of liquid

$$= \rho gh$$

$$= 1.53 \times 10^3 \times 9.81 \times 3$$

$$= 45027.9 \text{ N/m}^2$$

Gauge Pressure

$$\text{Absolute Pressure} = 100062 + 45027.9$$

$$= 145089.9$$

Pressure Measurement!-

There are two types of pressure measuring instrument

(i) Manometer

(ii) Mechanical gauge

1) Manometer :-

Manometer are the devices used for measuring pressure at a point in a fluid by balancing the column of fluid of the same fluid or another fluid.

Manometer are of two types

- (i) Simple manometer
- (ii) Differential manometer

Simple manometer :-

⇒ It is a pressure measuring instrument in which a glass tube having one end connected to a point where the pressure is to be measured and other end is open to atmosphere

⇒ These are of three type

- (i) Piezo meter
- (ii) U-tube meter
- (iii) single column manometer

2. Mechanical Gauge

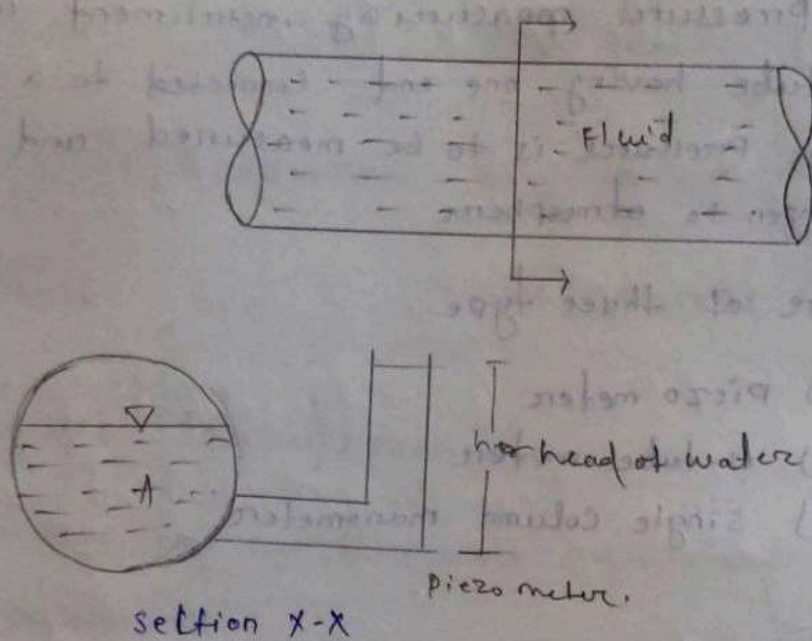
⇒ These are the devices used for measuring pressure at a point in a fluid by balancing the column of fluid by a spring or dead weight.

⇒ These are 4 type :-

- (i) Diaphragm pressure gauge
- (ii) Bourdon tube pressure gauge
- (iii) Dead weight pressure gauge
- (iv) Bellows pressure gauge

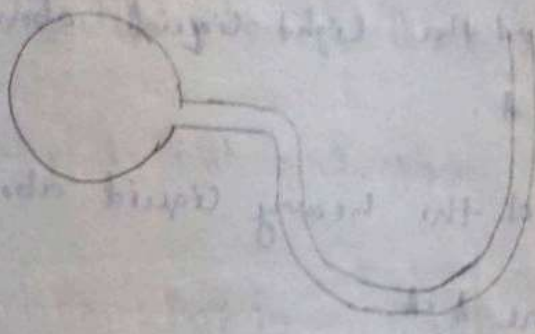
(i) Piezo meter:-

These are the simplest type of simple manometer in which a glass tube is connected to a point where the pressure is to be measured and other end of the glass tube is open to atmosphere.



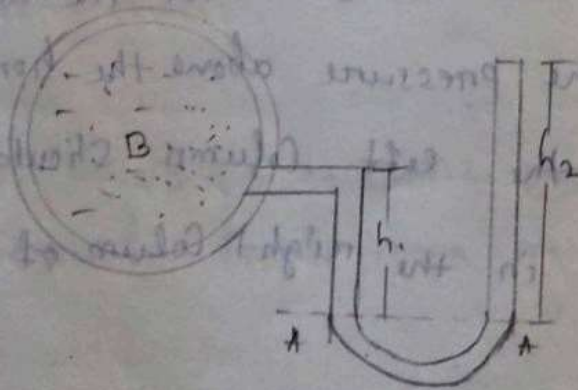
(ii) U-tube manometer

It consists of a glass tube bent in U-shape, and one end of which is connected to a point at which pressure is to be measured. The other end remains open to the atmosphere.



U tube manometer.

=> The tube generally contains mercury or any other fluid whose specific gravity is greater than the specific gravity of liquid whose pressure is to be measured.



P is the point at which pressure is to be measured whose value is P.

Let h_1 = height of the light liquid above the dotted line A A.

h_2 = height of the heavy liquid above the dotted line A A.

S_1 = Specific gravity of light liquid

S_2 = Specific gravity of heavy liquid

ρ_1 = Density of light liquid = $1000 \times S_1$

ρ_2 = Density of heavy liquid = $1000 \times S_2$

as the pressure is same for the horizontal surface, the pressure above the horizontal line A A in the left column should be equal to the pressure in the right column of U-tube manometer.

Pressure above A A the left column =

$$= P + \rho_1 g h_1$$

Pressure above A A the right column =

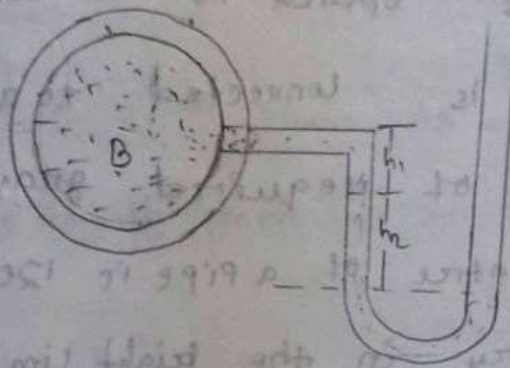
$$= P + \rho_2 g h_2$$

Now, equation both Column we have

$$P + \rho_1 g h_1 = \rho_2 g h_2$$

$$P = \rho_2 g h_2 - \rho_1 g h_1$$

Vacuum Pressure :-



For measuring the vacuum pressure, the level of heavy liquid in the manometer will be as shown in figure.

Pressure above a A A in the left Column

$$= P + \rho_1 g h_1 + \rho_2 g h_2$$

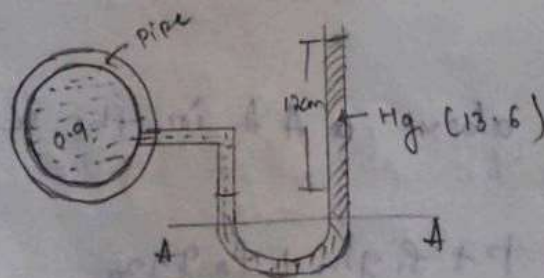
Pressure above A-A in right column above A-A = 0

Now

$$P + \rho_1 g h_1 + \rho_2 g h_2 = 0$$

$$P = -(\rho_1 g h_1 + \rho_2 g h_2)$$

The right limb of a simple U-tube manometer contains is opened to atmosphere while the left limb is connected to a pipe in which a fluid of required gravity 0.9 is flowing. Centre of a pipe is 12 cm below the level of mercury in the right limb. Find the pressure of the fluid in the pipe if the difference of mercury level in the two limbs is 20 cm.



Difference in mercury level in two limbs of

U-tube manometer = 20 cm.

Specific gravity of Fluid = $S_1 = 0.9$

Density of Fluid = $\rho_1 = 0.9 \times 1000 = 900 \text{ kg/m}^3$

Specific gravity of Hg = 13.6

Density of mercury = $13.6 \times 1000 \text{ kg/m}^3$

Ht of Fluid from AA = $20 - 12 = 8 \text{ cm} = 0.08 \text{ m}$

Let p is the pressure of fluid inside the pipe.

Left side pressure = Right side pressure

$$p + \rho_1 g h_1 = \rho_2 g h_2$$

$$p = \rho_2 g h_2 - \rho_1 g h_1$$

$$= (13.6 \times 1000 \times 9.8 \times 0.2) - (900 \times 9.8 \times 0.08)$$

$$= 25976.88 \text{ N/m}^2$$

Q A simple U-tube manometer containing mercury is connected to a pipe with fluid of specific gravity 0.8 and having vacuum pressure is flowing. The other end of the manometer is open to atmosphere. Find the vacuum pressure in the pipe if the difference of mercury level in the two pipe is 40 cm. and height of the fluid in the left limb from the centre of the pipe is 15 cm below.

Hydrostatic Force on Surfaces :- 08/04/202

When we submerged any plane object in a fluid mass the forces that are exerted on the ~~fluid~~ ^{plane} on the object due to the fluid is called Hydrostatic Force. (i)

This is due to the pressure of the fluid.

The Self rate of fluid particle

⇒ The fluid pressure will act normal to the surface on the plane.

Total Pressure :-

It is defined as the force exerted by a static fluid on a surface either plane or curve when the fluid comes in contact with the surfaces.

⇒ This force always acts normal to the surface.

Centre of Pressure :-

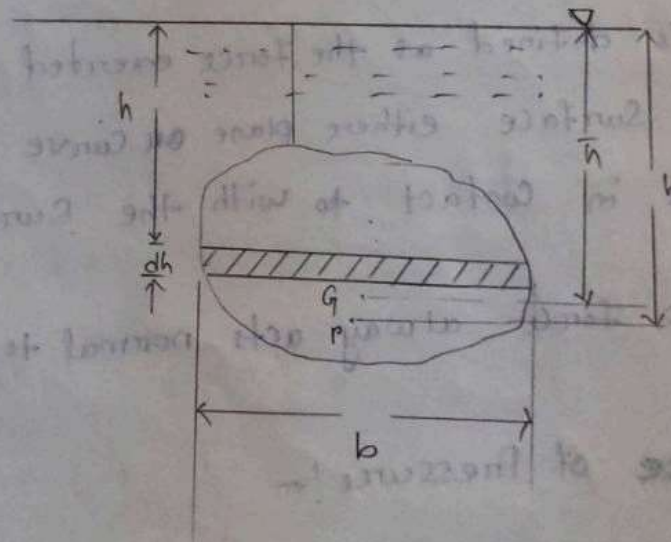
⇒ It is defined as the point of application of the total pressure on the surface.

Conditions of Submerged Surfaces :-

The Submerged Surfaces are of four type

- (i) Vertical plane Surfaces.
- (ii) Horizontal plane Surfaces.
- (iii) Inclined plane Surfaces.
- (iv) Curve Surfaces.

(i) Vertical Plane Surfaces ~~are~~ Submerged in Liquid :-



Let us consider a vertical Plane Surface of arbitrary shape submerged in a liquid as shown in figure.

Let,

⇒ A = Total Area of the Surface

⇒ $\bar{h}$ = Distance of CG of the Area from the Free Surface of liquid -

⇒ G = It is the Centre of gravity of the plane Surface.

⇒ P = It is the Centre of pressure

⇒ h^* = It is the Distance of Centre of pressure from free surface of liquid

(a) Calculation of total pressure → (F)

The total pressure on the surface can be determined by dividing the entire surface into No of small parallel strips.

The force on the small strip is then calculated and the total pressure on the whole area is calculated by integrating the force on small strip.

Let the thickness of the strip is dh
and width is b .

which is at a depth of h . from free surface of the liquid.

Pressure intensity on the small strip
 $= \rho gh$

Area of the strip $= dA = b \times dh$

The total pressure force on the strip $= \rho gh \times dA$
 $dF = \rho gh \times b \times dh$

Total Pressure on the plane =

$$\int dF = \int \rho gh \cdot b \cdot dh$$

Again $\int b h dh = \int h dA$

= moment of surface area about the free

surface of liquid.

= Area of the surface $\times$ Distance of the CG from the free surface.

$$= A \times \bar{h}$$

$$F = \rho g A \bar{h}$$

Calculation of Centre of Pressure :- (F^*)

The Centre of Pressure is calculated using the principle of moments. which states that the moment of the resultant force about an axis is equal to the sum of moment components about the same axis.

The resultant force F is acting at p which is at a distance h^* from the free surface of liquid as shown in figure.

So the moment of force F about Free surface

$$= F \times h^* \quad \text{--- (1)}$$

The moment of force dF about Free surface

$$= dF \times h$$

$$= \rho g h \times b \times dh \times h$$

$$= \rho g b^2 \times dh^2$$

$$= \rho g h^2 dA$$

Sum of the moment of all surface such forces about the free surface of liquid

$$= \int \rho g h^2 dA$$

$$= \rho g \int h^2 dA$$

But $\int h^2 dA =$ moment of inertia of the surface about the free surface of liquid

$$= I_0$$

Sum of moment about free surface

$$= \rho g I_0 \quad \text{--- (2)}$$

Now equating equation (1) and (2) we have

$$F \times h^* = \rho g I_0$$

$$\text{Again } F = \rho g \bar{h} A$$

$$\rho g \bar{h} A \times h^* = \rho g I_0$$

$$\Rightarrow \boxed{h^* = \frac{I_0}{A \bar{h}}}$$

applying parallel axis theorem we have

$$I_0 = I_G + A \bar{h}^2$$

Substituting I_0 in eqn (3)

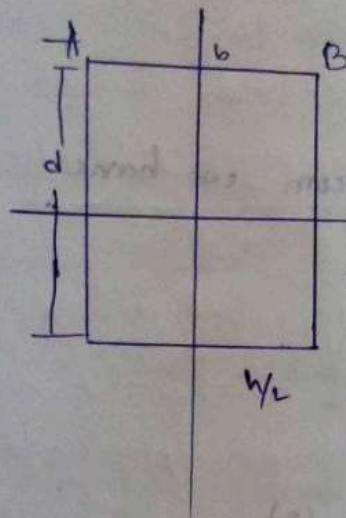
$$h^* = \frac{I_G + A \bar{h}^2}{A \bar{h}}$$

$$\boxed{h^* = \frac{I_G}{A \bar{h}} + \bar{h}}$$

From the above equation it is clear that the centre of pressure lies below the centre of gravity of the vertical surface.

The distance of Centre of pressure from the free surface of liquid is independent of the density of the liquid.

Location of CG Some Common Surface:-



$$I_{xx} = \frac{bd^3}{12}$$

$$I_{co} = I_{cg} + Ad^2$$

$$= \frac{bd^3}{12} + bd \times \left(\frac{d}{2}\right)^2$$

$$= \frac{bd^3}{12} + bd \times \frac{d^2}{4}$$

$$\frac{I_{xx} + I_{yy}}{A}$$

$$= \frac{bd^3}{12} + \frac{bd^3}{4}$$

$$\frac{I_{xx} + I_{yy}}{A} = \frac{bd^3 + 3bd^3}{12}$$

$$= \frac{4bd^3}{12}$$

from above equation it is clear that the

$$\frac{bd^3}{3}$$

is the moment of inertia of the vertical surface.

The distance of center of pressure from the vertical surface is

$$\bar{x} = \left(\frac{2a+b}{a+b} \right) \times \frac{h}{3} \text{ from b side.}$$

$$\bar{y} = \left(\frac{2b+a}{a+b} \right) \times \frac{h}{3} \text{ from a side.}$$

A rectangular plane surface of 2 m wide and 3 m deep lies in the vertical plane in water. determine the total pressure and position of centre of pressure on the plane surface. with the upward edge horizontal

(a) and coincide with water surface.

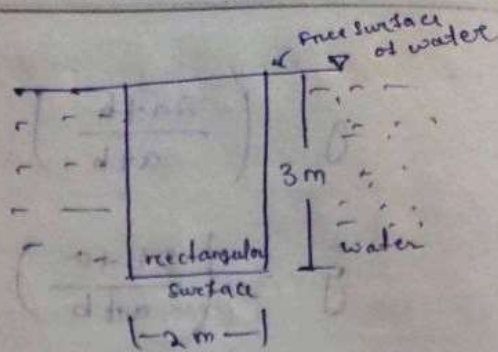
(b) 2.5 m below the free water surface.

Rectangular surface having width (b) = 2 m.

Depth (d) = 3 m.

