Control System

Control System -: A system which consist of a number of components connected together to perform a specific function, or task in which the output is controlled by the input is known as control system.

Ex-: Control of a room temperature is achieved by switching ON and switching OFF of a power supply to a heating appliance.

* When the room temperature is low, we can switch ON the power supply of heating appliance & switched off, when the desired temperature is reached.
* The above system can be modified, if the duration of application of power is predetermined to achieve the room temperature within desired limits.
* Error -: It is the difference betn actual value and desired output value.

Feed back component -: This component feed back from the o/p to compare it with the reference i/p.

* It is of two types.

a) Open loop control system -:

$\xrightarrow{i/p}$ $\boxed{\text{System}}$ $\xrightarrow{o/p}$

(Open loop control system)

* The open loop control system is that type of control system in which the output signal has no influence or effect on the control action of the input signal.
* Here i/p signal is directly connected to o/p signal via a ~~control~~ system.

Ex-: Traffic control system by glowing of light, washing machine, light switches

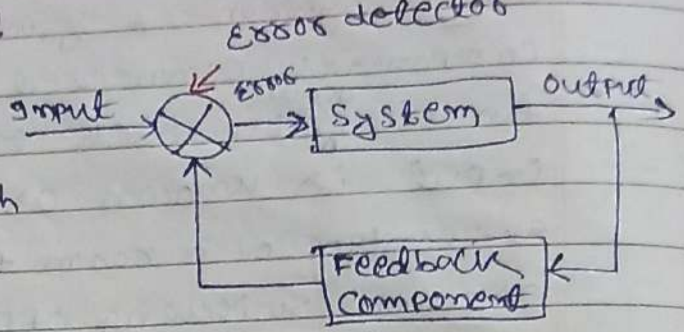
Advantages & disadvantages of open
b) ~~Closed~~ loop control system -:

Advantages -: 1. Simple & Economical.
2. Easier to construct.
3. Stable System.

Disadvantages -: 1. Inaccurate due to noise.
2. Changes in the output are not corrected automatically.

b> Closed loop control system -:

→ The closed loop control system is that type of control system ~~the~~ in which the output signal has an effect on the control action of the input signal via a feedback component.



Ex-: traffic controlled system by a police man.

Advantages & disadvantages -:

a> more Accurate.

b> less effected by noise

Disadvantage

1. complex & costiler
2. Feed back reduces the overall gain of the system.
3. stability is a main problem.

Difference betn open loop & closed loop system -:

open loop	Closed loop
* Feed back component is absent.	* feedback component is present.
* It is simple & easy to construct	* It is complex & difficult to construct.
* Less no. of components is used in this type of system.	* No. of components are more as compared to open loop system.
* The power requirement for this system is less.	* But for closed loop system power requirement is more.
* Installation cost is less for this type of system.	* The installation cost of closed loop system is more as compared to open loop system.

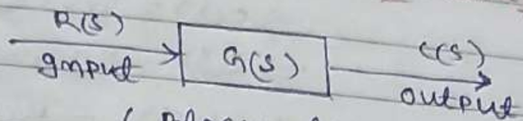
Block diagram -:

* In a control system many components are used, each component is represented by a block diagram.
* Generally a block represents the transfer function of the component.

Transfer function -:

* Transfer function is defined as the ratio of the Laplace transform of the output to the input, when all initial conditions of the system is zero.

Transfer function of open loop Control System :-



(Block diagram representation)

* Transfer function $G(S) = \frac{C(S)}{R(S)}$

$$\Rightarrow C(S) = G(S) \cdot R(S)$$

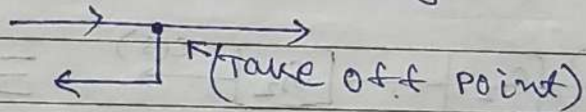
where $G(S)$ = Transfer function

$C(S)$ = Laplace transform of output variable.

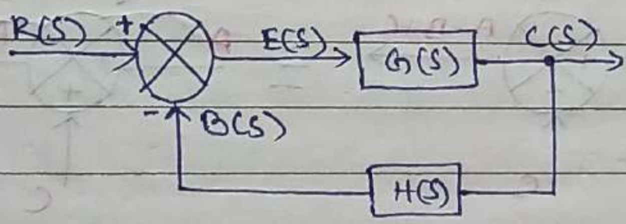
$R(S)$ = Laplace transform of input variable.

* The flow of system variables from one block to another block is represented by the arrow.
* The summing or differencing of signals is indicated by the symbol

* The "take off" point of a signal is represented by



Transfer function of closed loop control system :-



$R(S)$ = Reference i/p signal

$C(S)$ = output signal

$E(S)$ = error signal

$B(S)$ = feedback signal

$G(S)$ = forward path transfer function.

$H(S)$ = Feedback path transfer function.

$G(S)\,H(S)$ = Loop transfer function.

$M(S) = \frac{C(S)}{R(S)}$ = closed loop transfer function.

* From above figure, $E(S) = R(S) - B(S)$ — (i)

$B(S) = C(S) \cdot H(S)$ — (ii)

So $E(S) = R(S) - C(S) \cdot H(S)$ — (iii)

~~From open loop control system~~ $C(S) = G(S) \cdot E(S)$

$$\Rightarrow C(S) = G(S)\,[R(S) - C(S) \cdot H(S)]$$

$$\Rightarrow C(S) = R(S) \cdot G(S) - C(S) \cdot G(S) \cdot H(S)$$

$$\Rightarrow C(S) + C(S) \cdot G(S) \cdot H(S) = R(S) \cdot G(S)$$

$$\Rightarrow C(S)\,(1 + G(S) \cdot H(S)) = R(S) \cdot G(S)$$

$$\Rightarrow \boxed{M(S) = \frac{C(S)}{R(S)} = \frac{G(S)}{1 + G(S) \cdot H(S)}}$$

$$R(S) \longrightarrow \boxed{\frac{G(S)}{1 + G(S) \cdot H(S)}} \xrightarrow{C(S)}$$

(when the feedback is −ve)

(Reduced block diagram)

* when the feedback is +ve.

$$\boxed{M(S) = \frac{C(S)}{R(S)} = \frac{G(S)}{1 - G(S) \cdot H(S)}}$$

* so, in general $\boxed{M(S) = \frac{C(S)}{R(S)} = \frac{G(S)}{1 \pm G(S) \cdot H(S)}}$

Servomechanism -:

→ Automatic control of any physical quantity (position, velocity, displacement) is called servomechanism.

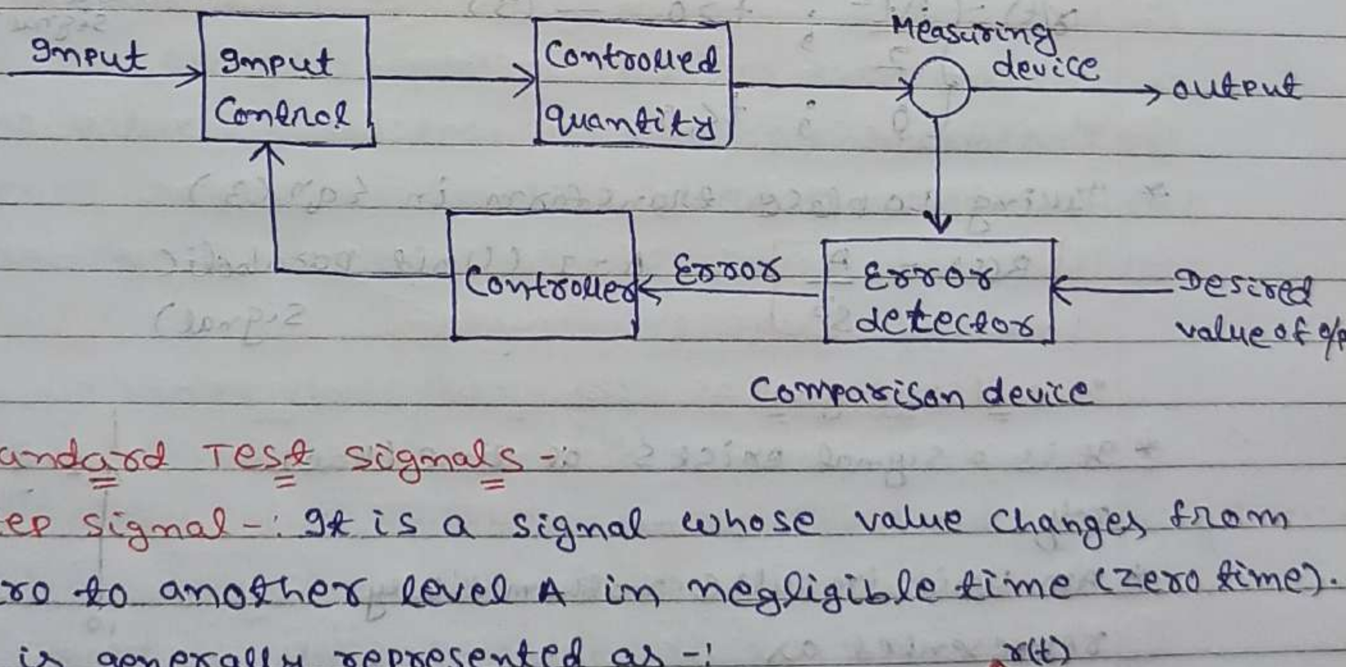
→ The word servo means controlling mechanical position or derivatives of position like velocity and acceleration.