→ kept in the vertical plane with top edge horizontal in water.





(a) Top edge coincide with free surface of water :-

$$\text{Total pressure force (F)} = \rho g \bar{h} A$$

$$= 1000 \times 9.81 \times \frac{3}{2} \times 2 \times 3$$

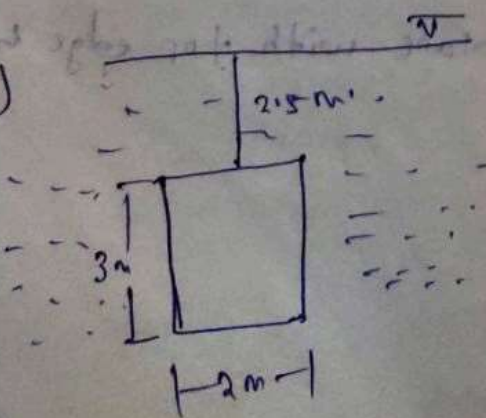
$$= 88290 \text{ N} \quad \text{--- (1)}$$

Position of Centre of pressure from free surface :-

$$h^* = \frac{I_{CG}}{A \bar{h}} + \bar{h} = \frac{\frac{2 \times 3^3}{12}}{2 \times 3 \times \frac{3}{2}} + \frac{3}{2}$$

$$= 2 \text{ m.} \quad \text{--- (2)}$$

(b)



Total pressure = $\rho g \bar{h} A$

$$= 1000 \times 9.81 \times 2.5 + \frac{3}{2} \times 3 \times 2$$

$$= 235440 \text{ N}$$

Position of Centre of Pressure from free surface :-

$$h^* = \frac{I_{CG}}{A \bar{h}} + \bar{h} = \frac{\frac{2 \times 3^3}{12}}{2 \times 3 \times (2.5 + \frac{3}{2})} + (2.5 + \frac{3}{2})$$

$$= 4.187 \text{ m.} \quad \text{--- (1)}$$

* Total Pressure horizontal surface in submerged condition :-

When the surface submerged in the liquid with horizontal position, every point on the surface is at same depth from free surface of the liquid.

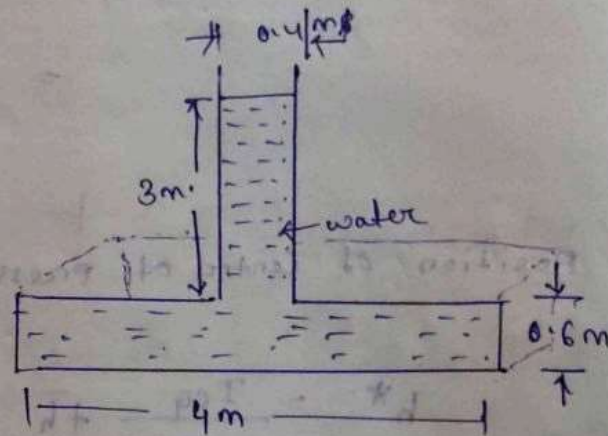
So the pressure intensity ~~should~~ ^{will} be equal to through out the entire surface & will be equal to

$$\rho = \rho g h$$

and the pressure force $F = \rho g A \bar{h}$

and $\boxed{\bar{h} = h^*}$

Q



Calculate

- total pressure force at the bottom of the dam
- weight of the water inside the dam.
- if the width of the dam is 2m, calculate the hydrostatic paradox between the results

of a & b.

Total pressure force at the bottom of tank/containers.

$$F = \rho g A \bar{h}$$

$$= 1000 \times 9.81 \times 4 \times 2 \times (3 + 0.6)$$

$$= 282528 \text{ N}$$

(b) wt of water inside the container

$$= \rho \times g \times \text{volume}$$

$$= 1000 \times 9.81 \times (3 \times 0.4 \times 2 + 4 \times 0.6 \times 2)$$

$$= 70632 \text{ N}$$

(c) Hydrostatic Paradox

The total pressure force at the bottom of the dam is much more than the weight of water inside the ~~dam~~ ^{dam}. This is called as Hydrostatic Paradox.

$$282528 - 70632$$

$$= 211896 \text{ N}$$

-! Kinematics of Flow!-

21/- 11/04/2022

Kinematics!-

Study of motion of body without knowing / Considering the force causing the motion.

Dynamics!-

Study of motion of body with knowing the force causing the motion.

Types of Fluid Flow!-

① Steady and unsteady flow!-

Steady!-

The flow of fluid in which the flow parameters like velocity, density etc. remain constant w.r.t time is called steady flow.

Unsteady!-

The flow in which the flow parameters like velocity, density etc. changes with time is called unsteady flow.

② (b) Uniform and non uniform!—

Uniform flow!—

The flow in which the velocity remains constant w.r.t. space that is called uniform flow.

Non uniform flow!—

The flow in which the velocity of flow changes w.r.t. space or distance is called non-uniform flow.

③ Lamina & turbulent flow

Lamina flow!—

The flow in which the fluid layer slides one above another layer without developing any restriction to flow is called lamina fluid. In this case the fluid particle just rolls over one another.

For lamina flow the value of Reynold's Number (Re) < 2000

turbulent flow!—

The flow of the fluid in which the fluid particle moves in zigzag fashion or random fashion without following a specified path. is called turbulent flow.

The value of Reynold's Number is > 4000 .

If the value of Reynold's Number is between 2000 to 4000 then the flow can be either lamina or turbulent.

$$Re = \frac{\rho v}{\gamma}$$

where,

ρ = density of fluid

v = velocity of flow

γ = coefficient of dynamic viscosity.

④ Compressible and ~~non~~ ~~can~~ Non Compressible!

Compressible :-

The flow of fluid in which the density is changing with w.r.t space or distance during its travel is called at point to point is called compressible flow.

~~no~~ The flow of gasses this is an example of compressible flow.

Non Compressible flow!:-

The flow of fluid in which the density remains constant throughout the sections of flow is called ~~non~~ non compressible flow.

The flow of liquid is an example of non compressible flow.

⑤ Rotational and Irrotational

Rotational flow!:-

The flow of liquid in which the fluid particles move in the direction of flow along with rotation about their own axis is called rotational flow.

Irrotational flow!:-

The flow of liquid in which the fluid particles move along the direction of flow with out any rotation about their own axis is called irrotational flow.

One Dimensional flow!:-

The flow of fluid particle in which the flow parameters can be expressed as a function of one dimension only by x axis or y axis or z axis then the flow is called one dimensional flow.

mathematically :-

$$V = f(x) \text{ or } f(y) \text{ or } f(z)$$

Two dimensional flow :-

The flow of fluid particles in which the flow parameters can be expressed as the function of two axis is called two dimensional flow.

mathematically,

$$V = f(x, y) \text{ or } f(y, z) \text{ or } f(z, x)$$

Three dimensional flow :-

The flow of fluid in which the flow parameters can be expressed as functions of three axis then it is called three dimensional flow.

mathematically,

$$V = f(x, y, z)$$

Equation of Continuity:-

Rate of Flow / Discharge of flow:-

The rate of flow or discharge is defined as a quantity of liquid flowing per second of a section of pipe or channel. is called discharge or rate of flow.

It is denoted by Q .

Mathematically,

$$Q = A v$$

where

A = Area of Cross-section of flow

v = velocity of flow.

The unit of discharge = m^3/sec in SI unit.

Continuity Equation:-

It is based on the principle of conservation of mass.

It states that the amount of liquid flowing or the discharge of a section remains constant.

different section of flow.

mathematically,

$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2$$

where,

$\Rightarrow \rho_1, \rho_2$ are the density of fluid before and after a particular section or at two different section.

$\Rightarrow A_1 \neq A_2$ Area of cross section of flow at two different section.

$\Rightarrow v_1 \neq v_2$ velocity of flow at two different section. (1)

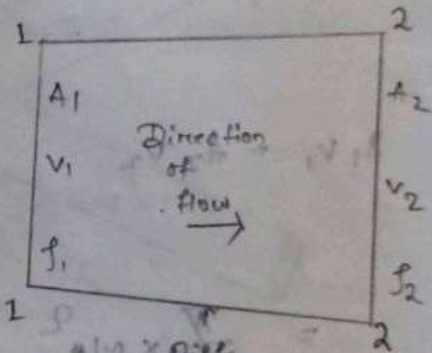
If the density is same for the liquid passing through the section that is i.e. $\rho_1 = \rho_2 = \rho_3$

the above equation become $\rho A_1 v_1 = \rho A_2 v_2$

$$\boxed{A_1 v_1 = A_2 v_2}$$

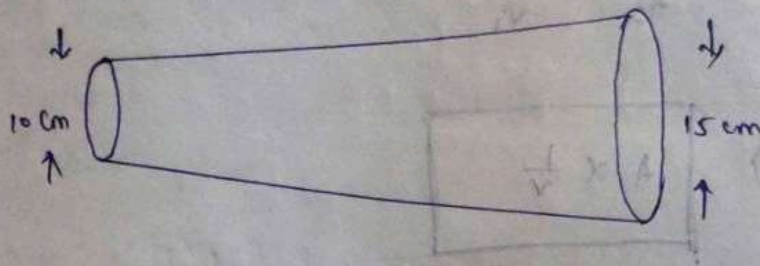
$$\frac{A_1}{A_2} = \frac{v_2}{v_1}$$

$$\Rightarrow A \propto \frac{1}{v}$$



- ① The Diameter of the Pipe of the Section 1 & 2
10 cm & 15 cm respectively.

Find the discharge to the pipe. if the velocity of
water flowing through the pipe at section 1 is 5 m/sec
determine the velocity of at section 2.



$$D_1 = 10 \text{ cm} \quad D_2 = 15 \text{ cm}$$

$$v_1 = 5 \text{ m/sec}$$

$$A_1 v_1 = A_2 v_2$$

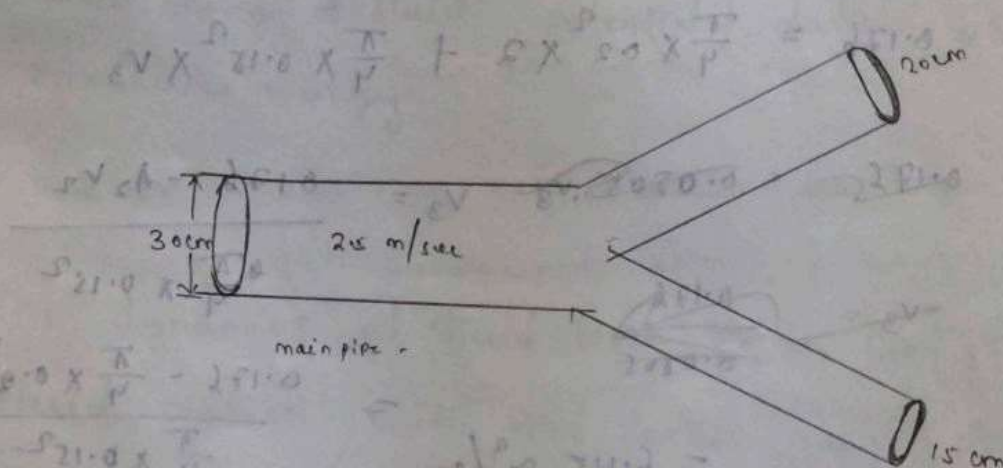
$$= \frac{\pi}{4} \times 0.1^2 \times 5 = \frac{\pi}{4} \times 0.15^2 \times v_2$$

$$= 0.05 \times 0.0225 \times v_2$$

$$v_2 = \frac{0.05}{0.0225}$$

$$v_2 = 2.22 \text{ m/sec}$$

A 30 cm diameter pipe conveying water branches into 2 pipes of diameter 20 cm and 15 cm respectively. If the average velocity in 30 cm diameter pipe is 2.5 m/sec. Find the discharge in this pipe also determine the velocity in 15 cm pipe if the average velocity in 20 cm pipe is 2 m/sec.



$$d_1 = 30 \text{ cm} \quad d_2 = 20 \text{ cm} \quad d_3 = 15 \text{ cm}$$

$$v_1 = 2.5 \text{ m/sec}$$

$$\rightarrow \frac{\pi}{4} \times d_1^2 \times v_1$$

$$\Rightarrow \frac{\pi}{4} \times 0.3^2 \times 2.5$$

$$\Rightarrow 0.176 \text{ m}^3/\text{sec}$$

$$Q_1 = Q_2$$

$$Q = Q_1 + Q_2$$

7. ~~0.176~~

$$\Rightarrow 0.176 = A_2 v_2 + A_3 v_3$$

$$\Rightarrow 0.176 = \frac{\pi}{4} \times 0.2^2 \times 2 + \frac{\pi}{4} \times 0.15^2 \times v_3$$

$$\Rightarrow \cancel{0.176} = \cancel{0.00805} v_3 \quad v_3 = \frac{0.176 - A_2 v_2}{\frac{\pi}{4} \times 0.15^2}$$

$$\Rightarrow v_3 = \frac{\cancel{0.176}}{\cancel{0.00805}} = \frac{0.176 - \frac{\pi}{4} \times 0.2^2 \times 2}{\frac{\pi}{4} \times 0.15^2} = 6.45 \text{ m}^3/\text{sec.}$$

Water flow to a pipe AB 1.2 m. diameter at 3 m. per/sec and then passes through then passes through a pipe 1.5 m. diameter at C, the pipe branchy CD is 0.8 in diameter and carries on $\frac{1}{3}$ of the flow in AB. the flow velocity in Branch CE is 2.5 m/sec Find the volume rate of flow and velocity in BC, velocity in CD & Diameter of CE.

Dynamics of fluid flow? —

- Euler's equation of motion.
- Bernoulli's equation.
- Application of Bernoulli's equation.
- Total energy of fluid, potential / kinetic, pressure energy.

The dynamics of fluid flow is the study of fluid motion with the forces gauging the motion.

Forces Acting on fluid element

In case of fluid flow the following forces are present.

- (i) gravity force
- (ii) Pressure force
- (iii) Forces due to viscosity
- (iv) Forces due to turbulence
- (v) Forces due to compressibility

→ If the forces due to compressibility is negligible then the equation of motion resulting from the above four forces are called Reynold's eqn of forces.

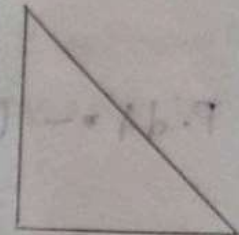
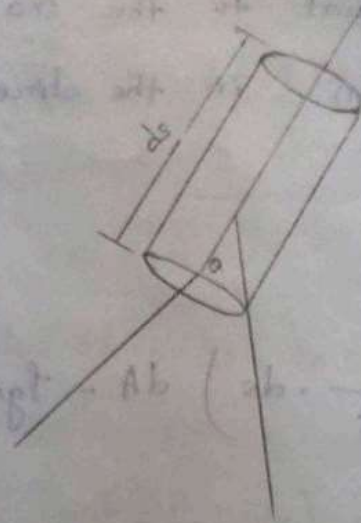
→ If the forces due to turbulence is negligible then the resulting equation of motion is called Navier - stoke's equation.

⇒ If the flow is assume in idea, viscous force is zero & the equation of motion is called Euler's equation of motion.

Euler's equation of motion:-

This equation of motion in which the forces due to gravity and pressure are taken into consideration.

This is derived by considering the motion of fluid element along a stream line.



Consider a stream line in which the flow is taking place in a s direction.

Consider a cylindrical element of cross section dA and length ds .

The forces acting on the cylindrical element are

- (i) The pressure force $p \times dA$ in the direction of flow.
- (ii) Pressure force $\left(p + \frac{\partial p}{\partial s} ds\right) dA$.
- (iii) Weight of the fluid element $\rho g dA ds$

Let θ be the angle between the direction of flow and the line of action of the weight of the element.

The resultant force on the fluid element in the direction of s must be equal to the mass of the element $\times$ ~~accel~~ acceleration in the direction of s

mathematically,

$$P \cdot dA = - \left(P + \frac{\partial P}{\partial s} \cdot ds \right) dA - \rho g ds dA \cos \theta = \rho dA ds \cdot a_s$$

where a_s is the acceleration in the direction of s .