→ It is an automatic device that uses the error sensing negative feedback to correction of the performance of mechanism.

→ A servodrive is a special electric amplifier used ~~to~~ for power electric servomechanisms.

→ Servo mechanism used negative fedback to control mechanical position.

→ It is used in automatic machine tools, statellite tracking antenna, Air craft system and Navigation system etc.



Comparison device

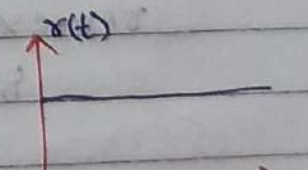
Standard Test signals -:

a) Step signal -: It is a signal whose value changes from zero to another level A in negligible time (zero time).

* It is generally represented as -:

$$r(t) = A\,u(t) \quad -(1)$$

where $u(t) = 1 ; t > 0$
$\quad\quad\quad\;\; = 0 ; t < 0$ (A = 1, for unit step signal)

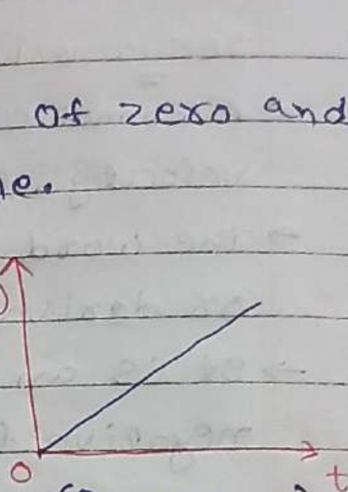


(Step signal)

* u(t) represents unit step function.
* Taking Laplace transform of Eqn (1) $R(S) = \frac{A}{S}$

Ramp Signal (constant velocity signal) :-

→ It is a signal which starts at a value of zero and increases linearly with respect to time.



(Ramp signal)

* It is generally represented mathematically as

$$r(t) = \begin{cases} At ; & t > 0 \\ 0 ; & t < 0 \end{cases} \quad \text{— (2)}$$

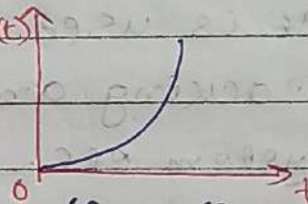
* Taking Laplace transform in Eqn (2)

$$\boxed{R(s) = \frac{A}{s^2}}$$

A = 1 (Unit ramp Signal)

Parabolic Signal (acceleration signal)

→ It is a signal which looks like a parabola and starts from zero.



(Parabolic signal)

* It is generally represented mathematically as

$$r(t) = \begin{cases} \frac{At^2}{2} ; & t > 0 \\ 0 ; & t \leq 0 \end{cases} \quad \text{— (3)}$$

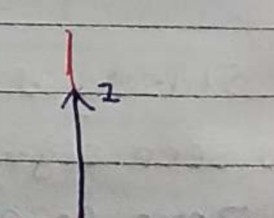
* Taking Laplace transform in Eqn (3)

$$\boxed{R(s) = \frac{A}{s^3}}$$

A = 1 (Unit parabolic Signal)

Impulse signal -

* It is a signal exists at time (t = 0) only.



* An Unit impulse mathematically represented as

$$\delta(t) = \begin{cases} 1 ; & t = 0 \\ 0 ; & t \neq 0 \end{cases} \quad \text{— (4)}$$

(Unit impulse signal)

* An impulse signal is the derivative of step signal

$$\delta(t) = \frac{d}{dt} u(t)$$

* taking Laplace transform of Eqn (4)

$$R(s) = \mathcal{L}^{-1} [\delta(t)] = 1$$

CP-2

Mathematical model of a system

Poles and zeros of a transfer function -:

* The transfer function of a control system can be written as

$$G(s) = \frac{C(s)}{R(s)} = \frac{a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + \cdots + a_n}{b_0 s^m + b_1 s^{m-1} + b_2 s^{m-2} + \cdots + b_m} \quad -(1)$$

* The numerator and the denominator can be factorized in terms of 'n' & 'm' respectively are given below.

$$G(s) = \frac{C(s)}{R(s)} = \frac{K(s-z_1)(s-z_2)\cdots\cdots(s-z_n)}{(s-p_1)(s-p_2)\cdots\cdots(s-p_m)} \quad -(2)$$

where $\boxed{K = \frac{a_0}{b_0}}$ is known as the gain factor of the transfer function.

Poles of the transfer function -:

* In the transfer function if 's' is put equal to $p_1, p_2 \cdots p_m$, the value of the transfer function becomes infinite.
* Hence $p_1, p_2 \cdots p_m$ are called the poles of the transfer function.

Zeros of the transfer function -:

* In the transfer function if 's' is put equal to $z_1, z_2 \cdots z_n$, the value of the transfer function becomes zero.
* Hence $z_1, z_2 \cdots z_n$ are called the zeros of the transfer function.

Impulse Response of Transfer function -:

* As we know that $G(s) = \frac{C(s)}{R(s)}$

$\Rightarrow C(s) = G(s) \cdot R(s) \quad -(1)$

* The output time response can be found out by taking inverse Laplace transform. ~~of eqn~~
* If input is unit impulse, so $R(s) = 1$

then $C(s) = G(s) \cdot 1$

$\Rightarrow C(s) = G(s) \longrightarrow (2)$

Taking inverse laplace transform of Eqn (2)

$c(t) = g(t) \quad —(3)$

* The inverse laplace transform of $G(s)$ is called the impulse response of a system.

Properties of transfer function -:

① Mathematical model of a transfer function expressing the differential equation that relates the output to the input of the system.

② It is independent of magnitude and the nature of input.

③ It does not provide any information about the physical structure of the system.

④ If the transfer function of a system is known, the output response can be studied for various inputs to understand the nature of the system.

Advantages of transfer function -:

① If the transfer function of a system is known, the response of the system to any input can be determined easily.

② A transfer function gives the mathematical model and ~~the~~ gain of the system.

③ The transfer function involves simple algebric expression and no differential term.

④ Poles and zeros of the system can be determine from the transfer function.

Disadvantages of transfer function -:

① Transfer function does not take any initial conditions into the account.

② It can be defined for linear system.

③ It is applicable for SISO (single input single output) system.

④ From transfer function, we can't determine the physical structure of the system.

Q) Determine the poles and zeros of the given transfer function

$$G(s) = \frac{(2s+6)(s+2)}{s(s+1)(s+4)}$$

Ans -: Poles -: $\boxed{s=0}$

$s+1=0 \Rightarrow \boxed{s=-1}$

$s+4=0 \Rightarrow \boxed{s=-4}$

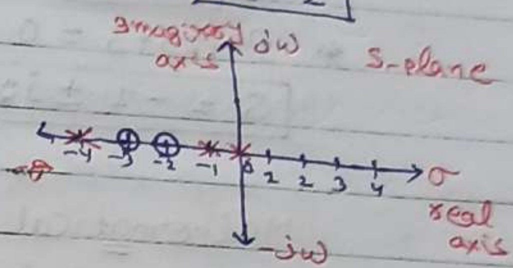
Zeros -: $2s+6=0$

$\Rightarrow 2s=-6$

$\Rightarrow \boxed{s=-6/2=-3}$

$s+2=0 \Rightarrow \boxed{s=-2}$

Note -: $s = \underbrace{\sigma}_{\text{Real part}} + \underbrace{j\omega}_{\text{imaginary part}}$



Q) For the transfer function

$$G(s) = \frac{1}{2} \cdot \frac{(s^2+4)(1+2.5s)}{(s^2+2)(1+0.5s)}$$

Plot the poles and zeros in s-plane and determine the value of the transfer function at $s=2$.

Ans: The poles are determined from the eqn

$(s^2+2)(1+0.5s)=0$

* $s^2+2=0$

$\Rightarrow s^2=-2 \Rightarrow \boxed{s=\pm j\sqrt{2}}$

* $1+0.5s=0$

$\Rightarrow 0.5s=-1 \Rightarrow \boxed{s=-1/0.5=-2}$

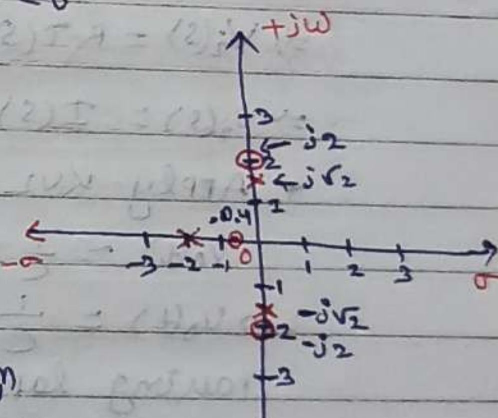
The zeros are determined from the eqn

$(s^2+4)(1+2.5s)=0$

* $s^2+4=0 \Rightarrow s^2=-4 \Rightarrow \boxed{s=\pm j2}$

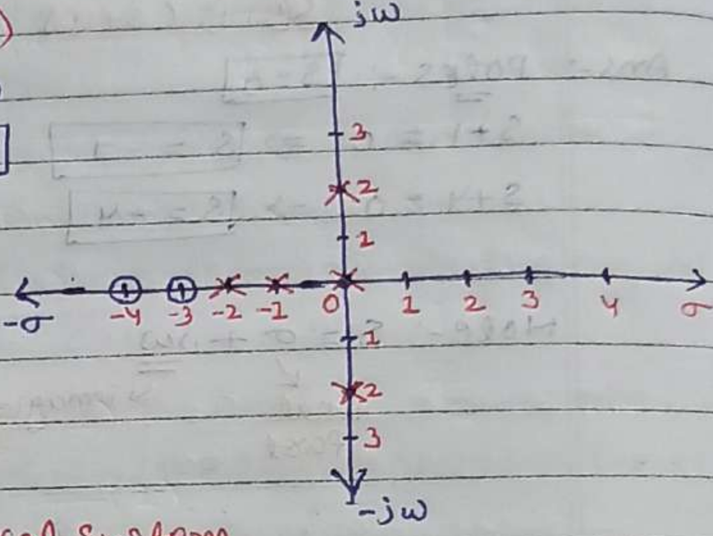
* $1+2.5s=0$

$\Rightarrow 2.5s=-1 \Rightarrow \boxed{s=-1/2.5=-0.4}$



Q// Determine the poles & zeros of transfer function.