Now $a_s = \frac{dv}{dt}$

where

v is the velocity of the fluid element which is a function of s and t .

$$a_s = \frac{dv}{dt} = \frac{\partial v}{\partial s} \cdot \frac{ds}{dt} + \frac{\partial v}{\partial t}$$

$$a_s = \frac{dv}{dt} = \frac{\partial v}{\partial s} \cdot \frac{ds}{dt} + \frac{\partial v}{\partial t}$$

$$= v \cdot \frac{\partial v}{\partial s} + \frac{dv}{dt}$$

if the flow is steady then $\frac{\partial v}{\partial t} = 0$

$$a_s = v \cdot \frac{\partial v}{\partial s}$$

Putting the value of a_s in ~~eqn~~ eqn (1) we get

$$\rightarrow p \, dA - \left(p + \frac{\partial p}{\partial s} \cdot ds \right) dA - \rho g \, ds \, dA \cos \theta = \rho \, dA \cdot ds \cdot a_s$$

$$\Rightarrow p \cdot dA - p \, dA - \frac{\partial p}{\partial s} \, ds \, dA - \rho g \, ds \, dA \cos \theta = \rho \, dA \cdot ds \cdot v \cdot \frac{\partial v}{\partial s}$$

$$\Rightarrow \frac{\partial p}{\partial s} \cdot ds \cdot dA = \rho g \cdot ds \cdot dA \cdot \cos \theta$$

$$= \rho \cdot dA \cdot ds \cdot v \cdot \frac{\partial v}{\partial s}$$

dividing the equation by $\rho \cdot ds \cdot dA$ we get

$$\frac{\frac{\partial p}{\partial s} \cdot ds \cdot dA}{\rho \cdot ds \cdot dA} = \frac{\rho g \cdot ds \cdot dA \cdot \cos \theta}{\rho \cdot ds \cdot dA} = \frac{\rho \cdot dA \cdot ds \cdot v \cdot \frac{\partial v}{\partial s}}{\rho \cdot ds \cdot dA}$$

$$\Rightarrow \frac{\frac{\partial p}{\partial s}}{\rho} - g \cos \theta = v \cdot \frac{\partial v}{\partial s}$$

$$\Rightarrow \frac{\partial p}{\rho \partial s} - g \cos \theta = v \cdot \frac{\partial v}{\partial s}$$

Again $\cos \theta = \frac{dz}{ds}$

$$\Rightarrow \frac{\partial p}{\rho \partial s} + g \frac{dz}{ds} + v \frac{\partial v}{\partial s} = 0$$

or

$$\Rightarrow \frac{dp}{\rho ds} + g \frac{dz}{ds} + v \frac{dv}{ds} = 0$$

$$\Rightarrow \boxed{\frac{dp}{\rho} + g dz + v dv = 0} \quad \text{--- (2)}$$

Euler's equation of motion.

derivation

* Derivation of Bernoulli's eqⁿ from Euler's eqⁿ

The Euler's eqⁿ is as follow

$$\frac{dp}{\rho} + g dz + v dv = 0$$

Integrating the above equation.

$$\int \frac{dp}{\rho} + \int g dz + \int v dv = \text{Constant}$$

The fluid is incompressible then ρ is constant.

Then the equation become

$$\frac{p}{\rho} + g \cdot z + \frac{v^2}{2} = \text{Constant}$$

$$\Rightarrow \frac{p}{\rho} + g \cdot z + \frac{v^2}{2} = \text{Constant}$$

$$\Rightarrow \frac{p}{\rho g} + \frac{g \cdot z}{g} + \frac{v^2}{2g} = \text{Constant}$$

$$\Rightarrow \frac{P}{\rho g} + \frac{v^2}{2g} + z = \text{Constant}$$

This is Bernoulli's equation of motion.

where

$\frac{P}{\rho g}$ is the Pressure energy or unit weight of fluid or pressure head.

$\frac{v^2}{2g}$ kinetic energy per unit weight or kinetic head.

z potential energy per unit or potential head.

The sum of pressure head, kinetic head and potential head is called the total head or total energy per unit weight of fluid element.

$$\text{Total head} = \frac{P}{\rho g} + \frac{v^2}{2g} + z$$

Assumption made in the derivation of Bernoulli's equation.

- (i) The fluid is ideal (viscosity is zero)
- (ii) The flow is steady.
- (iii) The flow is incompressible.
- (iv) The flow is irrotational.

DTL- 19/04/2022

Q Water is flowing through a pipe of 5 cm diameter under a pressure of 29.43 N/cm^2 and with a mean velocity of 2 m/sec . Find the total head or total energy per unit weight of the water at a cross-section, which is 5 m above the datum line.

Given:-

$$\frac{P}{\rho g} = 29.43 \text{ N/cm}^2$$

$$\text{diameter of pipe} = 5 \text{ cm} = 0.05 \text{ m}$$

$$\text{velocity of water in pipe} = 2 \text{ m/sec} = v$$

$$\text{Pressure in the pipe} = 29.43 \text{ N/cm}^2$$

$$= 29.43 \times 10^4 \text{ N/m}^2$$

Pressure head :-

$$\frac{p}{\rho g} = \frac{29.43 \times 10^4}{1000 \times 9.81}$$
$$= 30 \text{ m}$$

velocity head :-

$$\frac{v^2}{2g} = \frac{2^2}{2 \times 9.81} = 0.20 \text{ m}$$

Daton head :- (Z) = 5 m

Total head =

$$= 30 + 0.2 + 5$$

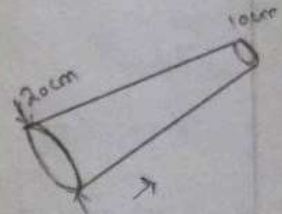
$$= 35.2 \text{ m} \quad \underline{\text{Ans.}}$$

Q. A pipe through which water is flowing, is having diameters 20 cm and 10 cm at cross section one and two respectively. The velocity of water at section one is given 4 m/sec. Find the velocity head at section one and two and also the discharge.

Given:

Diameter at one section = 20 cm

Diameter at two section = 10 cm.



Section - 1

$$v_1 = 4 \text{ m/sec}$$

$$d_1 = 20 \text{ cm} = 0.2 \text{ m}$$

$$A_1 = \frac{\pi}{4} \times d_1^2$$

$$= \frac{\pi}{4} \times 0.2^2$$

$$= 0.03 \text{ m}^2$$

Section - 2

$$v_2 = 16 \text{ m/sec}$$

$$d_2 = 0.1 \text{ m}$$

$$A_2 = \frac{\pi}{4} \times d_2^2$$

$$= \frac{\pi}{4} \times 0.1^2 = 0.00785 \text{ m}^2$$

Applying equation of continuity we have

$$A_1 v_1 = A_2 v_2$$

$$v_2 = \frac{A_1 v_1}{A_2}$$

$$= \frac{0.03 \times 4}{0.00785}$$

$$\approx 16 \text{ m/sec}$$

velocity head at section (1)

$$\frac{v_1^2}{2g} = \frac{4^2}{2 \times 9.81}$$

$$= 0.8154$$

velocity head at section (2)

$$\frac{v_2^2}{2g} = \frac{16^2}{2 \times 9.81}$$

$$= 13.04 \text{ m}$$

Discharge

$$= A_1 v_1$$

$$= 0.12 \text{ m}^2/\text{sec}$$

* Bernoullies equation for real fluid

The bernoullies equation for real fluid between point one and 2 is given by

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + z_2 + h_L$$

where

where h_L = loss of Energy or head between point one and two.

Real fluid :-

The real fluids are viscous and offers resistance to flow. giving rise to some losses of energy in fluid flow.

Practical applications of Bernoulli's equation

Bernoulli's theorem is applicable in all problem of incompressible fluid flow where the energy consideration are involved.

It is applicable in the following measuring devices.

(i) Venturimeter

(ii) Orifice meter

(iii) Pitot-tube

Venturimeter :-

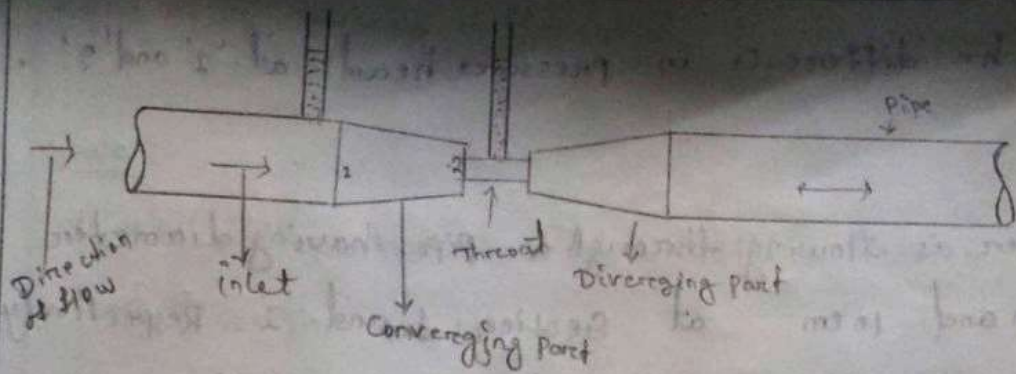
A venturimeter is a device which is used for measuring the rate of flow of a fluid flowing through a pipe.

It consists of three parts :-

(a) A short converging part

(b) Throat

(c) A diverging part



The Theoretical Discharge $Q_{th} =$

$$Q_{th} = \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$$

This equation gives the discharge under ideal condition and is called theoretical discharge, but actual discharge will be less than the theoretical discharge.

$$Q_{act} = C_d \cdot \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh} \quad (\text{actual discharge})$$

where

C_d is Coefficient of venturi meter and its value is less than 1.

a_1, a_2 = area of cross section of section 1 and 2
~~difference~~

$h =$ difference in pressure head at '1' and '2'.

water is flowing through a pipe having diameter 20cm and 10cm at section 1 and 2 respectively. the rate of flow through pipe is 354 sec .

the section 1 is 6m above datum and section 2 is 4m above datum.

if the pressure at section 1 is 39.24 N/cm^2 , find the intensity of pressure at section 2.

This equation gives the discharge under ideal condition and is called theoretical discharge, but actual discharge will be less than the theoretical discharge.

$$Q_{act} = C_d \cdot \frac{a_1 a_2 \sqrt{h}}{\sqrt{a_1^2 - a_2^2}} \quad (\text{actual discharge})$$

C_d is coefficient of discharge and its value is less than 1.

Value of h given by differential U-tube manometer :-

Case I :-

If the differential manometer contains heavier liquid than that through the pipe

$$h = \alpha \left(\frac{S_h}{S_a} - 1 \right)$$

S_h - Specific gravity of heavier liquid.

S_a = Specific gravity of liquid flowing through pipe.

α = Difference of heavier liquid column in U-tube.

Case II :-

If the differential manometer contains a liquid lighter than the liquid flowing through the pipe.

$$h = \left(1 - \frac{S_h}{S_a} \right) \alpha$$

S_L = Specific gravity of lighter liquid in U-tube

Q. A horizontal venturimeter with inlet and throat diameters 30 cm and 15 cm respectively.

It is used to measure the flow of water. The reading of a differential manometer connected to the inlet & the throat is 20 cm of mercury. Determine the rate of flow, take C_d is 0.98.

Solve:-

Given:-

For the horizontal venturimeter

$$d_1 = \text{at inlet} = 30 \text{ cm} = 0.3 \text{ m}$$

$$d_2 = \text{at throat} = 15 \text{ cm} = 0.15 \text{ m}$$

$$a_1 = \frac{\pi}{4} \times 0.3^2$$

$$= 0.0706 \text{ m}^2$$

$$a_2 = \frac{\pi}{4} \times 0.15^2$$

$$= 0.0176 \text{ m}^2$$

$$C_d = 0.98$$

$$h = 20 \text{ cm}$$

$$h = a \left(\frac{S_h}{S_a} - 1 \right)$$

$$= 0.2 \left(\frac{13.6}{1} - 1 \right)$$

$$= 2.52 \text{ m.}$$

$$Q_{act} = C_d \cdot \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$$

$$= 0.98 \frac{0.0706 \times 0.0176}{\sqrt{(0.0706)^2 - (0.0176)^2}} \times \sqrt{2 \times 9.81 \times 2.52}$$

$$= 0.125 \text{ m}^3/\text{sec}$$

② In a 100 mm diameter horizontal pipe a venturi meter of 0.5 contraction ratio has been fixed. The head of water on the meter when there is no flow is 3 m. (gauge) Find the rate of flow for which the throat pressure will be 2 m. of water absolute. C_d is 0.98. Take atmospheric pressure head = 10.3 m of water.

given :-

diameter of pipe = 100 mm

$$= 0.1 \text{ m}$$

Contraction ratio = 0.5
= $a_2 = 0.5 a_1$

$$\frac{a_1}{a_2} \text{ or } \frac{a_2}{a_1}$$

head of water = 8 m

$$= a_2 = 0.5 a_1$$

gauge pressure = 3 m

$$a_1 = \frac{\pi}{4} \times 0.1^2$$

$$= 0.00785 \text{ m}^2$$

$$a_2 = 0.5 \times 0.00785$$

$$= 0.003925 \text{ m}^2$$

atmospheric

$$P_1$$

absolute pressure = atmospheric pressure

gauge pressure

gauge pressure

$$= 10.3 + 3$$

$$= 13.3 \text{ m}$$

$$h = P_1 - P_2$$

$$= 13.3 - 2$$

$$= 11.3 \text{ m}$$

$$C_d \cdot \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$$

$$= 0.97 \cdot \frac{0.00785 \times 0.00392}{\sqrt{0.00785^2 - 0.00392^2}} \times \sqrt{2 \times 9.81 \times 11.3}$$

$$= 0.0658 \text{ m}^3/\text{sec}$$

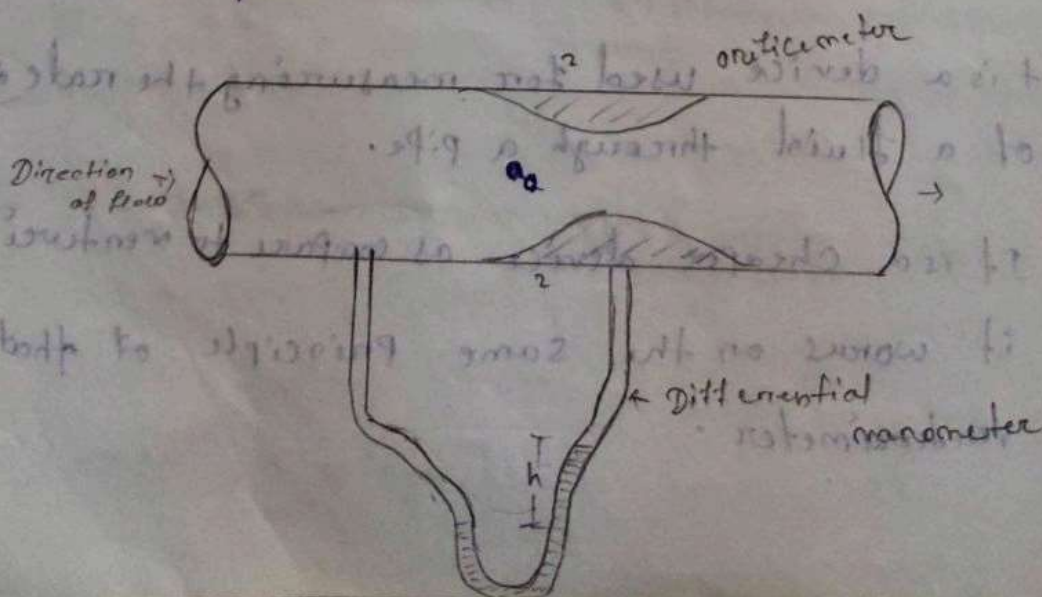
Orifice meter:-

- It is a device used for measuring the rate of flow of a fluid through a pipe.
- It is a cheaper device as compare to venturimeter.
- It works on the same principle of that of venturimeter.

→ It consists of a ~~extra~~ flat circular plate which has a circular sharp edge ~~whole~~ called orifice, which is concentric with the pipe.

→ The orifice diameter is kept generally 0.5 times the diameter of the pipe, though it may vary from 0.4 to 0.8 times the pipe diameter,

→ A differential manometer is connected as section one which is at a distance of 1.5 to 2 times the diameter the string for the orifice plate and at section 2 which is at a distance of about half the diameter ~~from~~ the orifice on the down stream side from the orifice plate.



a_0 = Area of orifice meter

a_1 = Area of c/s of pipe at section 1-1.

a_2 = Area ~~at~~ at vena contracta.

$$\text{Coefficient of Contraction (Cc)} = \frac{a_2}{a_0}$$

The discharge of orifice meter = $Q =$ ~~not~~

$$Q = \frac{C_d a_0 a_1 \sqrt{2gh}}{\sqrt{a_1^2 - a_0^2}}$$

C_d = Coefficient of discharge of orifice meter having
~~also~~ value is less than that the venturi meter.

An orifice meter with orifice diameter 10 cm is inserted in a pipe of diameter 20 cm. the pressure gauges fitted ~~at~~ up stream and down stream of orifice meter

give reading of 19.62 N/cm² and 9.81 N/cm²

respectively. $C_d = 0.6$ Find the discharge through the pipe.

Given :-

$$d_0 = 10 \text{ cm} = 0.1 \text{ m}$$

$$d_1 = 20 \text{ cm} = 0.2 \text{ m}$$

$$a_0 = \frac{\pi}{4} \times 0.1^2 = 0.00785 \text{ m}^2$$

$$a_1 = \frac{\pi}{4} \times 0.2^2 = 0.0314 \text{ m}^2$$

upside pressure =

$$\text{up stream pressure} = 19.62 \text{ m}$$

$$\text{down stream pressure} = 9.81$$

$$p = \rho g h$$

$$19.62 = 1000 \times 9.81 \times h$$

$$h = \frac{19.62}{9810} = 0.002$$

$$h_1 = \frac{P}{\rho g}$$

$$= \frac{19.62 \times 10^4}{1000 \times 9.81}$$

$$= 20$$

for water with a pipe of diameter 100 mm and a length of 10 m, the head loss due to friction is 10 m.

$$h_2 = \frac{P}{\rho g}$$

$$= \frac{9.81 \times 10^4}{1000 \times 9.81}$$

$$= 10$$

$$m_1 = 10$$

$$m_2 = 10$$

$$h = h_1 - h_2$$

$$= 20 - 10$$

$$= 10 \text{ m.}$$

$$Q = \frac{C_d \cdot a_0 \cdot a_1 \cdot \sqrt{2gh}}{\sqrt{a_0^2 - a_1^2}}$$

$$= \frac{0.6 \times 0.00785 \times 0.0314 \times \sqrt{2 \times 9.81 \times 10}}{\sqrt{0.00785^2 - 0.0314^2}}$$

$$= 0.068 \text{ m}^3/\text{sec}$$

Assumption

2. An orifice meter with orifice diameter 15 cm is inserted with a pipe of 30 cm diameter. The pressure difference measured by a mercury oil differential manometer on the two side of orifice meter gives a reading gives a reading 150 cm of mercury and the rate of flow of oil of specific gravity 0.9. If C_d of orifice meter is 0.64.

Ans

$$d_o = 15 \text{ cm}$$

$$d_i = 30 \text{ cm}$$

$$a_o = \frac{\pi}{4} \times 0.15^2$$

$$= 1.76 \text{ m}^2$$

$$a_o = \frac{\pi}{4} \times 0.15^2$$

$$= 0.0176 \text{ m}^2$$

$$a_i = \frac{\pi}{4} \times 0.3^2$$

$$a_i = \frac{\pi}{4} \times 0.3^2$$

$$= 0.0706 \text{ m}^2$$

$$h = 15 \text{ cm.}$$

$$= 0.15 \text{ m.}$$

$$C_d = 0.64$$

$$g = 9.8$$

$$Q = \frac{C_d a_0 a_1 \sqrt{2gh}}{\sqrt{a_0^2 - a_1^2}}$$

$$\sqrt{a_0^2 - a_1^2}$$

$$= \frac{0.64 \times 0.0176 \times 0.0706 \times \sqrt{2 \times 9.8 \times 0.15}}{\sqrt{(0.0706)^2 - (0.0176)^2}}$$

$$\sqrt{(0.0706)^2 - (0.0176)^2}$$

$$= 0.00604 \text{ m}^3/\text{sec}$$

Q Find the velocity of flow of an oil through a pipe when the difference of mercury level in a differential U-tube manometer connected to two tapping of the pitted tube is 100 mm take coefficient of pitted tube 0.08 . the specific gravity of oil is 0.8 .

Solⁿ

1. differential manometer is inserted to the pipe.

~~the manometer~~

It contains Hg x mercury having $sg = 13.6$

$$C_v = 0.08$$

$$g = 0.8$$

Difference in the level of U-tube manometer

$$= 100 \text{ mm}$$

$$= 0.1 \text{ m} = h$$

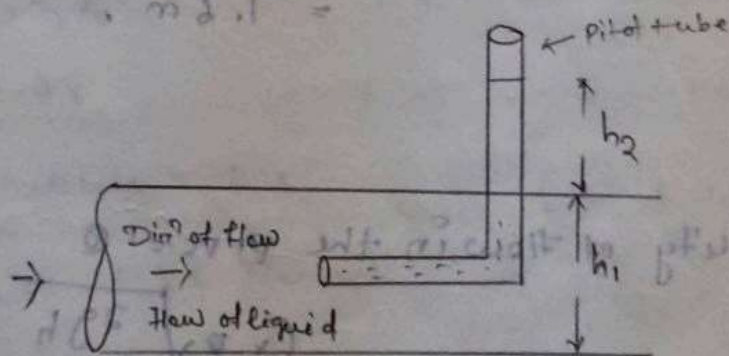
$$h =$$

Pitot-tube:-

It is a device used for measuring the velocity of flow at any point in a pipe or channel.

→ It is based on the principle that if the velocity of flow at a point becomes 0 at a point, the pressure at that point is increased due to conversion of kinetic energy into pressure energy.

→ In the simplest form the pitot tube consists of a glass tube bent at right angle to the direction of flow.



h_1 - Depth of pitot tube in liquid.

h_2 Rise of liquid above free surface

$$V_{th} = \sqrt{2gh_2}$$

$$V_{act} = C_r \sqrt{2gh_2}$$

where,

C_r is the Coefficient of pitot tube.

$$h = m \left(\frac{S_1}{S_0} + 1 \right) = 0.1 \left(\frac{13.6}{6.8} + 1 \right) = 1.6 \text{ m}$$

velocity of flow in the pipe = @

$$C_r \sqrt{2gh}$$

$$= 0.98 \sqrt{2 \times 9.81 \times 1.6}$$

$$= 5.49 \text{ m/sec}$$

2 A Pitot static tube is placed in the centre of a 300 mm pipe line has one orifice point of stream and other perpendicular to it. the mean velocity in the pipe is 0.8 of the central velocity. Find the discharge through the pipe, if pressure difference between the two orifices is 60 mm of water
 loss coefficient of pitot-tube $C_v = 0.98$

$$\text{Difference in pressure} = 60 \text{ mm}$$

$$= 0.06 \text{ m}$$

$$= h$$

$$C_v = 0.98$$

$$\text{Mean velocity} = 0.8$$

$$v_{\text{mean}} = 0.8 \times v_{\text{central}}$$

$$v_{\text{central}} = C_v \sqrt{2gh}$$

$$= 0.98 \sqrt{2 \times 9.81 \times 0.06}$$

$$= 1.06 \text{ m/s}$$

$$v_{\text{mean}} = 0.8 \times 1.06 \\ = 0.85 \text{ m/s.}$$

$$\text{discharge} = v_{\text{mean}} \times \text{Area}$$

$$Q = 0.85 \times \frac{\pi}{4} \times 0.3^2 \\ = 0.06 \text{ m}^3/\text{s.}$$

Discharge of the pipe.

A Pitot tube is used to measure the velocity of water in a pipe. the stagnation pressure head is 6m. and static pressure head is 5m.

Calculate the velocity of flow, assuming

$$C_v = 0.98.$$

A submarine moves horizontally in a sea and has its axis is 15m below the surface of water.

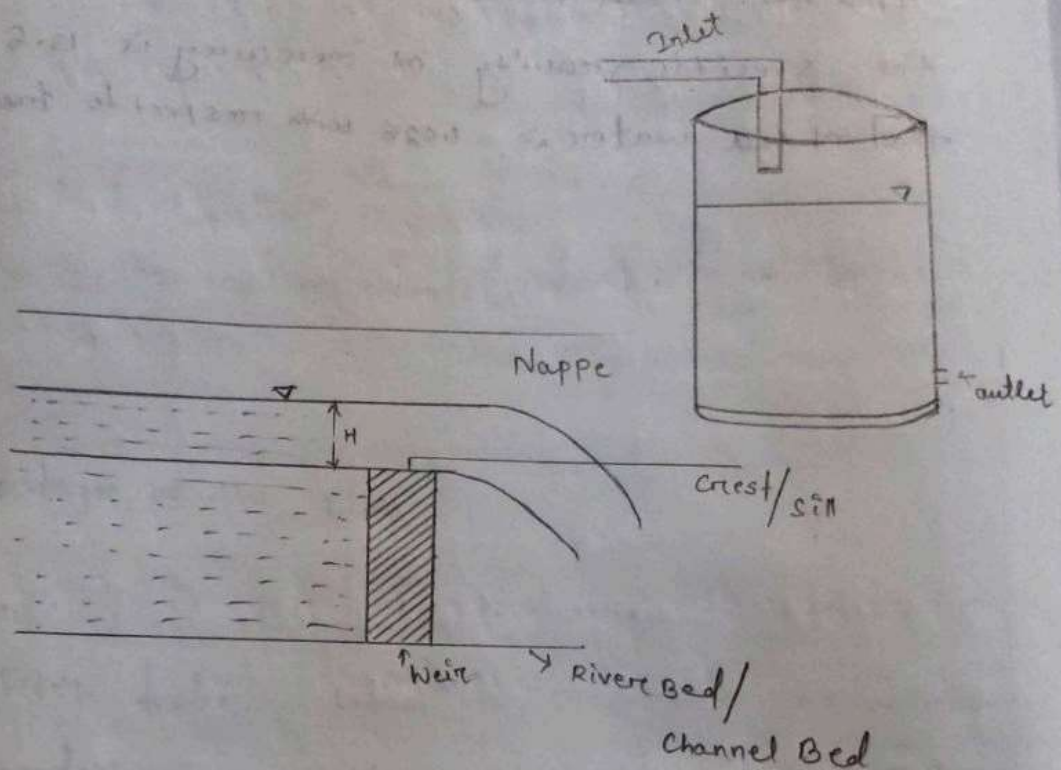
a pitot tube properly placed just in front of the submarine and along its axis is connected to the two limbs of a U-tube manometer containing mercury.

the difference of mercury level is found to be 170 mm
find the speed of the submarine knowing that
the specific gravity of mercury is 13.6 and
that of sea water is 1.026 with respect to fresh water.

ep: 2.2

Flow Through Notches & Weirs

Notch :-



It is a device for measuring the rate of flow of liquid in a small channel or a tank.

It may be defined as a small opening in the side of a tank or a small channel in such a way that the liquid surface in the tank is below the top edge of the opening.

It is made up of metallic plate of smaller size.

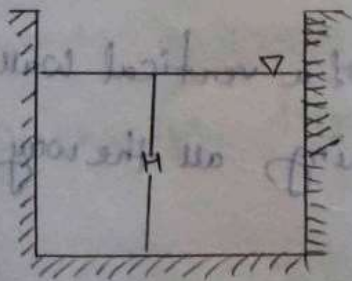
Classification of Notch:-

(a) according to the shape of the opening

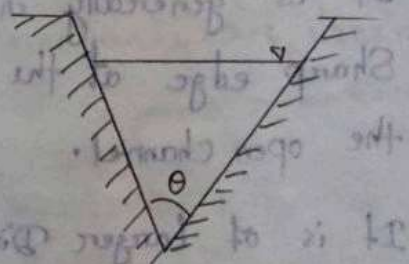
- (i) rectangular Notch.
- (ii) Triangular Notch.
- (iii) Trapezoidal Notch.
- (iv) stepped Notch.

(b) According to the effect of sides of Nappe

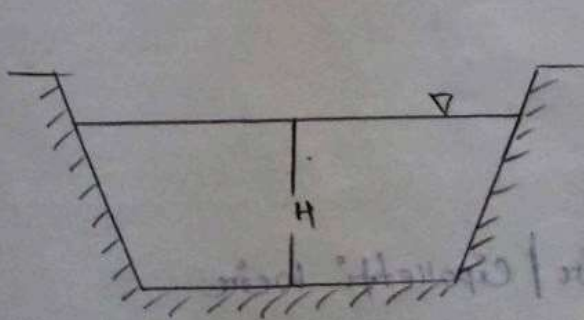
- (i) Notch with end contraction.
- (ii) Notch without end contraction. (Suppressed Notch)



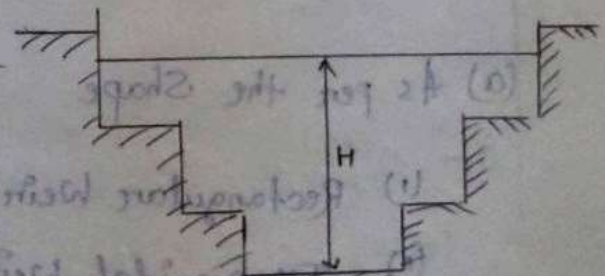
Rectangular Notch



Triangular Notch



Trapezoidal Notch



Stepped Notch

Nappe / Vein :-

It is the sheet of water flowing through the notch or over the weir.

Crest / Sill :-

It is the bottom edge of the notch or top of the weir in which water flows.

Weir :-

It is a concrete or masonry structure placed in an open channel over which the flow occurs.

It is generally in the form of a vertical wall with a sharp edge at the top running all the way across the open channel.

It is of larger dimension.

Classification of weirs :-

(a) As per the shape

- (i) Rectangular weir
- (ii) Trapezoidal weir / Cipolletti weir
- (iii) Triangular weir

(b) As per the Shape of Crest

(i) Sharp Crested Weir.

(ii) Broad Crested Weir.

(iii) Narrow Crested Weir.

(iv) Ogee shaped Weir.

(c) Effect of Side on the Emerging Nappe.

(i) Weir with end contraction.

(ii) Weir without end contraction.

Discharge through a rectangular weir :-

$$Q = \frac{2}{3} C_d L \sqrt{2g} H^{3/2}$$

$$Q = \frac{8}{15} c d \tan \frac{\theta}{2} \cdot 2g H^{5/2}$$

h = head of water at notch

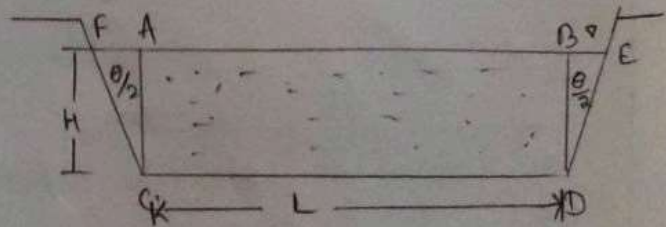
θ = angle of notch

If $\theta = 90^\circ$ called right angled notch.

$$Q = 1.417 \times h^{5/2}$$

Discharge over a :

Discharge through a trapezoidal notch/weir



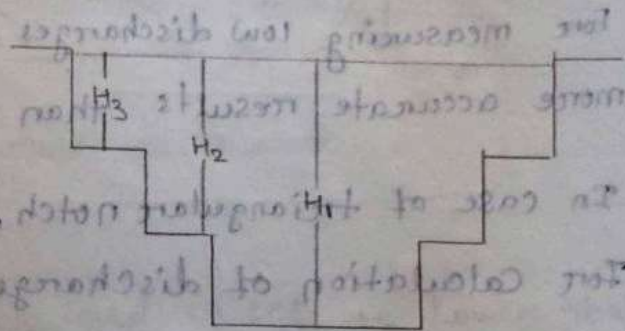
$$Q = Q_1 + Q_2$$

(ABCD) + FAE + BDE

= $Q_1 + Q_2$

$$\Rightarrow Q = \frac{2}{3} cd_1 L \sqrt{2g} H^{3/2} + \frac{8}{15} cd_2 + \tan \frac{\theta}{2} \sqrt{2g} H^{5/2}$$

Discharge over a Stepped Notch :-



$$Q = Q_1 + Q_2 + Q_3$$

$$Q = \frac{2}{3} cd_1 L_1 \sqrt{2g} (H_1^{3/2} - H_2^{3/2}) +$$

$$\frac{2}{3} cd_2 L_2 \sqrt{2g} (H_2^{3/2} - H_3^{3/2}) +$$

$$\frac{2}{3} cd_3 L_3 \sqrt{2g} H_3^{3/2}$$

2 mark Advantages of triangular notch over weir or rectangular notch or weir :-

Expression for discharge for a right angle V notch is very simple.