$$G(S) = \frac{8(S+3)(S+4)}{S(S+2)^2(S^2+2S+3)}$$



Ans:- Poles $\boxed{S=0}$

* $(S+2)^2 = 0$

$\Rightarrow S+2 = 0$

$\Rightarrow \boxed{S=-2}$

* $S^2+2S+3 = 0$

$\boxed{S = -1 \pm j2}$

zeros $S+3=0$

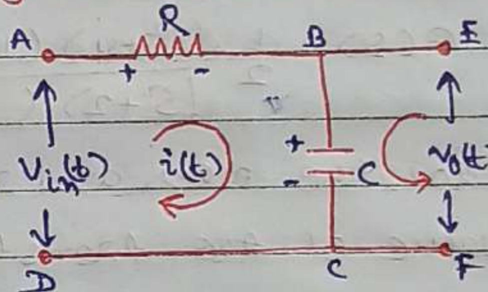
$\Rightarrow \boxed{S=-3}$

$S+4=0$

$\Rightarrow \boxed{S=-4}$

Mathematical Model of electrical system

Q// Find the transfer function of given network.



Ans:- * Apply KVL in mesh ABCDA

$$V_i(t) - R\,i(t) - \frac{1}{C}\int i(t).dt = 0 \quad -(1)$$

Taking laplace transform of Eqn (1)

$$V_i(S) - R\,I(S) - \frac{1}{C}\cdot\frac{I(S)}{S} = 0$$

$$\Rightarrow V_i(S) = R\,I(S) + \frac{1}{CS} I(S)$$

$$\Rightarrow V_i(S) = I(S)\left[R + \frac{1}{CS}\right] \quad -(2)$$

* Apply KVL in mesh EBCFE

$$V_0(t) - \frac{1}{C}\int i(t).dt = 0 \quad -(3)$$

$$\Rightarrow V_0(t) = \frac{1}{C}\int i(t).dt$$

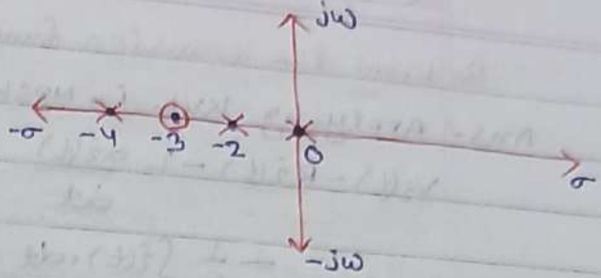
Taking laplace transform

$$V_0(S) = \frac{1}{C}\cdot\frac{I(S)}{S} \quad -(4)$$

* Then $G(S) = \dfrac{V_0(S)}{V_i(S)} = \dfrac{\frac{1}{CS}\cdot \cancel{I(S)}}{\cancel{I(S)}\left[R + \frac{1}{CS}\right]} = \dfrac{\frac{1}{CS}}{\frac{RCS+1}{CS}}$

$$\Rightarrow \boxed{G(S) = \frac{V_0(S)}{V_i(S)} = \frac{1}{1+RCS}}$$

Q// The value of the transfer function at $s=1$ is found to be 3.2. Determine the transfer function G(S) and Gain factor (K).



Ans-: zeros -:

$S=-3$

$\Rightarrow \boxed{S+3=0}$

Poles -:

$\boxed{S=0}$, $S=-2$, $S=-4$

$\Rightarrow \boxed{S+2=0}$ $\Rightarrow \boxed{S+4=0}$

$$G(S) = \frac{K(S+3)}{S(S+2)(S+4)}$$

$$\Rightarrow 3.2 = \frac{K(1+3)}{1(1+2)(1+4)}$$

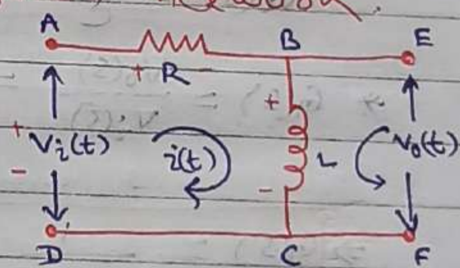
$$\Rightarrow 3.2 = \frac{4K}{15} \Rightarrow K = \frac{3.2 \times 15}{4}$$

$\Rightarrow \boxed{K=12}$ (Gain factor)

∴ The original transfer function

$$\boxed{G(S) = \frac{12(S+3)}{S(S+2)(S+4)}}$$

Q// Find the transfer function of given network.



Ans-: Apply KVL in mesh ABCDA

$$V_i(t) - R\,i(t) - L\frac{di(t)}{dt} = 0 \quad \text{— (1)}$$

Taking laplace transform of Eqn (1)

$$V_i(S) - R\,I(S) - SL\,I(S) = 0$$

$$\Rightarrow V_i(S) = R\,I(S) + SL\,I(S)$$

$$\Rightarrow V_i(S) = I(S)\,[R+SL] \quad \text{— (2)}$$

* Apply KVL in mesh EBCFE

$$V_0(t) - L\frac{di(t)}{dt} = 0$$

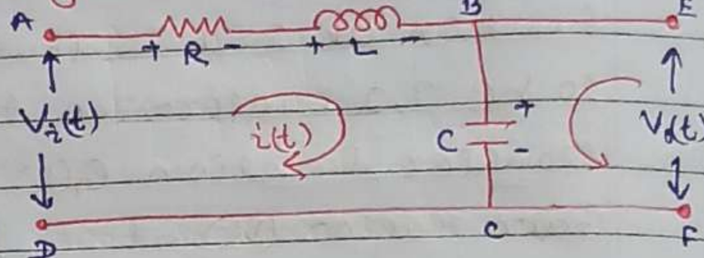
$$\Rightarrow V_0(t) = L\frac{di(t)}{dt} \quad \text{—— (3)}$$

Taking laplace transform

$$\Rightarrow V_0(S) = SL\,I(S) \quad \text{—— (4)}$$

$$G(S) = \frac{V_0(S)}{V_i(S)} = \frac{SL\,\cancel{I(S)}}{\cancel{I(S)}\,[R+SL]} = \frac{SL}{R+SL} \quad \text{(Ans)}$$

Q1/ Find the transfer function of given network.



Ans-: Applying KVL in Mesh ABCDA

$$V_i(t) - R\,i(t) - L\frac{di(t)}{dt} - \frac{1}{C}\int i(t)\cdot dt = 0 \quad -(1)$$

Taking Laplace transform function of Eqn (1)

$$V_i(S) - R\,I(S) - SL\,I(S) - \frac{1}{C}\cdot\frac{I(S)}{S} = 0$$

$$\Rightarrow V_i(S) = R\,I(S) + SL\,I(S) + \frac{1}{CS}I(S)$$

$$\Rightarrow V_i(S) = I(S)\left[R + SL + \frac{1}{CS}\right] \quad -(2)$$

* Apply KVL in mesh EBCFE

$$V_0(t) - \frac{1}{C}\int i(t)\cdot dt = 0$$

$$\Rightarrow V_0(t) = \frac{1}{C}\int i(t)\cdot dt \quad -(3)$$

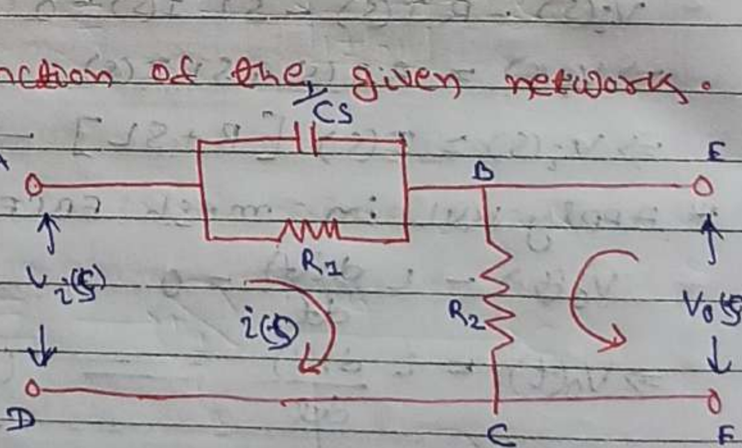
Taking Laplace transform function of Eqn (3)

$$V_0(S) = \frac{1}{C}\cdot\frac{I(S)}{S} \quad -(4)$$

* $$G(S) = \frac{V_0(S)}{V_i(S)} = \frac{\frac{1}{CS}\cdot \cancel{I(S)}}{\cancel{I(S)}\left[R + SL + \frac{1}{CS}\right]} = \frac{\frac{1}{\cancel{CS}}}{\frac{RCS + S^2LC + 1}{\cancel{CS}}}$$

$$\Rightarrow \boxed{G(S) = \frac{1}{RCS + S^2LC + 1}}$$

Q2 Derive the transfer function of the given network.



Ans-: By applying KVL in Mesh ABCDA

$$V_i(S) - I(S)\left[\left[\frac{R_1\cdot\frac{1}{CS}}{R_1 + \frac{1}{CS}}\right] + R_2\right] = 0 \quad -(1)$$

$$\Rightarrow V_i(S) = \left[\frac{R_1/\cancel{CS}}{\frac{R_1CS + 1}{\cancel{CS}}} + R_2\right]I(S) \Rightarrow V_i(S) = \left[\frac{R_1}{1 + R_1CS} + R_2\right]I(S) \quad -(2)$$

* Apply KVL in mesh EBCFE

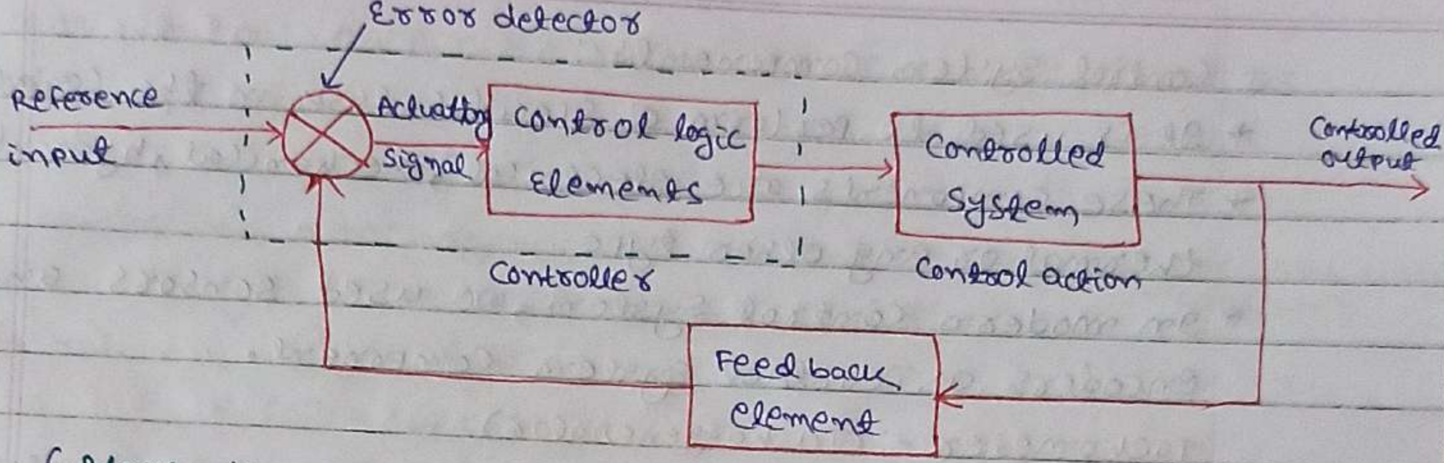
$V_0(s) - R_2 I(s) = 0 \Rightarrow V_0(s) = R_2 I(s)$

* $G(s) = \dfrac{V_0(s)}{V_i(s)} = \dfrac{R_2 \cancel{I(s)}}{\left[\dfrac{R_1}{1+R_1Cs} + R_2\right]\cancel{I(s)}} = \dfrac{R_2(1+R_1Cs)}{R_1 + R_2(1+R_1Cs)}$

$\Rightarrow G(s) = \dfrac{R_2 + R_1R_2Cs}{R_1 + R_2 + R_1R_2Cs}$ (Ans)

CP-3

Control System Components



(Block diagram representation of closed loop control System)

* In this type of control System, there are three basic elements. a) Feedback element b) controller c) control system

Feedback element -:

* The feedback element is used to feedback the output signal to the error detector for comparision with the input signal.

Controller -:

* controller is consist of error detector and control logic elements.

Error detector -:

* It receive the feedback/output signal and compare it with the reference input signal. So that the error can be determine and we can get the actuating signal.

Actuating signal-:

* It is a low power level signal which is not sufficient to operate the control system.
* There fore, we need an intermediate device between error detector & control system, which can manipulate the actuating signal as desired level.
* To achieve the desired level value, we can use the amplification process.
* By the help of control logic element or control system component, the manipulation of signal is done.

Control System Components -:

* It is used to perform a specific task in the control system.
* These components are electrical, mechanical, hydraulic thermal or any other type.
* In modern control system, we used sensors and encoders as control system component.

Tachometer -: (Tacho-generator)

* Tachometer is nothing but a small motor and develop an output voltage proportional to its shaft speed.
  ($V_0 \propto$ Shaft speed)
* To increase the performance of such system, generally tachometric feedback is used in control system.

Application of Tachometer -:

a) velocity control b) position control c) speed reading of machine

* The advantage of using this type of feedback is to control the control system component.
* This type of design are easy to made and inexpensive.

→ It is of two types.

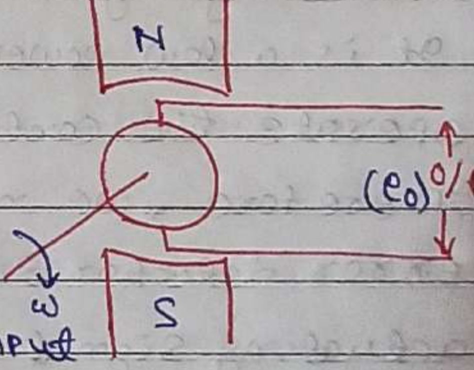
a) DC tachometer b) AC tachometer.

DC tachometer

→ A DC tachometer is generally a DC Generator.

→ It is used to convert the rotational speed into a proportional D.C o/p voltage.

→ It uses a permanent magnet for producing magnetic field.



→ The tachogenerator is coupled to a shaft of which speed is to be measured.

⇒ The output of DC tachogenerator can be written as

$$e_0(t) \propto \omega(t)$$

$$\Rightarrow e_0(t) = K_T\,\omega(t) \quad \text{——(1)}$$

where K_T = tachogenerator constant (Tachogenerator sensitivity) V/(rad/sec)

ω = Angular Speed of the Shaft (rad/sec)

* Taking Laplace transform of Eqn (1)

$$E_0(S) = K_T\,\omega(S)$$

$$\Rightarrow G(S) = \frac{E_0(S)}{\omega(S)} = K_T \quad \text{—(2)} \quad \text{(Transfer function)}$$

$$\omega(S) \longrightarrow \boxed{K_T} \longrightarrow E_0(S)$$

(Block diagram of d.c. tachogenerator)

Advantage -:

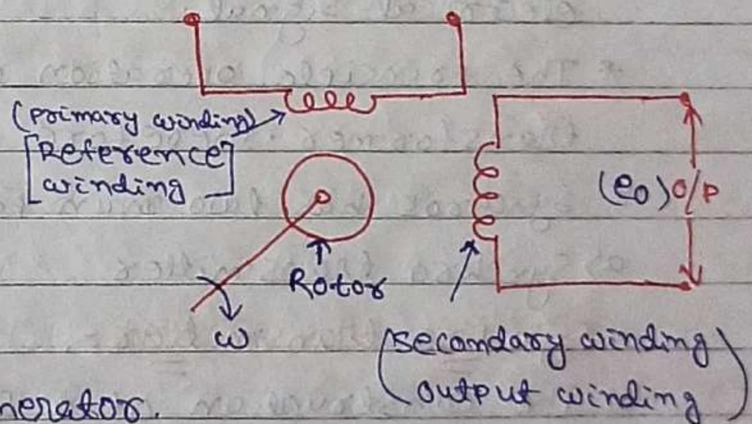
* It is possible to generate high voltage in small size machine
* There is no residual voltage present at zero Speed

Disadvantage -:

* Due to bad commutation, noise is generated by arcing at the brushes.

A.C. Tachogenerator -:

* An A.C. Tachogenerator is used to convert a rotational Speed into a proportional A.C voltage and works on the principle of induction generator.



* The tachogenerator is coupled to a Shaft, of which speed is to be measured.
* The reference winding is supplied by a reference voltage and the output voltage (e_0) is induced across the output winding.