For measuring low discharges, a triangular notch gives more accurate results than rectangular notch.

In case of triangular notch, only h is required for calculation of discharge.

ventilation of triangular notch is not necessary.

Effect on discharge due to error in measurement of head H .

For rectangular weir :-

An error of one % in measuring H will produce 1.5% error in discharge over a rectangular weir or notch.

For triangular weir or notch :-

An error of 1% in measuring H will produce 2.5% error in discharge over a triangular weir or notch.

Application :- (Application of ~~weir~~ weir or notch)

(a) The time required to empty a reservoir or tank with rectangular weir or notch.

$$T = \frac{3A}{cdL\sqrt{2g}} \left(\frac{1}{\sqrt{H_2}} - \frac{1}{\sqrt{H_1}} \right)$$

where,

T = the time required in sec to lower the height of liquid from H_1 to H_2

A = Crosssectional Area of the reservoir.

H_1 = Initial height of liquid ^{above} the crest.

H_2 = Final height of liquid above the crest or notch.

(b) Time required to empty reservoir with a triangular notch or weir.

$$T = \frac{5A}{4cd \tan \frac{\theta}{2} \sqrt{2g}} \left(\frac{1}{H_2^{3/2}} - \frac{1}{H_1^{3/2}} \right)$$

(1) velocity of approach (V_a) :-

It is defined as the velocity with which the water approaches the weir or notch before it close over it.

$$V_a = \frac{Q}{\text{Area of the channel}}$$

$$h_a = \frac{V_a^2}{2g}$$

$$\text{again } Q = \frac{2}{3} C_d \times L \times \sqrt{2g} \left((H_1 + h_a)^{3/2} - h_a^{3/2} \right)$$

Discharge equation with velocity of approach

$$\left(\frac{1}{H_1} - \frac{1}{H_2} \right) \frac{H_2}{\sqrt{2g} \frac{Q}{L}} = P$$

- ② Find the discharge of water flowing over a rectangular notch of 2m length when the crest head over the notch is 300 mm. take $C_d = 0.6$.

$$C_d = 0.6$$

$$L = 2 \text{ m.}$$

$$H = 300 \text{ mm} \\ = 0.3 \text{ m.}$$

$$Q = \frac{2}{3} C_d L \sqrt{2g} H^{3/2}$$

$$= \frac{2}{3} \times 0.6 \times 2 \text{ m.} \times \sqrt{2 \times 9.81} \times 0.3^{3/2}$$

$$= 0.58 \text{ m}^3/\text{sec.}$$

- ③ Determine the height of a rectangular weir of length 6m. to be built across a rectangular channel. the maximum depth of water on the upstream side of the weir is 1.8 m. and discharge is 2000 l/sec. take $C_d = 0.6$ and neglect end contraction.

Find the discharge over a triangular notch of angle 60° when the head over the V notch is 0.3 m .
Take $C_d = 0.6$

$$Q = \frac{8}{15} C_d \tan \frac{\theta}{2} \sqrt{2g} H^{3/2}$$

a)

Solⁿ

Given data:-

For the rectangular weir, to be built across a rectangular channel.

$$\text{length } (L) = 6\text{ m.}$$

maximum depth of water on the upstream side of

$$\text{weir} = 1.8\text{ m.}$$

$$\text{Discharge in } Q = 2000\text{ m}^3/\text{s}$$

$$C_d = 0.6$$

$$H = ?$$

we have discharge equation

$$Q = \frac{2}{3} C_d \sqrt{2g} h^{3/2}$$

$$\Rightarrow 2 = \frac{2}{3} \times 0.6 \times 6 \sqrt{2 \times 9.81} (1.8 - h)^{3/2}$$

$$\Rightarrow 2 = 10.63 (1.8 - h)^{3/2}$$

$$\Rightarrow (1.8 - h)^{3/2} = \frac{2}{10.63}$$

$$\Rightarrow (1.8 - h)^{3/2} = 0.188$$

for

$$\Rightarrow 4 = 112.99 (1.8 - h)^{3/2}$$

$$\Rightarrow (1.8 - h)^{3/2} = \frac{4}{112.99}$$

$$\Rightarrow (1.8 - h)^{3/2} = 0.035$$

$$\Rightarrow 1.8 - h = (0.035)^{2/3}$$

$$\Rightarrow h = 1.8 - (0.035)^{2/3}$$

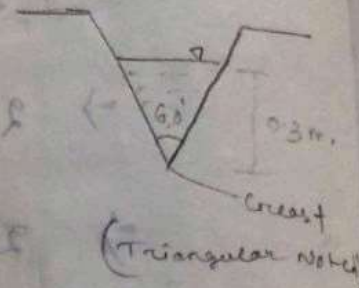
$$h = 1.47 \text{ m.}$$

3 Given data, For the triangular notch

$$\theta = 60^\circ$$

head over the notch = 0.3 m.

$$H = 0.3 \text{ m.}$$



$$Q = \frac{8}{15} C_d + \tan \frac{\theta}{2} \sqrt{2g} (H)^{5/2}$$

$$Q = \frac{8}{15} \times 0.6 \times \tan \frac{60}{2} \sqrt{2 \times 9.81} (0.3)^{5/2}$$

$$= 0.04 \text{ m}^3/\text{s} \text{ Ans}$$

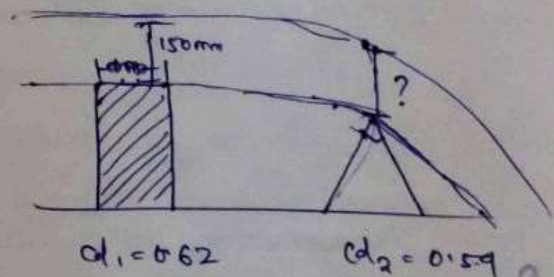
4) Water flows over a rectangular weir 1 m. wide and at a depth of 150 mm and after wards passes through a triangular right angle weir. take C_d for the rectangular and triangular weir as 0.62 and 0.59 respectively find the depth of water over the triangular weir

⑤ find the time required to lower the water level from 3m. to 2m. in a reservoir of diameter 80 m x 80 m. by a rectangular notch of length 1.5 m. take $C_d = 0.62$.

⑥ find the discharge over a rectangular weir of length 100 m. the head of water over the weir is 1.5 m. the velocity of approach is given as 0.5 m/s. take $C_d = 0.6$.

1

$$L = 1m.$$



$$Q_{Rec} = \frac{2}{3} C_{d1} L \sqrt{2g} H^{3/2}$$

$$= \frac{2}{3} \times 0.62 \times 1 \times \sqrt{2 \times 9.81} (0.15)^{3/2}$$

$$= 0.106$$

$$H^{5/2} = 0.076$$

$$H = 0.3565$$

$$H = 0.3565$$

Area

$$0.106 = \frac{8}{15} C_{d2} \tan \frac{\theta}{2} \sqrt{2g} H^{5/2}$$

$$0.106 = \frac{8}{15} \times 0.59 \times 1 \times \sqrt{2 \times 9.81} \times H^{5/2}$$

$$0.106 = 1.393 H^{5/2}$$

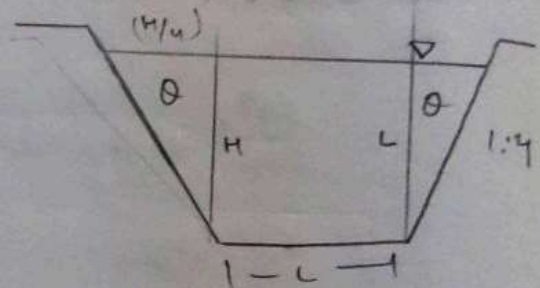
Cipolletti Weir / Notch :-

It is a type of trapezoidal weir in which the side slope horizontal to vertical is 1:4.

The discharge

$$Q = \frac{2}{3} C_d L \sqrt{2g} (H)^{3/2}$$

$$H:V = 1:4$$



$$2 \times 3 = 6$$

$$4 \times 1 = 4$$

10 marks

5 Given :-

$$h_1 = 3\text{m.}$$

$$h_2 = 2\text{m.}$$

$$\text{diameter} = 80\text{m} \times 80\text{m}$$

$$\text{Area} = 6400$$

$$L = 1.5\text{m}$$

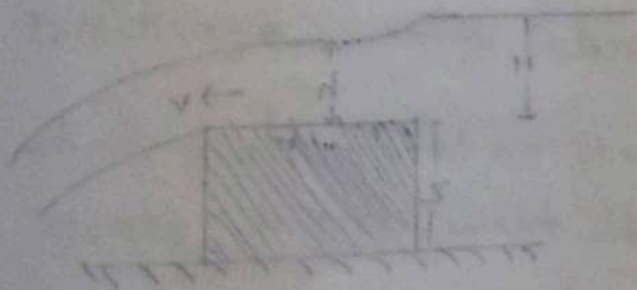
$$C_d = 0.62$$

$$t = \frac{3A}{cd \sqrt{2g}} \left(\frac{1}{\sqrt{H_2}} - \frac{1}{\sqrt{H_1}} \right)$$

$$= \frac{3 \times 6400}{0.62 \times 1.5 \times \sqrt{2 \times 9.81}} \left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{3}} \right)$$

$$= 604.780 \text{ Sec.}$$

h = head of water at the nozzle or weir height



Velocity of flow over the weir

$$Q_{max} = 1.48 C_d L H^{3/2}$$

$$H = \frac{2}{3} h$$

Discharge over a narrow crested weir:-

$$Q = \frac{3}{2} C_d L \sqrt{2g} H^{3/2}$$

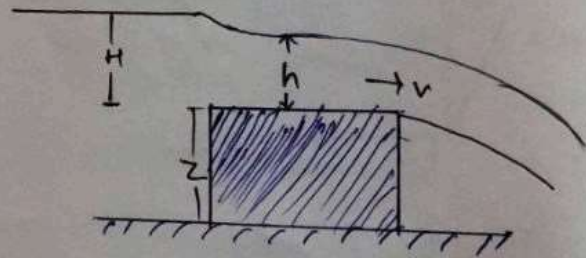
Discharge over a broad crested weir :-

$$Q = Cd \cdot L \times \sqrt{2g} (Hh^2 - h^3)$$

if $2L > H$ broad crested weir

$2L < H$ narrow crested weir

h = head of water at the middle of weir within crest



v = velocity of flow over the water

$$Q_{max} = 1.705 Cd \cdot L \times H^{3/2} \quad \text{if } h = \frac{2}{3}H$$

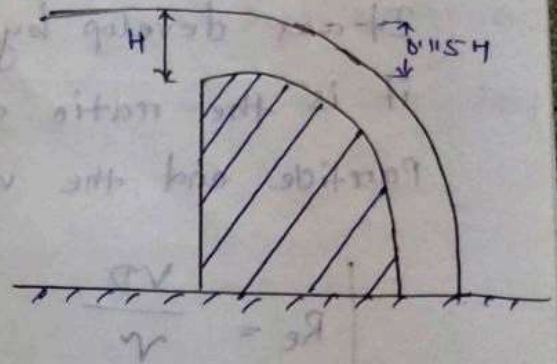
Discharge over a narrow crested weir :-

$$Q = \frac{2}{3} Cd \cdot L \times \sqrt{2g} H^{3/2}$$

Discharge over a Ogee weir:-

Here crest of the weir rises upto to $0.115H$ and falls as shown in figure

$$Q = \frac{2}{3} C_d L \sqrt{2g} H^{3/2}$$

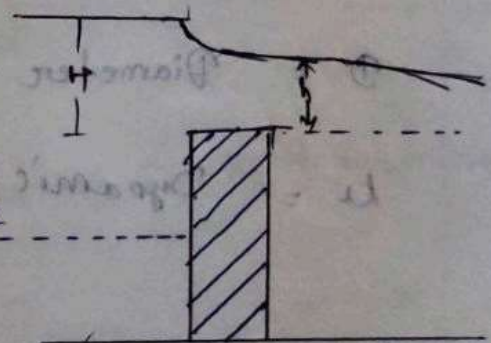


Discharge over a Submerged/weir
when water level on the d/s side of weir is above the crest level of weir, it is called submerged drowned weir.

$$Q = \frac{2}{3} C_{d1} L \sqrt{2g} (H-h)^{3/2} + C_{d2} L h \sqrt{2g} (H-h)$$

C_{d1} = Coefficient of discharge for
free portion.

C_{d2} = Coefficient of discharge
for drowned portion.



~~A broad crested weir~~

Reynold's Number (Re) :-

It was develop by Reynold in 1883

it is the ratio of inertial force of flowing Particle and the viscous force of fluid particle.

$$Re = \frac{VD}{\nu}$$

$$Re = \frac{\rho v D}{\mu}$$

Where,

ρ = Density of the Fluid through pipe

v = Average velocity of the fluid

D = Diameter of the pipe.

μ = Dynamic viscosity of the fluid.

ν = Kinematic viscosity.

if $Re < 2000$ the flow is called lamina flow.

if $Re > 4000$ the flow is called turbulent flow

if the value of Re is in between 2000 to 4000 then the flow changes from lamina to turbulent flow.

Energy losses in Pipe flow! -

(i) Major Energy losses (Energy loss due to friction)

(ii) Minor Energy losses

Energy loss due to friction! -

The energy loss due to the friction between the fluid and walls of the pipe contributes the major loss in energy of the fluid.

The energy loss due to friction can be determined by two equations.

(a) Darcy - Weisbach's Formula

(b) Chezy's Formula.

Darcy - Weisbach's Formula :-

$$h_f = \frac{4fLv^2}{2gd}$$

where,

h_f = the head loss due to friction

f = Coefficient of friction

$$= \frac{16}{Re} \quad \text{if } Re < 2000$$

$$= \frac{0.079}{Re^{1/4}} \quad \text{if } Re > 4000 \text{ to } 10^6$$

Chezy's Formula :-

$$H_f = \frac{F'}{fg} \cdot \frac{P}{A} L \times v^2$$

where,

F' = the friction factor

P = wetted perimeter

A = Area of cross-section of pipe

v = mean velocity of flow

$$\frac{A}{p} = m \quad \frac{d}{4} \text{ (for circular pipe)}$$

m = hydraulic mean depth /
hydraulic radius.

$\frac{h_f}{L} = i$ = loss of head per meter length of the pipe.

$$C = \sqrt{\frac{fg}{f'}}$$

= Chezy's Constant

$$v = C \sqrt{mi}$$

Q Find the head loss due to friction in a pipe of diameter 300mm and length 50m. velocity of flow is 3 m/sec use darcy formula and chezy's formula take $C = 60$ and ν of water is 0.01 stoke

Solⁿ ① use darcy, weisbach's formula

$$h_f = \frac{4fLv^2}{2gd}$$

$$Re = \frac{VD}{\nu}$$

$$Re = \frac{3 \times 0.3}{0.01 \times 10^{-4}}$$

$$= 900000$$

$$\nu = 0.01 \text{ stoke}$$

$$= 0.01 \text{ cm}^2/\text{s}$$

$$= 0.01 \times 10^{-4} \text{ m}^2/\text{s}$$

$$f = \frac{0.079}{Re^{1/4}}$$

$$= \frac{0.079}{900000^{1/4}}$$

$$= 2.56 \times 10^{-3}$$

$$h_f = \frac{4 \times 2.56 \times 10^{-3} \times 50 \times 3^2}{2 \times 9.81 \times 0.3}$$

$$= 0.78 \text{ m.}$$

$$\frac{(v^2 - u^2)}{2g} = \dots$$

use chezy's Formula :-

$$v = 60 \sqrt{\frac{0.3}{4} \times \frac{h_f}{L}}$$

$$3 = 60 \sqrt{\frac{0.3}{4} \times \frac{h_f}{50}}$$

$$h_f = \dots$$

$$h_f = \dots$$

$$q = 3600 \left[\frac{0.3}{4} \times \frac{h_f}{50} \right]$$

$$h_f = \frac{9 \times 4 \times 50}{3600 \times 0.3}$$

$$= \frac{36 \times 50}{3600 \times 0.3}$$

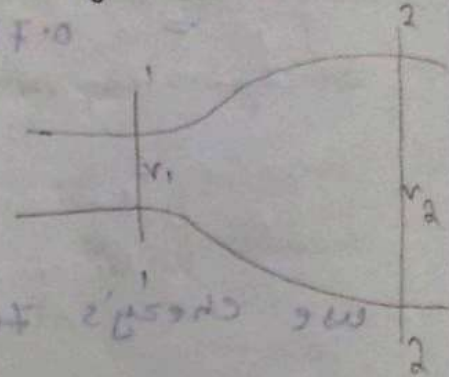
$$= \frac{1}{0.6} = \frac{10}{6}$$

$$= 1.66$$

Minor losses in Pipe Flow :-

① loss of head due to Sudden enlargement :-

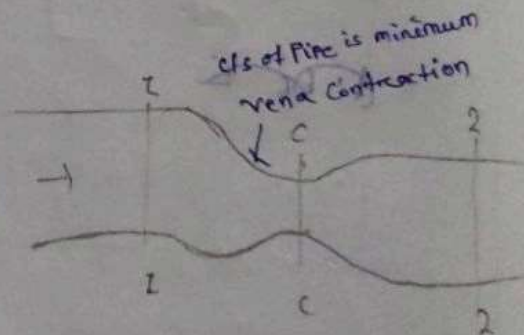
$$h_e = \frac{(v_1 - v_2)^2}{2g}$$



v_1 = velocity of flow at section 1,1
 v_2 = velocity of flow at section 2,2

(ii) Loss of head due to Sudden Contraction :-

$$h_c = \left(\frac{A_1}{A_2} - 1 \right)^2 \frac{v_2^2}{2g}$$



$$C_c = \frac{A_c}{A_2}$$

$$\frac{0.1}{1} = \frac{1}{1.0} \frac{A_2}{A_2}$$

C_c = Coefficient of Contraction

$$h_c = 0.5 \frac{v_2^2}{2g}$$

$$h_c = 0.375 \frac{v_2^2}{2g}$$

Loss of head at entrance of pipe :-

$$h_e = 0.5 \frac{v^2}{2g}$$

Loss of head at exit of the pipe :-

$$h_o = \frac{v^2}{2g}$$

Loss of head due to obstruction in pipe :-

$$h_e = \frac{v^2}{2g} \left(\frac{A}{C_c(A-a)} - 1 \right)^2$$

where,

a = maximum area of obstruction

A = Area of cross-section of the pipe

v = velocity of flow in pipe.

C_c = Coefficient of contraction.

Loss of head due to bend in pipe:-

$$h_b = \frac{kv^2}{2g}$$

k = Coefficient of bend which depends upon angle of the bend, Radius of Curvature of the bend and diameter of the pipe.

$$\frac{5}{f_b} = 0.1$$

Loss of head due to pipe fittings:-

$$h_p = \frac{kv^2}{2g}$$

$$\frac{5}{f_p} \left(1 - \frac{A}{A_0} \right) \frac{v}{2g} = 0.1$$

where,

k = Coefficient of pipe fittings

Loss of head due to valves, Pipe fitting, and couplings.

Hydraulic Gradient ~~Pipe~~ Line :- (HGL)

i) It is the line formed by joining the datum head and pressure head along the direction of flow of fluid at various locations.

ii) It is also formed by joining the ordinates of the sum of datum head and pressure head from the centre line of the pipe or a reference datum.

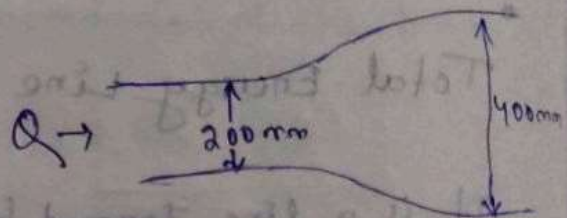
Total Energy Line (TEL) :-

It is a line formed by joining the datum head, pressure head and kinetic head from the centre line of the pipe along the direction of flow of fluid.

① Find the loss of head of pipe of diameter 200 mm is suddenly enlarged to a diameter of 400 mm the rate of flow through the pipe is 250 l/sec

(ii) At a sudden enlargement of a watermain from 240 mm to 480 mm diameter, the hydraulic gradient rises by 10 mm. Estimate the rate of flow.

Solⁿ
$$h_e = \frac{K V^2}{2g}$$



$$Q = 250 \text{ l/sec} = 0.25 \text{ m}^3/\text{sec}$$

discharge is constant

applying equation of Continuity

$$h_e = \frac{(V_1 - V_2)^2}{2g}$$

$$a_1 v_1 = a_2 v_2$$

$$a_1 = 0.0314$$

$$a_2 = 0.1256$$

$$v_1 = \frac{0.25}{0.0314}$$

$$= 7.96 \text{ m/sec}$$

$$v_2 = \frac{0.25}{0.1256}$$

$$= 1.99$$

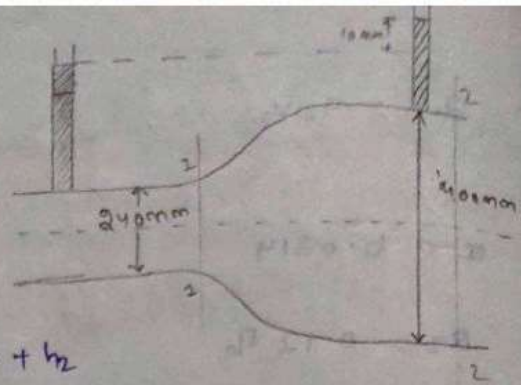
$$h_e = \frac{(7.96 - 1.99)^2}{2 \times 9.81}$$

$$= 0.3081 \text{ } 1.81 \text{ m}^2$$

① Ans

Apply Bernoulli's equation

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + z_2 + \frac{v_2^2}{2g} + h_L$$



$$\frac{v_1^2}{2g} - \frac{v_2^2}{2g} = h_2 = 100 \text{ mm} = (0.1 \text{ m})$$

$$v_1^2 - v_2^2 = 2g (0.1)$$

$$v_1^2 - v_2^2 = 2 \times 9.81 \times 0.1 = 0.196$$

$$v_1^2 = 0.196 + v_2^2$$

We have $a_1 v_1 = a_2 v_2$

$$\Rightarrow \frac{\pi}{4} \times 0.24^2 \times v_1 = \frac{\pi}{4} \times 0.1^2 \times \sqrt{v_1^2 - 0.196}$$

$$\Rightarrow v_1 \times 0.24^2 = 0.1^2 \times \sqrt{v_1^2 - 0.196}$$

$$\Rightarrow v_1^2 (0.24^2)^2 = 0.1^4 (v_1^2 - 0.196)$$

$$\Rightarrow v_1^2 \times 0.0033 = 0.0001 (v_1^2 - 0.196)$$

$$\Rightarrow v_1^2$$

Water is flowing through a pipe diameter 200 mm at a velocity of 3 m/sec. A circular solid plate of diameter 150 mm is placed in the pipe to obstruct the flow. Find the loss of head due to obstruction in the pipe. Take $C_c = 0.62$

$$h_{re} = \frac{V^2}{2g} \left(\frac{A}{C_c(A-a)} - 1 \right)^2$$

$$A = 0.000314$$

$$a = 0.0176$$

$$= \frac{3^2}{2 \times 9.81} \left(\frac{\frac{\pi}{4} \times (0.2)^2}{0.62 \left(\frac{\pi}{4} \times (0.2)^2 - \frac{\pi}{4} \times (0.15)^2 \right)} - 1 \right)^2$$

$$= 0.02 = 20 \text{ mm}$$

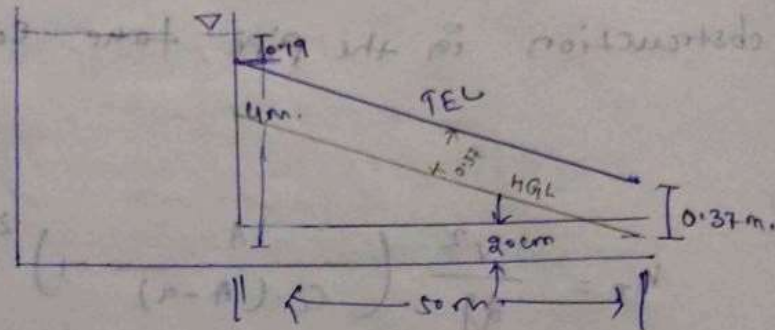
$$\text{Diameter of pipe} = 200 \text{ mm}$$

$$9 \text{ m} = 9000 \text{ mm}$$

Determine the Rate of flow of water through a pipe of diameter 20 cm and length 50 m. when one end of the pipe is connected to a tank and the other end of the pipe is open to atmosphere. The pipe is horizontal at the height of the water tank is 9 m. above the centre of the pipe. Consider all minor losses and take $F = 0.009$ in the formula

$$h_f = \frac{4 f L v^2}{2 g d}$$

Also draw the ~~h_f~~ HGL and TEL



Given data:

length of pipe = 50 m = L

Diameter of pipe = 20 cm

$d = 200 \text{ mm} = 0.2 \text{ m}$

HA of water in the tank from the centre line of

the pipe = 1 m

Apply Bernoulli's theorem at top surface of water and

at the exit of pipe with respect to

horizontal line of the water tank

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + h_1 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + h_2 + \text{losses}$$

$$0 + 0 + 4 = \frac{v^2}{2g} + 0 + h_e + h_f$$

$$4 = \frac{v^2}{2g} + 0.5 \frac{v^2}{2g} + \frac{4 f l v^2}{2g d}$$

$$4 = \frac{v^2}{2g} \left(1 + 0.5 + \frac{4 f l}{d} \right)$$

$$= \frac{v^2}{2 \times 9.81} \left(1 + 0.5 + \frac{4 \times 0.009 \times 50}{0.2} \right)$$

$$4 = 0.535 v^2$$

$$v^2 = 7.47$$

$$v = 2.73 \text{ m/s}$$

$$\text{Rate of Flow} = Q = A \cdot v$$

$$= \frac{\pi}{4} \times 0.2^2 \times (2.73)$$

$$= 0.085 \text{ m}^3/\text{s}$$

Consider A, B, C are the points at top of water surface at pipes & at the entrance of pipe with the tank respectively.

$$\text{Total energy at A} = 0 + 0 + 4 \\ = 4 \text{ m.}$$

$$\text{Total energy at C} = \text{Total energy at A} - h_f$$

$$= 4 - 0.5 \frac{v^2}{2g}$$

$$4 - \frac{0.5 \times 2.7^2}{2 \times 9.81}$$

$$= 3.81 \text{ m.}$$

$$\text{Total energy at B} = 0 + \frac{v^2}{2g} + 0$$

$$= \frac{2.7^2}{2 \times 9.81}$$

$$= 0.37$$

hydraulic gradient line (HGL)

If we deduct the kinetic head at TEL, we will get HGL

$$\text{kinetic head} = \frac{v^2}{2g}$$

$$= 0.37 \text{ m}$$

Re < 2000 - laminar flow

Re > 2000 - turbulent flow

$$Re = \frac{vD}{\nu}$$

Transition zone

Critical flow Re = 2300

Subcritical flow Re < 2300

Supercritical flow Re > 2300

Flow number (Fr)

$$Fr = \frac{v}{\sqrt{gd}}$$

Open Channel Flow

- Water surface will be exposed to atm pressure.
- Flow occurs due to gravity / slope of channel
- In case pipe, if water flows half / not full

$$Re < 500-600$$

$$Re > 2000 - \text{turbulent flow}$$

$$Re = 500 - 2000$$

transition state.

Critical flow $Fr = 1$

Subcritical flow $Fr < 1$

Super critical flow $Fr > 1$

Froude's Number (Fr)

$$Fr = \frac{VE}{\sqrt{gd}}$$

Discharge through Open Channel :-

(a) Chezy's Formula :-

$$Q = A \sqrt{m i}$$

where, $C = \sqrt{\frac{w}{f}}$

$$m = \frac{A}{P}$$

C = Bed slope

w = unit weight of water / liquid

f = Frictional resistance per unit velocity.
Per unit area

A = Area of flow

m = Hydraulic mean depth

P = wetted perimeter

(b) Mannings Formula :-

$$C = \frac{1}{N} m^{1/6}$$

N = Mannings' Constant

Q Find the velocity of flow and rate of flow, to a rectangular channel, having 6m width and 3m depth and it is running full the bed slope of the channel is $1:2000$ and $C = 55$

Given:-

width of rectangular channel (w) = 6m

Depth of channel $d = 3m$

Bed Slope = $1:2000$

$C = 55$

hydraulic mean depth = $m = \frac{A}{p}$

$$\frac{3 \times 6}{2 \times 3 + 6} = \frac{3 \times 6}{12}$$

$$= 1.5m$$

$$Q = C \sqrt{m i}$$

$$= 55 \sqrt{1.5 \times \frac{1}{2000}}$$

$$= 1.50 \text{ m}^3/\text{s}$$

$$Q = AV$$

$$1.5 = 3 \times 6 \times V$$

$$1.5 = 3 \times 6 \times V$$

$$V = \frac{1.5}{18}$$

$$= 0.083 \text{ m/s}$$

An arthen channel with 3m wide base and side slope 1:1 carries water with a depth of 1m. the bed slope is 1 in 1600. Estimate the discharge using Manning's equation and take $N = 0.04$

$$w = 3 \text{ m}$$

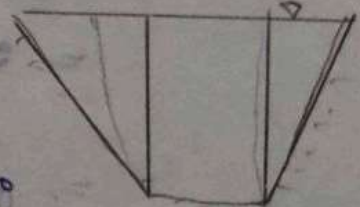
$$d = 1 \text{ m}$$

depth of water

$$= 1 \text{ m}$$

$$N = 0.04$$

$$\text{Bed Slope} = i = 1:1600$$



$$m = \frac{A}{P}$$

$$= \frac{3 \times 1}{3 + 2 \times 1}$$

$$= \frac{3}{5}$$

$$= 0.6$$

$$m = \frac{A}{P}$$

$$= \frac{\frac{1}{2} \times 5 \times 1}{2 + \sqrt{2} \times 3}$$

$$= 0.428$$

$$\approx 0.43$$

$$C = \frac{1}{0.64} \times 0.43^{1/6}$$

$$= 21.71$$

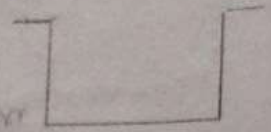
$$Q = C \sqrt{mi}$$

$$= 21.71 \sqrt{0.43 \times \frac{1}{1600}}$$

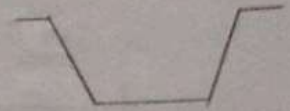
$$= 0.35 \text{ m}^3/\text{s}$$

Types of channel section :-

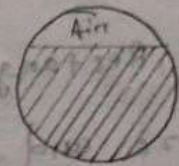
(a) Rectangular channel section



(b) Trapezoidal channel section



(c) Circular channel section



Most economical section :-

→ Section of the channel for which the cost of construction will be minimum and we will have max^m discharge.