* The amplitude or RMS value & the phase of the output voltage depends on the direction of rotation.
* The output voltage (e_0) is thus related to the shaft speed as follows -: $e_0(t) \propto \omega(t)$

$$\Rightarrow e_0(t) = K_T\,\omega(t) \quad -(1)$$

where K_T = Techogenerator constant V/(rad/sec)

Taking laplace transform of eqn (1)

$$E_0(s) = K_T\,\omega(s)$$

$$\Rightarrow G(s) = \frac{E_0(s)}{\omega(s)} = K_T \quad -(2)$$

$$\xrightarrow{\omega(s)} \boxed{K_T} \xrightarrow{E_0(s)}$$

(Block diagram of a.c tachogenerator)

* An a.c tachogenerator can also be used on d.c. systems by using a phase sensitive rectifier.

Synchros -:

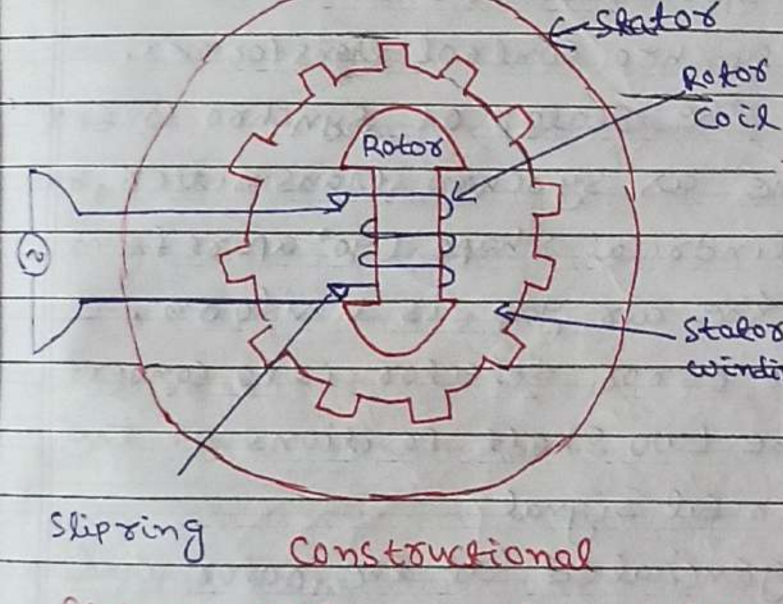
* A Synchro is an electromagnetic transducer which converts the angular position of a shaft into an electrical signal.
* The principle operation of a Synchro is same as a transformer. Therefore it is an a.c device.
* Synchros has two main parts.

a) Synchro transmitter b) Synchro Receiver (or detector)

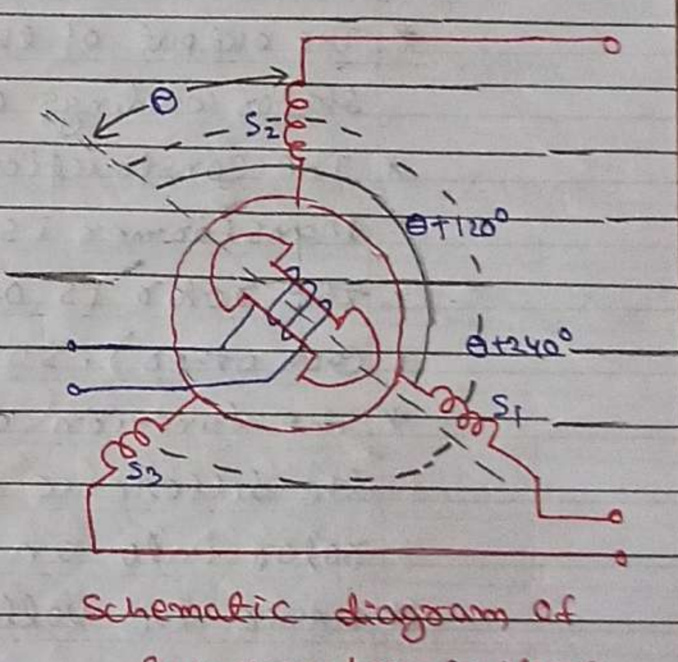
Synchro transmitter -:

* The construction of a Syncho-transmitter is similar to a three phase alternator.
* The stator is of laminated silicon steel in which a balanced three phase windings is connected in star fashion with 120° apart from each other.
* The rotor is of dumbbell shape and a coil is wound on the rotor.

* An a.c. voltage is applied to the rotor winding through slip rings.
* Let us assume that A.C input signal $v_s(t) = V_m \sin\omega t$ is applied to the rotor of the synchrotransmitter.
* This applied voltage produces a flux directed along its axis and distributed in the air gap along the stator periphery.
* Due to transformer action, voltages are induced in each of the stator coils.
* Here, the flux is linking to any stator coil which is proportional to the cosine angle between the rotor and stator coil axis & also the voltage induced in the stator coil.
* Here we can observe that the synchro-transmitter acting like a rotating single phase transformer in which the rotor coil is the primary and the stator coils are the secondaries.



Constructional features of Synchro transmitter



Schematic diagram of Synchro transmitter

* Let the voltage in the phase windings are $V_{S_1}, V_{S_2}, V_{S_3}$

$V_{S_2} = K V_m \sin\omega t \cdot \cos\theta$

$V_{S_1} = K V_m \sin\omega t \cdot \cos(120^\circ + \theta)$

$V_{S_3} = K V_m \sin\omega t \cdot \cos(240^\circ + \theta)$

* The three terminal voltages of the stator are

$V_{S_1S_2} = V_{S_1} - V_{S_2} = \sqrt{3}\, K V_m \sin(240^\circ + \theta) \cdot \sin\omega t$

$V_{S_2S_3} = V_{S_2} - V_{S_3} = \sqrt{3}\, K V_m \sin(120^\circ + \theta) \cdot \sin\omega t$

$V_{S_3S_1} = V_{S_3} - V_{S_1} = \sqrt{3}\, K V_m \sin\theta \cdot \sin\omega t$

* If $\theta = 0$, maximum phase voltage is in the stator coil (S_2) and terminal voltage $V_{S_3S_1}$ is zero.
* This is known as the "electrical zero" position of the transmitter & is used as reference for specifying the angular position of the rotor.
* It is also observed that the input to the synchro transmitter is the angular position of its rotor shaft and the output is a set of three single phase voltages.

Synchro Receiver (Error detector) :-

* A synchro receiver is a combination of synchro control transformer and synchro transmitter.
* The output of the synchro transmitter is applied to the stator windings of a synchro control transformer.
* The construction of the stator of synchro control transformer is same as synchro transmitter, but the rotor is of cylindrical shape (90° apart from each other). So that the air gap is uniform.
* The function of the error detector is to convert the difference of the two shaft positions of the rotor into an electrical signal.
* Hence the voltage generated at the rotor winding of the synchro control transformer is proportional to the cosine angle between the two shafts and is represented by $\boxed{e(t) = K' V_m \sin\omega t \cos\phi}$ —(1)
* where ϕ = Angular displacement between the two rotors.

* When $\phi = 90^\circ$ i.e. two rotors are at right angles, then the voltage induced is zero. This position is known as "Electrical zero" position of the control transformer.
* Let's assume that rotor of transmitter rotate by an angle θ in the direction indicated and control transformer rotor rotate by an angle α in the same direction.
* So, the net angular displacement between two rotors

$$\boxed{\phi = [90^\circ - (\theta + \alpha)]} \Rightarrow \boxed{\phi = (90^\circ - \theta - \alpha)} \quad \text{--- (2)}$$

Put Eqn (2) in Eqn (1)

$$e(t) = K' V_m \sin \omega t \cos(90^\circ - \theta - \alpha)$$

$$\Rightarrow \boxed{e(t) = K' V_m \sin \omega t \cdot \sin(\theta - \alpha)} \text{ --- (3) } (\because \cos(90^\circ - \theta) = \sin\theta)$$