For rectangular section :-

For the most economical section which is having minimum wetted perimeter so that the discharge will be maximum

In case of rectangular channel the two conditions

are

For

$$\boxed{b = 2d}$$
$$m = \frac{d}{2}$$

For max^m Q +
max^m V

$$Q_{\max} = C \sqrt{mi}$$

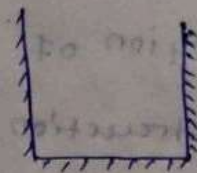
$$m \uparrow = \frac{A \uparrow}{P \downarrow}$$

A Rectangular has Crosssection 8 m^2 find its Size and discharge through the most economical Section if the bed slope is 1 in 1000 and $C = 55$

$$A = 8 \text{ m}^2$$

$$i = \frac{1}{1000}$$

$$C = 55$$



$$m = \frac{A}{P}$$

$$m = \frac{d}{2}$$

$$b \times d = 8$$

$$b \times d = 8$$

$$b \times \frac{b}{2} = 8$$

$$b \times \frac{b}{2} = 8$$

$$b^2 = 16$$

$$b = \sqrt{16}$$

$$b = 4$$

$$d = \frac{8}{4} = 2$$

$$m = \frac{2}{3}$$

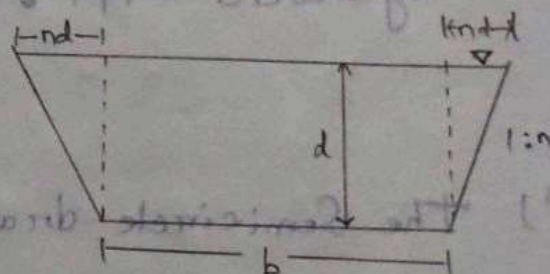
$$m = 1$$

$$Q = 55 \sqrt{1 \times \frac{1}{1000 + 0.01}} = \frac{b \sqrt{g d}}{2}$$

$$= 1.73$$

$$\frac{b}{d} = m \quad (2)$$

Most economical Section for trapezoidal channel :-



$$\sqrt{n^2 d^2 + d^2}$$

$$= \sqrt{n^2 d^2 + d^2}$$

$$= \sqrt{d^2 (n^2 + 1)}$$

$$= d \sqrt{n^2 + 1}$$

$$\text{Side slope} = d \sqrt{n^2 + 1}$$

$$\text{wetted perimeter} = P = b + 2d \sqrt{n^2 + 1}$$

$$\text{Area} = A = \frac{1}{2} (b + b + 2nd) \times d$$

$$m = \frac{A}{P}$$

The half of the top width of the trapezoidal section is = the length of the side slope

mathematically,

$$\frac{b + md}{2} = d \times \sqrt{n^2 + 1}$$

$$(2) \quad m = \frac{d}{2}$$

hydraulic depth is = ~~depth~~ half of the depth

(3) The Semicircle drawn from O with a radius = the depth of the flow will touch the three sides of the channel.

Q A trapezoidal channel has a side slope of one horizontal to two vertical and the bed slope of channel is 1:1500. The area of the section is 40 m^2 . Find the dimension of the section if it is economical. Determine the discharge of most economical section if $C = 50$.

Ans:- Side Slope = 1:2 $= 1/2$
 bed slope = 1:1500

$$A = 40 \text{ m}^2$$

$$C = 50$$

$$\text{Side Slope} = d \sqrt{n^2 + 1}$$

$$A = d \sqrt{\left(\frac{1}{2}\right)^2 + 1}$$

$$d = 1.8$$

$$= \frac{1}{2} (ab + 2nd + b) \times d = 40$$

$$\Rightarrow \frac{1}{2} (ab + 2nd) \times d = 40$$

$$\Rightarrow (b + nd) d = 40$$

$$\Rightarrow bd + nd^2 = 40$$

For most economical Section,

$$\frac{b + 2nd}{2} = n \sqrt{d^2 + 1}$$

$$m = \frac{d}{2}$$

$$\Rightarrow \frac{A}{P} = \frac{d}{2}$$

$$\Rightarrow \frac{40}{b + 2d\sqrt{n^2+1}} = \frac{d}{2} \quad \text{--- (1)}$$

$$\Rightarrow \frac{40}{b + 2d\sqrt{\left(\frac{1}{2}\right)^2+1}} = \frac{d}{2}$$

$$\Rightarrow 80 = bd + 2d^2\sqrt{\left(\frac{1}{2}\right)^2+1}$$

$$\Rightarrow 80 = bd + 2d^2 \times \frac{\sqrt{5}}{2}$$

$$\Rightarrow 80 = bd + 2.22d^2 \quad \text{--- (11)}$$

$$bd + 0.5d^2 = 40 \quad \text{--- (10)}$$

$$bd + 2.22d^2 = 80 \quad \text{--- (11)}$$

$$1.72d^2 = 40$$

$$d^2 = \frac{40}{1.72}$$

$$d^2 = 23.25$$

$$d = \sqrt{23.25}$$

$$= 4.82$$

$$4.82 \times b + \frac{1}{2} \times 4.82^2 = 40$$

$$4.82 \times b = 40 - 11.61$$

$$b = \frac{28.39}{4.82} = 5.89$$

$$= \frac{b + a \sin \theta}{2}$$

$$= \frac{5.89 + 2 \times \frac{1}{2} \times 4.82}{2}$$

$$= 5.355 \text{ m}$$

$$\frac{1}{2} \sqrt{4.82^2 + 1}$$

$$= 2.46 \text{ m}$$

$$\frac{b + 2 \times \frac{1}{2} \times d}{2} = \frac{1}{2} \sqrt{d^2 + 1}$$

$$= \frac{b + d}{2} = \frac{1}{2} \sqrt{d^2 + 1}$$

$$= b + d = \sqrt{d^2 + 1}$$

$$= b + 4.82 = \sqrt{4.82^2 + 1}$$

$$= b = 4.92 - 4.82$$

$$b = 0.1$$

$$Q = AC \sqrt{mi}$$

$$= 40 \times 50 \sqrt{\frac{4.82}{2}} \times \frac{1}{1500}$$

$$= 80.166 \text{ m}^3/\text{s}$$

Most economical Section for Circular channel:-

There are two conditions for most economical section in case of circular open channel flow, as the area of flow can not be maintained in case of circular cross section.

(i) For maximum velocity $\theta = 128^\circ 45'$

$$d = 0.81D$$

$$m = 0.3D$$

(ii) For maximum discharge $\theta = 154^\circ$

$$d = 0.95D$$

$$\theta = \frac{360^\circ}{2} = 180^\circ$$

$$\theta = 180^\circ$$

$$\theta = 180^\circ$$

Flow through Circular Cross Section:-

Let the depth of water in the circular channel
in = 'd'

Diameter of channel = ϕD

$$Q = A c \sqrt{m i}$$

A = Area of flow = wetted area = Area of Sector
APBQA

Let 2θ in the angle made by the top surface of water
at the centre Circle 'O'.

$$360^\circ = 2\pi = 2\pi R^2$$

$$1^\circ = \frac{\pi R^2}{2\pi}$$

$$2\theta = \frac{\pi R^2}{2\pi} \times 2\theta$$

$$= R^2 \theta$$

$$= OABO$$

The rate of flow of water through a circular channel of diameter 0.6m. is 150 liter/second. Find the bed slope for maximum velocity. take $c = 60$

$$C = 60$$

$$D = 0.6m$$

$$Q = 150 \text{ ltr/sec}$$

$$= 0.15 \text{ m}^3/\text{s}$$

For maximum velocity

$$\theta = 128.45^\circ = 128.45^\circ \times \frac{\pi}{180} = 2.25$$

$$m = 0.3D = 0.3 \times 0.6 =$$

$$d = 0.18 \text{ m}$$

$$A = R^2 \left(\theta - \frac{\sin 2\theta}{2} \right)$$

$$= \left(\frac{0.6}{2} \right)^2 \left(2.25 - \frac{\sin 2 \times 128.45^\circ}{2} \right)$$

$$= 0.246 \text{ m}^2$$

$$0.15 = 0.246 \times 60 \times \sqrt{0.18 \times i}$$

$$0.15 = 0.246 \times 60 \times \sqrt{1 \times 0.18}$$

$$0.45 = 14.76 \times 0.421$$

$$0.421 = \frac{0.15}{14.76} \quad 0.15 = 14.76 \times \sqrt{0.18 \times i}$$

$$\sqrt{0.18 \times i} = \frac{0.15}{14.76} = 0.0101 \quad \sqrt{0.18 \times i} = 0.0101$$

$$i = 0.0001$$

$$0.18 \times i = (0.01)^2$$

$$i = 0.0001$$

A Concrete line Circular Channel of diameter 3m, has a bed slope of 1:500 Calculate the velocity and discharge

For maximum velocity

maximum discharge

Take $C = 50$

Pump

Pump :- Hydraulic machine which convert mechanical energy to hydraulic energy.

(1) Reciprocating Pump

(2) Centrifugal pump

~~Step 1 to~~

The hydraulic energy is in the form of pressure energy if the mechanical energy is converted into pressure energy by means of centrifugal force acting on the fluid, the hydraulic machine is called centrifugal pump.

Working Principle of Centrifugal Pump :-

→ The centrifugal pump works on the principle of forced vortex flow which means when a certain mass of liquid is rotated by a external torque, the rise in pressure head of the rotating liquid takes place.

→ The rise in pressure head at any point of the rotating liquid is proportion to the square of tangential velocity of the liquid of that point.

Mathematically :-

$$\text{Rise in pressure head} = \frac{v^2}{2g} \quad \text{or} \quad \frac{\omega^2 r^2}{2g}$$

Thus at the outlet of the impeller, where the radius is more, the rise in pressure head will be more and the liquid will be discharged at the outlet with a high pressure.

Due to this high pressure head, the water can be lifted to a high level.

Main parts of a Centrifugal Pump :-

The main parts,

(i) Impeller

(ii) Casing

(iii) Suction Pipe with foot valve & strainer

(iv) Delivery pipe

Impeller :-

The rotating part of a centrifugal pump is called impeller.

It consists of a series of backward curved vanes.

The impeller is mounted on a shaft which is connected to the shaft of an electric motor.

In case of centrifugal pump the power is transmitted from the shaft of electric motor to the shaft of the pump and then to the impeller. From the impeller the power is given to water. Thus the power is decreasing from the shaft of the pump to the impeller then to water.

⇒ The following are the important efficiencies of a centrifugal pump.

(i) Manometric Efficiency

(ii) Mechanical Efficiency

(iii) Overall Efficiency

Manometric Efficiency (η_{max}) :-

The ratio of the manometric head imparted to the head by the impeller to the water is called as manometric efficiency.

Mathematically,

$$\eta_{max} = \frac{\text{Manometric head}}{\text{head imparted by the impeller to water}}$$

$$= \frac{H_m}{\frac{V_{w2} U_2}{g}}$$

$$= \frac{g H_m}{V_{w2} U_2}$$

The Ratio of the power given to the water at outlet of the pump to the power available at the impeller is known as manometric efficiency.

The given at the outlet of the pump of water

$$= \frac{W H_m}{1000} \rightarrow \text{KW}$$

where,

$$U_2 = \text{tangential velocity of impeller at outlet}$$

$$= \frac{\pi D_2 N}{60}$$

where

D_2 = Diameter of the impeller at outlet, m

W = weight of the water

$$= \rho g Q$$

Q = volume of water

V_{w2} =

The velocity of the water at the outlet of the impeller

The power at the impeller =

work done by the impeller / sec

kw

$$= \frac{W \times V_{w2} U_2}{g \times 1000} \text{ kw}$$

$$\eta_{man} = \frac{\frac{W H_m}{1000}}{\frac{W V_{w2} U_2}{g \times 1000}} = \frac{g H_m}{V_{w2} U_2}$$

Mechanical ~~eff~~ efficiency :-

The ratio of the power available at the Impeller to the power at the shaft of centrifugal pump is called as mechanical efficiency.

it is written

$$\eta_m = \frac{\text{Power at the Impeller}}{\text{Power at the shaft}}$$

Power at the impeller in kW = work done by the

$$= \frac{W \cdot V_{w2} U_2}{g \times 1000}$$

S.P - power at shaft

Overall efficiency :- η_o

It is defined as the ratio of power output of the pump to the power input of the pump.

The power output of the pump in kW

$$= \frac{\text{Weight of water lifted} \times H_m}{1000}$$

$$= \frac{W H_m}{1000}$$

The power input of the pump = The power supplied by the electric motor

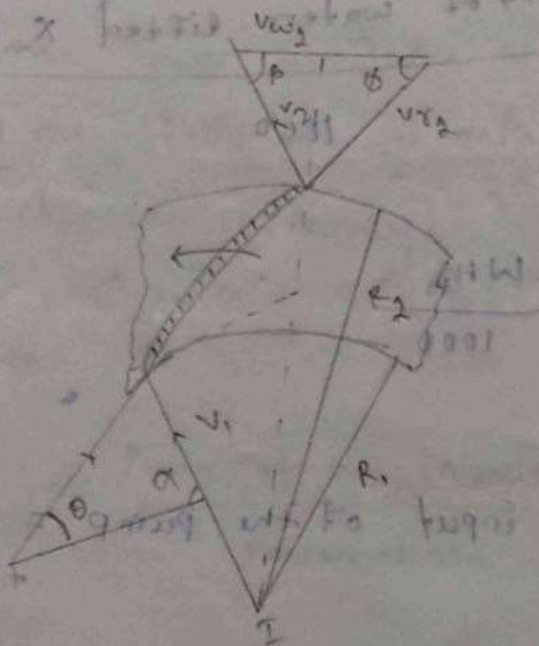
Mathematically,

$$\eta_o = \frac{W H_m}{1000}$$

S.p. of the pump

$$\eta_o = \eta_{man} \times \eta_m$$

The internal and the external diameter of the impeller of a centrifugal pump are 200 mm and 400 mm respectively. The pump is running at 1200 RPM. The vane angle of the impeller at the inlet and outlet are 20° and 30° respectively. The water enters the impeller radially and velocity of flow is constant. determine the velocity of flow per meter Sec.



N = speed of the impeller

D_1 = Diameter of the impeller at the inlet

u_1 = Tangential velocity of the impeller at inlet

$$= \frac{\pi D_1 N}{60}$$

D_2 = Diameter of the impeller at the outlet.

V_2 = Tangential velocity of impeller at the outlet.

$$= \frac{\pi D_2 N}{60}$$

V_1 = The absolute velocity of water at the inlet

V_{r1} = is the relative velocity of water at inlet

α = angle made by absolute velocity V_1 at inlet with the direction of motion of vane.

θ = angle made by the relative velocity V_{r1} at inlet with direction of motion of vane

V_2 , V_{r2} , β and ϕ at the corresponding value of outlet.

V_{f2} , V_{f1} the velocity of flow at inlet and outlet.

Given data :-

$D_1 =$

Internal diameter of impeller = 200mm
= 0.2m.

External diameter of impeller $D_2 = 400\text{mm}$
= 0.4m

$N = 1200 \text{ rpm}$

vane angle at inlet = $\theta = 20^\circ$

vane angle at outlet = $\phi = 30^\circ$

Water enters the impeller radially means

$$\alpha = 90^\circ$$

$$V_{w1} = 0$$

velocity of flow =

$$V_{f1} = V_{f2}$$

$$V_{f1} = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.2 \times 1200}{60} = 12.56$$

4000

$$\tan \phi = \frac{V_{t2}}{u_1}$$

$$= \frac{V_{t2}}{12.56}$$

$$\tan 20^\circ = \frac{V_{t2}}{12.56}$$

$$V_{t2} = 12.56 \times \tan 20^\circ$$

$$= 4.56 \text{ m/s}$$

Delivery head :- (m)

Static head (m) :-

It is the sum of

Heads :-

- (i) Suction Head (h_s)
- (ii) Delivery head (h_d)
- (iii) Manometric head (h_m)

1. Suction head (h_s) :-

It is the vertical height of the centreline of the centrifugal pump above the water surface in the water tank or the Sump from which water is to be lifted this height is called Suction lift and is denoted by (h_s).

Delivery head :- (h_d)

The vertical distance between the centre line of the pump and the water surface in the tank to which water is delivered is called as delivery head.

Static head (h_s') :-

It is the sum of the Suction head and delivery head.

$$h_s' = h_s + h_d$$

Manometric head :-

The manometric head is defined as the head against which a centrifugal pump has to work. it is denoted by (h_m)

(a) h_m = head imparted by the impeller to the water + the loss of head in the pump.

$$= \frac{V_{w2} V_2}{g} - \text{loss of head, in impeller + casing.}$$

(b) h_m = The total head at the outlet of the pump - The total head at the inlet of the pump

$$= \left(\frac{P_o}{\rho g} + \frac{V_o^2}{2g} + Z_o \right) - \left(\frac{P_i}{\rho g} + \frac{V_i^2}{2g} + Z_i \right)$$

$$(c) \quad h_m = h_s + h_d + h_{fs} + h_{fd} + \frac{V_d^2}{2g}$$

where,

h_s = Suction head

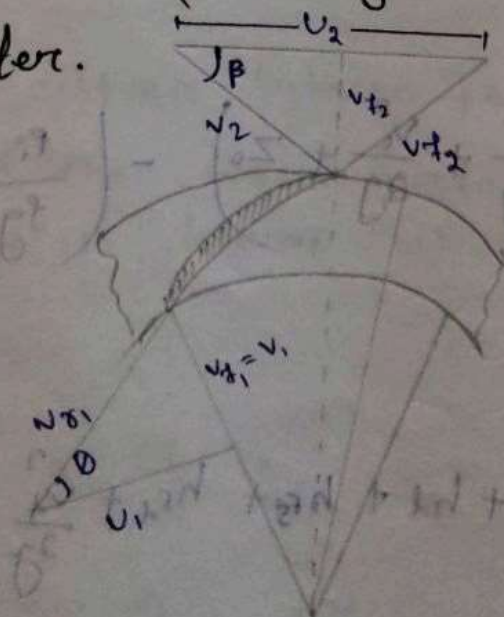
h_d = Delivery head

h_{fs} = Friction head loss in the Suction Pipe

h_{fd} = Friction head loss in the delivery pipe.

V_d = velocity of water in the delivery pipe

Q The internal and external diameters of the impeller of a Centrifugal pump are 200 mm and 400 mm respectively. The pump is running at 1200 rpm. The vane angles at inlet and outlet are 20° and 80° respectively. The water enters the impeller radially and the velocity of flow is constant. determine the work done by the impeller per unit weight of water.



$$\theta = 20^\circ \phi 30^\circ$$

$$V_{t1} = V_{t2} = \text{Constant}$$

$$D_1 = 200 \text{ mm} \approx 0.2 \text{ m}$$

$$D_2 = 400 \text{ mm} \approx 0.4 \text{ m}$$

$$N = 1200 \text{ rpm}$$

$$V_1 = 12.57 \text{ m/s}$$

$$V_2 = 25.13 \text{ m/s}$$

$$V_{t1} = V_{t2} = 4.57 \text{ m/s}$$

from the outlet velocity triangle

$$\tan \phi = \frac{V_{t2}}{V_2 - V_{w2}}$$

$$\Rightarrow \tan 30^\circ = \frac{4.57}{12.57 - V_{w2}}$$

$$\Rightarrow \tan 20^\circ = \frac{4.57}{12.57 - V_{w2}}$$

$$\Rightarrow 0.57 \times 4.57 = 12.57 - V_{w2}$$

$$\Rightarrow 0.57 \times 12.57 - V_{w2} = 4.57$$

$$\Rightarrow V_{w2} = \frac{4.57 - 0.57 \times 25.13}{-0.57} = 17.11 \text{ m/s}$$

work done by impeller per kg of water per

$$\text{Second} = \frac{1}{g} \cdot V_{w2} U_2$$

$$= \frac{1}{9.81} \times 17.11 \times 2513 = 43.83 \text{ Nm}$$

Q A Centrifugal pump is to discharge $0.118 \text{ m}^3/\text{s}$ at a speed of 1450 rpm against a head of 25 m . the impeller diameter is 250 mm , and the width at the outlet is 50 mm and the manometric efficiency is 75% . determine the vane angle at the outlet periphery of the impeller

$$\frac{U_2}{V_{w2}} = \frac{1}{\tan \theta}$$

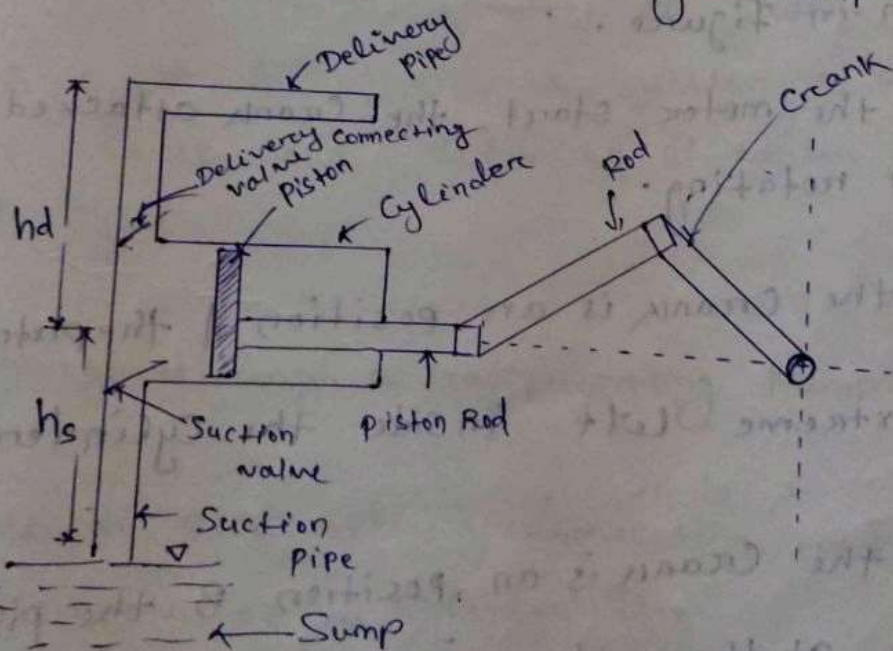
$$\frac{F_2 \cdot R}{U_2 V_{w2}} = \frac{1}{\tan \theta}$$

$$\frac{F_2 \cdot R}{U_2 V_{w2}} = \frac{1}{\tan \theta}$$

$$\tan \theta = \frac{F_2 \cdot R}{U_2 V_{w2}}$$

Reciprocating Pump :- If the mechanical energy is used to convert into hydraulic energy or (Pressure energy) by shaking the liquid into a cylinder in which a piston is reciprocating. (Moving forward & backward) which exerts the thrust on liquid and increases its hydraulic energies the pump is called a reciprocating pump.

Main Parts of Reciprocating Pump :-



Single
vacuum pressure created inside the

- (1) A cylinder with piston rod connecting rod & crank
- (2) Suction pipe
- (3) Delivery pipe
- (4) Suction valve
- (5) Delivery valve

Working Principle of Reciprocating Pump:-

The crank is attached to the shaft of motor as shown in figure.

when the motor starts, the crank attached to it starts rotating.

when the crank is at position A the piston will move to extreme left inside the cylinder.

when the crank is at position B the piston will have at the mid way inside the cylinder.

when the crank is at position C the piston will move to extreme right inside the cylinder.
the total distance covered by the piston inside the

Crank

$$\text{Cylinder } l = 2\pi r$$

where,

r is the length of the crank as shown in Fig.

When crank is at position C a vacuum is created inside the cylinder whose pressure is less than the atmospheric pressure acting on the surface of water in the sump. This causes suction of water from the sump into inside the cylinder by opening of the suction valve.

When the crank will move to position A it forces the water inside the cylinder to go out through the delivery pipe by opening the delivery valve.

So for one complete rotation of crank ~~there~~ i.e.

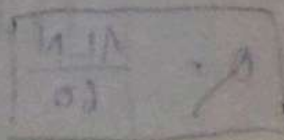
A, B, C, D, A, there is suction of water in half cycle and delivery of water in another half cycle.

now
Discharge through a reciprocating Pump:-

Consider a single acting reciprocating pump in which the diameter of the cylinder D .

$$\text{Area of cross-section of piston or cylinder} = \frac{\pi}{4} \times d^2$$

$$\text{radius of the crank} = r$$



N = rpm of the crank

L = Length of the Stroke (m)

h_s $\rightarrow$ height of the axis of the Cylinder from the water Surface in Sump
 $\rightarrow$ it is also called Suction head

h_d = height of the delivery outlet above the axis of the Cylinder.

$\rightarrow$ it is also called delivery head

In one Complete revolution the volume of water discharged = Area $\times$ length of the stroke
 $= A \times L$

No of Revolution per second = $\frac{N}{60}$

Discharge of the pump per second = Q

Q = volume $\times$ No. of revolution per second

$$Q = A \times L \times \frac{N}{60}$$

$$Q = \frac{A L N}{60}$$

Workdone per second = the weight of water delivered per second = w

$$w = \rho g Q$$

$$w = \frac{\rho g A L N}{60}$$

Workdone by the reciprocating pump :-

$$\text{Workdone per second} = \frac{\rho g A L N}{60} \times (h_s + h_d)$$

Power required to drive the pump in kW.

$$P = \frac{\text{Workdone per second}}{1000}$$

$$P = \frac{\rho g A L N}{60 \times 1000} \times (h_s + h_d)$$

$$P = \frac{\rho g A L N}{60000} \times (h_s + h_d)$$

Classification of reciprocating Pump :-

(a) according to the water being in contact with one side or both sides of the piston.

(i) Single acting Pump

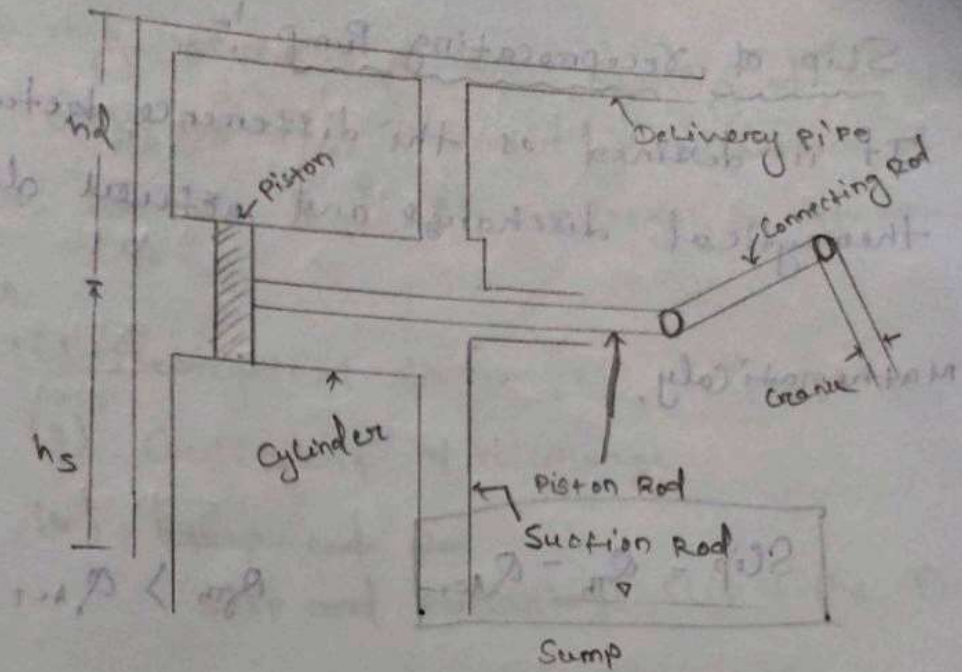
(ii) double acting Pump

(b) according to the no of cylinders provided,

(i) Single Cylinder pump

(ii) double Cylinder Pump

(iii) triple Cylinder Pump



Discharge of Double acting reciprocating pump

$$Q = \frac{2ACLN}{60}$$

$$\text{Work done per Second} = \frac{2 \rho g A C N (h_s + h_d)}{60}$$

$$\text{Power in kW} = \frac{2 \rho g A C N (h_s + h_d)}{60000}$$

Slip of Reciprocating Pump :-

It is defined as the difference between the theoretical discharge and actual discharge

Mathematically,

$$\text{Slip} = Q_{th} - Q_{act}$$

$$Q_{th} > Q_{act}$$

Percentage Slip

$$= \frac{Q_{th} - Q_{act}}{Q_{th}} \times 100$$

$$Q_{th} < Q_{act}$$

$$Q_{th} < Q_{act}$$

In this case $\therefore$ slip will be Negative called Negative slip

Negative slip occurs when the suction pipe is long and delivery pipe is short and the pump is running at high speed.

A single acting reciprocating Pump Running at 50 rpm delivers $0.01 \text{ m}^3/\text{sec}$ of water the diameter of the piston is 200 mm and stroke length is 400 mm. determine

(i) Theoretical discharge

(ii) Coefficient of discharge

(iii) ~~Slip and~~ per

Slip and Percentage Slip of the Pump.

$$Q_{\text{act}} = 0.01 \text{ m}^3/\text{sec}$$

$$D = 200 \text{ mm} \\ = 0.2 \text{ m}$$

$$\text{Speed of pump} = N = 50 \text{ rpm}$$

$$\text{Stroke Length } (L) = 400 \text{ mm} = 0.4 \text{ m}$$

$$\text{Area of piston} = A = \frac{\pi}{4} \times 0.2^2 \\ = 0.031$$

$$Q_{\text{Th}} = \frac{A L N}{60} \\ = \frac{0.031 \times 0.4 \times 50}{60} = 0.0103 \text{ m}^3/\text{s}$$

(ii) Coefficient of discharge =

$$C_d = \frac{Q_{Act}}{Q_{th}}$$

$$= \frac{0.0103}{0.01} = 1.03$$

$$= 0.97$$

(iii) Slip = $Q_{th} - Q_{Act}$

$$= 0.0103 - 0.01$$

$$= 0.0003 \text{ m}^3/\text{s}$$

(iv) Percentage slip

$$= \frac{Q_{th} - Q_{Act}}{Q_{th}}$$

$$= \frac{0.0003}{0.0103} \times 100$$

$$= 2.91 \%$$

$$\% \text{ slip} = (1 - C_d) \times 100$$

A double acting reciprocating pump is running at 40 rpm is discharging 1 m^3 of water per min. the pump has a stroke of 400 mm. the diameter of the piston is 200 mm. the delivery and suction head are 20 m and 5 m. respectively. Find the slip of the pump and the power required to drive the pump.

$$Q_{\text{act}} = 1 \text{ m}^3/\text{min}$$

$$D = 200 \text{ mm} \\ = 0.2 \text{ m.}$$

$$\text{Stroke of pump} = 400 \text{ mm} \\ L = 0.4 \text{ m.}$$

$$N = 40 \text{ rpm}$$

$$\text{delivery head} = 20 \text{ m}$$

$$\text{suction head} = 5 \text{ m.}$$

Sol

$$\text{Area of piston} = \frac{\pi}{4} \times 0.2^2$$

$$= 0.031$$

$$Q_{act} = \frac{1 \text{ m}^3}{60}$$

$$= \frac{1}{60} \text{ m}^3/\text{sec}$$

$$Q_{th} = \frac{2 A L N}{60}$$

$$= \frac{2 \times 0.031 \times 0.4 \times 40}{60}$$

$$= 0.0165$$

$$\text{Slip} = Q_{th} - Q_{act} = 0.0165 - \frac{1}{60}$$

$$= 0.0001$$

Power required to run the pump

$$= \frac{2 \rho g A L N (h_s + h_d)}{60000}$$

$$= \frac{2 \times 1000 \times 9.81 (0.031 \times 0.4 \times 40 (5 + 20))}{60000}$$

$$= 4.05$$

Indicator diagram :-

It is a graph between the pressure head in the cylinder and the distance traveled by the piston from the inner dead centre for one complete revolution of the crank.

In the graph, the pressure head is taken as ordinate (along y axis) and stroke length is taken as abscissa (along x axis).

The maximum distance travelled by the piston
= the stroke length

The ~~diff~~

Centrifugal pump	Reciprocating Pump
The discharge is continuous and smooth.	The discharge is fluctuating and pulsating.
It can handle large quantity of liquid.	It can handle small quantity of liquid.
It can be used for highly viscous liquid.	It can be used for less viscous liquid as water.
It can be used for large discharges through small heads.	It can be used for small discharges with high head.

the Cost is less

It runs at high speed

The operation is smooth
without much noise.

Its efficiency is high

Its installation and
maintainance Cost is low

the cost is 4 times than
that of Centrifugal Pump

It runs at low Speed.

The operation is
complicated and
produces more noise.

Its efficiency is low.

The installation and
maintainance is high