* For small angular displacement

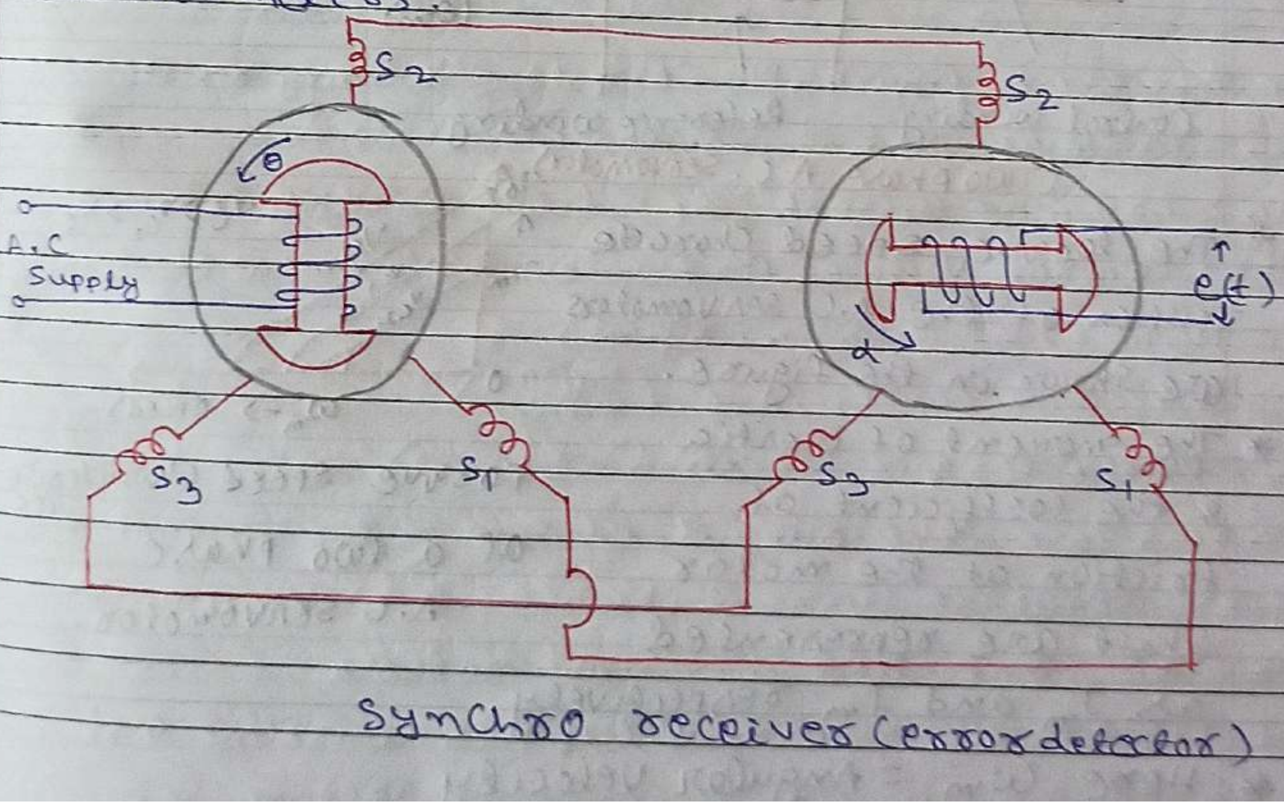
$$\boxed{e(t) = K' V_m \sin \omega t \cdot (\theta - \alpha)} \quad \text{--- (4)}$$

Advantages:-

a) Syncho system is highly reliable

b) The Synchro systems can be operated at much higher speeds than potentiometers.

c) The accuracy of syncho system is much higher than potentiometers.



Synchro receiver (error detector)

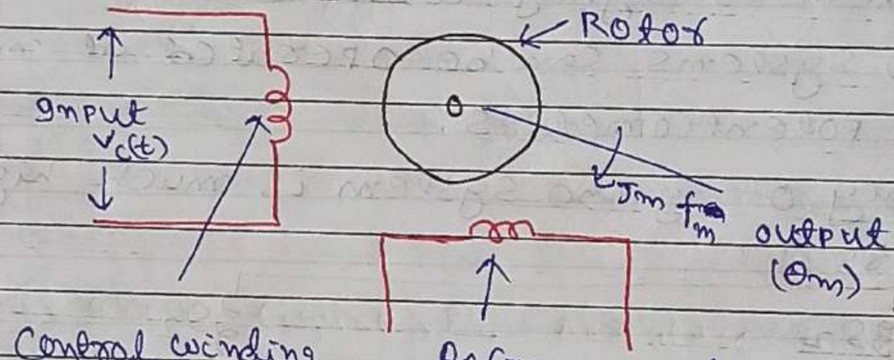
Servomotor -:

* A motor which is used in control system is known as a servomotor. (Motor having low power rating)
* These are of two types.

a) D.C servomotor b) A.C servomotor

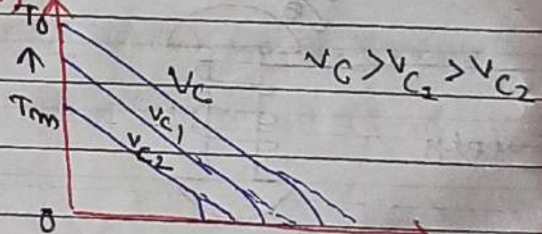
A.C servomotor

* A.C servomotor is a specially designed two phase induction motor.
* The control voltage $v_c(t)$ is applied to the control winding and a fixed voltage having a phase difference of 90° w.r.t control winding voltage is applied to the reference winding.
* The control voltage results in the development of the motor torque (T_m).



(TWO Phase A.C. Servomotor)

* The torque speed characteristics of the A.C servomotors are shown in the figure.
* The moment of inertia & the coefficient of friction at the motor shaft are represented as J_m and f_m respectively.



Torque speed characteristics of a two phase A.C servomotor.

* Here ω_m = Angular velocity

θ_m = Angular shift in the motor shaft.

* The torque is proportional to both speed as well as the control winding voltage (V_c).

$T_m(t) \propto V_c(t)$, $T_m(t) \propto \omega_m(t)$ $\quad \because \omega_m(t) = \frac{d\theta(t)}{dt}$

$\Rightarrow \boxed{T_m(t) = K V_c(t)}$ — (1), $\boxed{T_m(t) = m \frac{d\theta(t)}{dt}}$ — (2)

$\Rightarrow$ ~~Tm~~ where m = Slope of torque-speed curve.

* Combined Eqn (1) & Eqn (2)

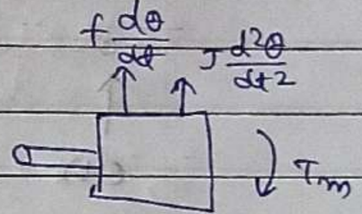
$$\boxed{T_m(t) = K V_c(t) + m \frac{d\theta(t)}{dt}} \quad \text{— (3)}$$

Taking laplace transform of Eqn (3)

$$\boxed{T_m(S) = K V_c(S) + m S \theta(S)} \quad \text{— (4)}$$

* Due to load, torque Eqn is written as

$$\boxed{T_m(t) = J_m \frac{d^2\theta(t)}{dt^2} + f_m \frac{d\theta(t)}{dt}} \quad \text{— (5)}$$



Taking laplace transform of Eqn (5)

$$\boxed{T_m(S) = J_m S^2 \theta(S) + f_m S \theta(S)} \quad \text{— (6)}$$

Put the value of Eqn (4) in Eqn (6)

$\Rightarrow K V_c(S) + m S \theta(S) = J_m S^2 \theta(S) + f_m S \theta(S)$

$\Rightarrow K V_c(S) = J_m S^2 \theta(S) + f_m S \theta(S) - m S \theta(S)$

$\Rightarrow K V_c(S) = \theta(S) [J_m S^2 + f_m S - mS]$

$$\Rightarrow \boxed{\frac{\theta(S)}{V_c(S)} = \frac{K}{J_m S^2 + f_m S - mS}} \quad \text{— (7)}$$

Advantages -:

i) It has no brush contacts like d.c. servomotor.

ii) Effective damping is increased.

Disadvantages -:

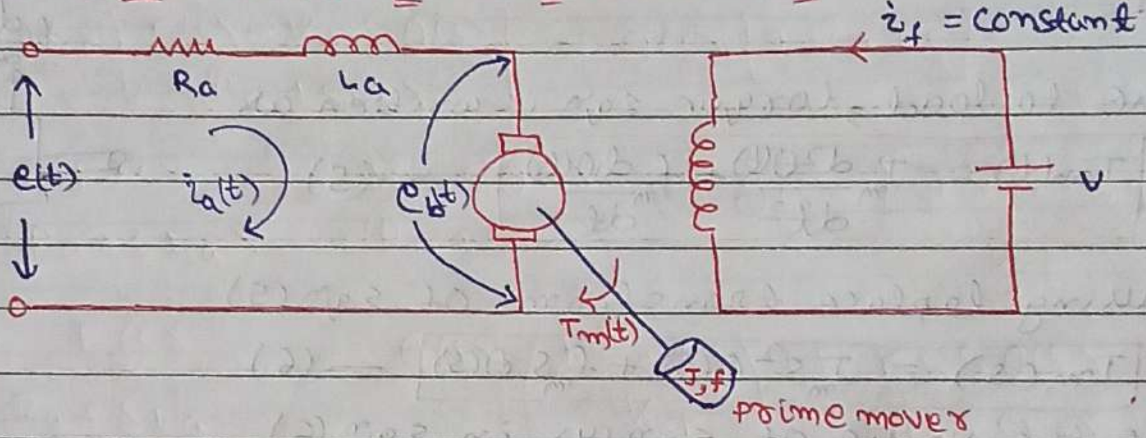
i) It is operated at low speed.

ii) The controlled phase voltage is supplied through an amplifier.

D.C. Servomotor :-

* D.C. Servomotors are generally used in control System to get rapid acceleration from Stand Still condition.
* The requirements of the d.c. motor are low inertia and high starting torque.
* A d.c. servomotor is an ordinary d.c. motor for some minor changes.
* Depending on the modes of operation, they are classified as
  a) Armature Controlled d.c. Servomotor
  b) Field controlled d.c. servomotor

a) Armature controlled d.c. Servomotor :-



* In this above figure, R_a = Armature winding resistance
  L_a = Armature winding inductance
  I_a = Armature current, I_f = field winding current
  e(t) = Control voltage, θ = Angular displacement (Radians)
  J = Moment of inertia ($Kg\text{-}m^2$)
  f = Friction coefficient ($\frac{N-m}{rad/sec}$)
  T_m = Motor torque (N-m)
* In this case, the output of the motor is controlled by the armature current, hence it is known as armature controlled d.c. servomotor.
* The field winding current (i_f) is constant and is obtained by applying a constant voltage source (V).

* The current in the field winding produces a flux in the air gap. $\phi \propto I_f \Rightarrow \boxed{\phi = K_f I_f}$ — (1)

* The torque developed in the motor is proportional to both the flux as well as the armature current.

$$T_m(t) \propto \phi I_a(t) \Rightarrow T_m(t) = K \phi I_a(t)$$

$$\Rightarrow T_m(t) = K K_f I_f I_a(t) \quad (\because \phi = K_f I_f)$$

$$\Rightarrow \boxed{T_m(t) = K_T I_a(t)} \text{ — (2)} \quad (\because K_T = K K_f I_f)$$

* Taking laplace transform of Eqn (2)

$$\Rightarrow \boxed{T_m(S) = K_T I_a(S)} \text{ — (3)}$$

* Now the back emf (e_b) is directly proportional to the angular velocity.

$$e_b(t) \propto \frac{d\theta(t)}{dt} \Rightarrow \boxed{e_b(t) = K_b \frac{d\theta(t)}{dt}} \text{ — (4)}$$

* Taking laplace transform of Eqn (4)

$$\Rightarrow \boxed{E_b(S) = K_b S \theta(S)} \text{ — (5)}$$

* So, the equivalent circuit for armature winding is shown in figure (1)

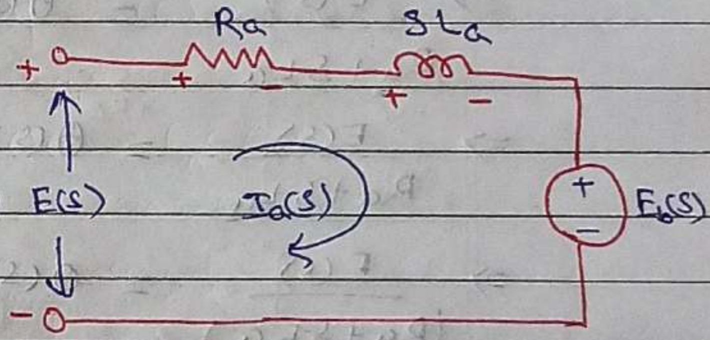


figure (1)

* By applying KVL to the armature winding, we get

$$E(S) - R_a I_a(S) - S L_a I_a(S) - E_b(S) = 0$$

$$\Rightarrow E(S) - E_b(S) = R_a I_a(S) + S L_a I_a(S)$$

$$\Rightarrow E(S) - E_b(S) = I_a(S)\left[R_a + S L_a\right]$$

$$\Rightarrow I_a(S) = \frac{E(S) - E_b(S)}{(R_a + S L_a)} \quad \left(\because E_b(S) = K_b S \theta(S) \text{ from Eqn (5)}\right)$$

$$\Rightarrow \boxed{I_a(S) = \frac{E(S) - K_b S \theta(S)}{(R_a + S L_a)}} \text{ — (6)}$$

* when the torque is developed on the motor, the load is displaced by an amount $\theta(t)$.
* The free body diagram of load (prime mover) is shown in figure (2)
* The torque eqn is given by

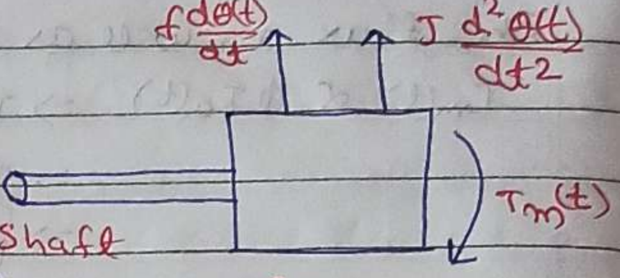


figure (2)

$$\boxed{T_m(t) = J\frac{d^2\theta(t)}{dt^2} + f\frac{d\theta(t)}{dt}} \quad -(7)$$

* Taking laplace transform of eqn (7)

$$T_m(s) = J s^2 \theta(s) + f s\, \theta(s)$$

$$\Rightarrow K_T I_a(s) = \theta(s)[J s^2 + f s] \qquad (\because T_m(s) = K_T I_a(s))$$

$$\Rightarrow K_T\left[\frac{E(s) - K_b s\, \theta(s)}{(R_a + sL_a)}\right] = \theta(s)[J s^2 + f s]$$

$$\Rightarrow \frac{E(s)}{(R_a + sL_a)} - \frac{K_b s\, \theta(s)}{(R_a + sL_a)} = \theta(s)\frac{[J s^2 + f s]}{K_T}$$

$$\Rightarrow \frac{E(s)}{R_a + sL_a} = \theta(s)\left(\frac{J s^2 + f s}{K_T}\right) + \frac{K_b s\, \theta(s)}{(R_a + sL_a)}$$

$$\Rightarrow \frac{E(s)}{R_a + sL_a} = \theta(s)\left[\frac{J s^2 + f s}{K_T} + \frac{K_b s}{R_a + sL_a}\right]$$

$$\Rightarrow \frac{E(s)}{(R_a + sL_a)} = \theta(s)\left[\frac{(J s^2 + f s)(R_a + sL_a) + K_b K_T s}{K_T(R_a + sL_a)}\right]$$

Transfer function is given by

$$\boxed{G(s) = \frac{\theta(s)}{E(s)} = \frac{K_T}{(J s^2 + f s)(R_a + sL_a) + K_b K_T s}} \quad -(8)$$

* τ_{me} = mechanical time constant = J/f

  τ_a = armature time constant = L_a/R_a

$$G(s) = \frac{\theta(s)}{E(s)} = \frac{K_T}{s[(Js+f)(R_a+sL_a)+K_bK_T]}$$

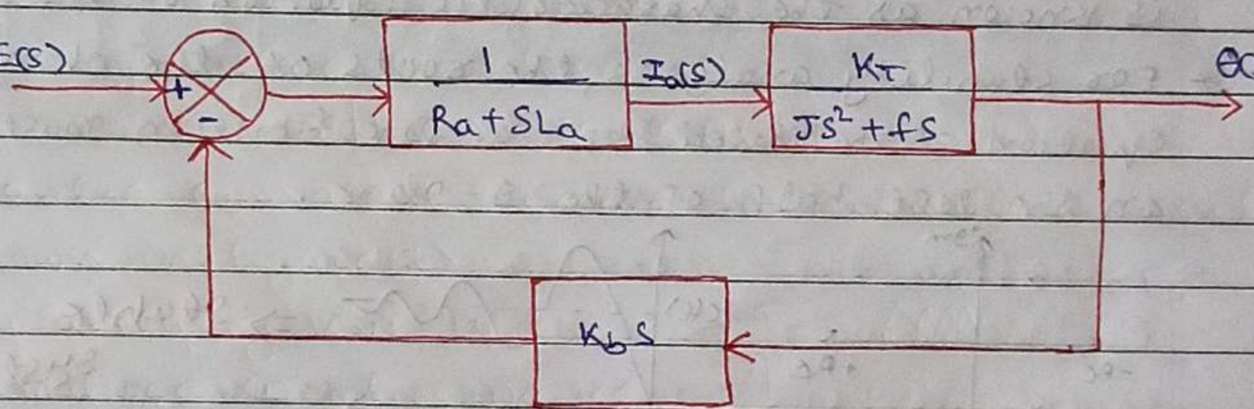
$$\Rightarrow G(s) = \frac{K_T}{s[f(1+s\frac{J}{f})R_a(1+s\frac{L_a}{R_a})+K_bK_T]}$$

$$\Rightarrow \boxed{G(s) = \frac{K_T}{s[f(1+s\tau_{me})R_a(1+s\tau_a)+K_bK_T]}} \quad -(9)$$

Case-1-:

* For an armature controlled motor, $\boxed{\tau_a = 0}$

$$\boxed{G(s) = \frac{K_T}{s[fR_a(1+s\tau_{me})+K_bK_T]}} \quad \text{——} (10)$$



(Block diagram representation)

Advantages-:

1) The back emf produces additional damping besides friction.
2) The time constant is less.

Disadvantages-:

1) It requires additional power amplifiers to increase the power.

(ii) Field controlled D.C. Servomotor

A field controlled d.c. motor is shown in fig. 5.1(e). In a field control d.c. servomotor the armature current is supplied from a constant current source. The field current is varied by the voltage applied across the field winding.

R_f = field winding resistance.

L_f = field winding inductance.

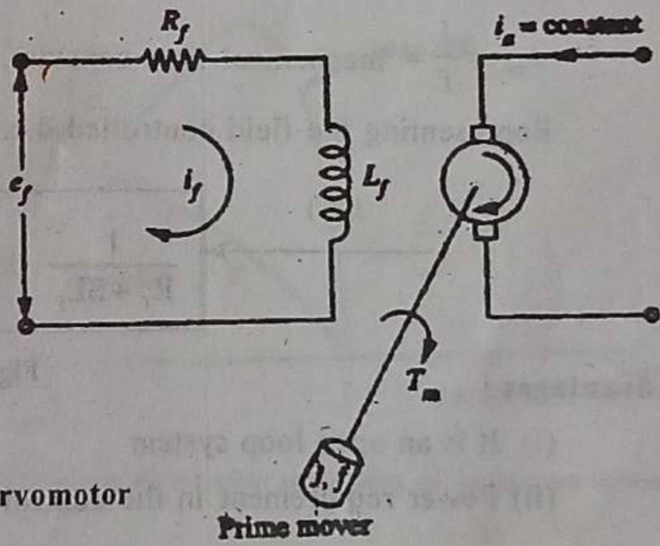


Fig.5.1(e) field controlled D.C. servomotor

The torque developed is proportioned to the field current, since the armature current is constant.

$$T_m(t) = k_T i_f(t) \quad \text{....(5.9)}$$

Taking Laplace transform of equation (5.9)

$$T_m(s) = K_T \; I_f(s) \quad \text{...(5.10)}$$

The equivalent circuit for the field winding is given below in Fig. 5.2

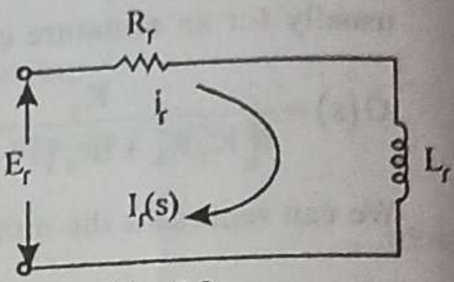


Fig. 5.2

Taking Laplace transform of Fig. 5.2 and applying K.V.L to find $I_f(s)$

$$I_f(s) = \frac{E_f(s)}{(R_f + sL_f)} \quad \text{....(5.11)}$$

The torque developed is given by, $T_m(t) = J\dfrac{d^2\theta}{dt^2} + f\dfrac{d\theta}{dt}$ ....(5.12)

Taking Laplace transform $T_m(s) = (Js^2 + fs)\,\theta(s)$ ....(5.13)

$$\Rightarrow \quad K_T \,.\, I_f(s) = (Js^2 + fs)\,\theta(s) \;\Rightarrow\; K_T \,.\, \frac{E_f(s)}{(R_f + sL_f)} = (Js^2 + fs)\,\theta\,(s)$$

The transfer function is given by

$$G(s) = \frac{\theta(s)}{E(s)} = \frac{K_T}{s(Js + f)\,(R_f + sL_f)} \quad \text{....(5.14)}$$

$$= \frac{K_T}{fs\left[1 + s\left(\frac{J}{f}\right)\right] R_f\left[1 + s\left(\frac{L_f}{R_f}\right)\right]} = \frac{K_T^1}{s(1 + \tau_{me})\,(1 + s\tau_f)} \quad \text{....(5.15)}$$

$\tau_{me} = \dfrac{J}{f} =$ mechanical time constant, $\quad \tau_f = \dfrac{L_f}{R_f} =$ Field time constant

Representing the field controlled d.c. servomotor into block diagram.

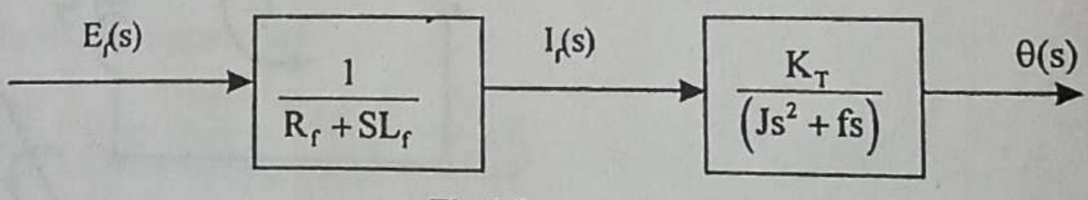


Fig.5.3

Advantages :

(i) It is an open loop system

(ii) Power requirement in the control field circuit is small

Disadvantages

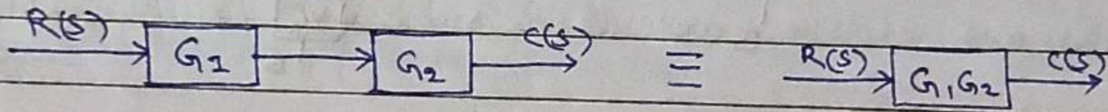
(i) It requires a constant current source which is difficult to realise physically.

(ii) Additional damping is necessary due to the absence of back emf.

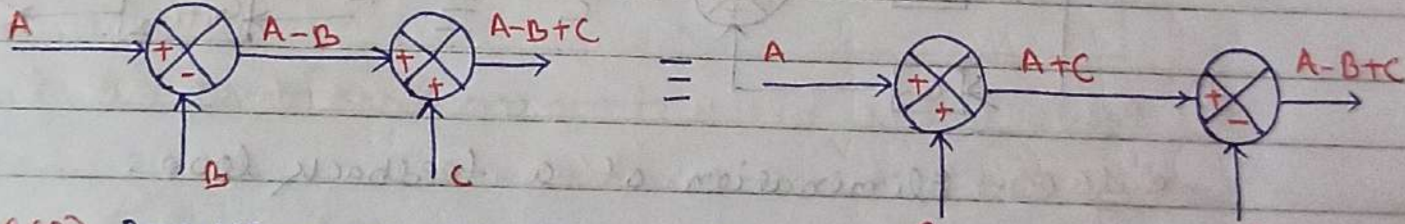
CP-4 **Block diagram Algebra & Signal flow Graphs**

Block diagram Reduction Technique -:

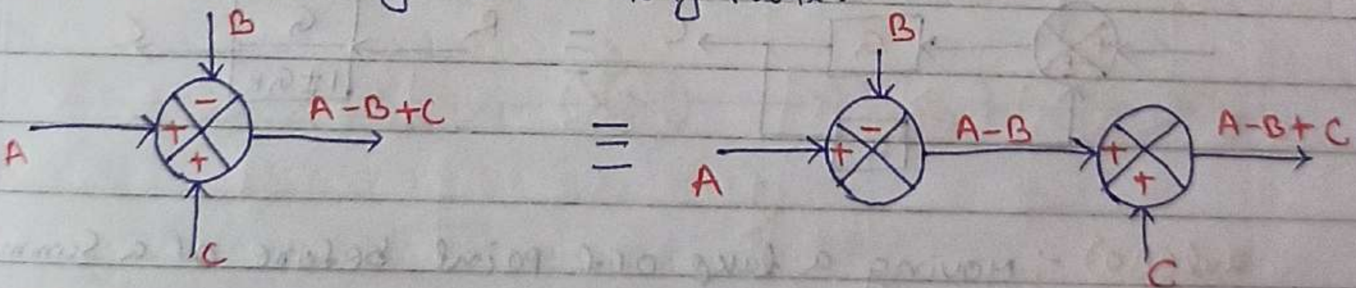
Rule (1) -: Block connected in cascade.



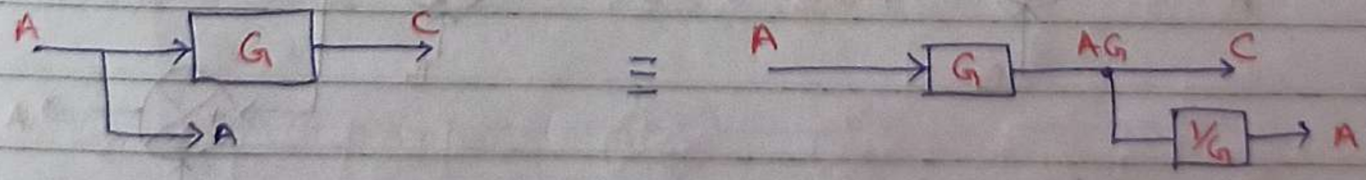
Rule (2) -: Interchanging of summing point.



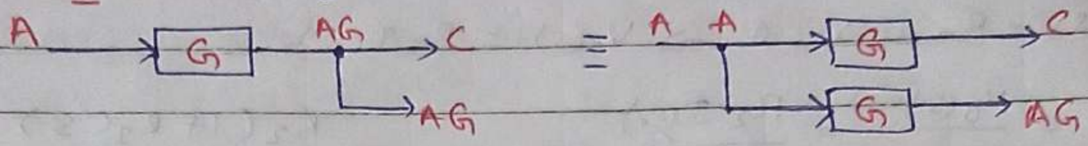
Rule (3) Breaking up summing point.



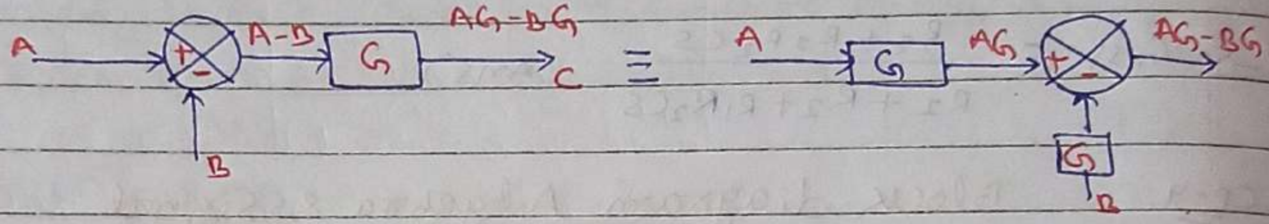
Rule (4) -: Moving a take off point before a block.



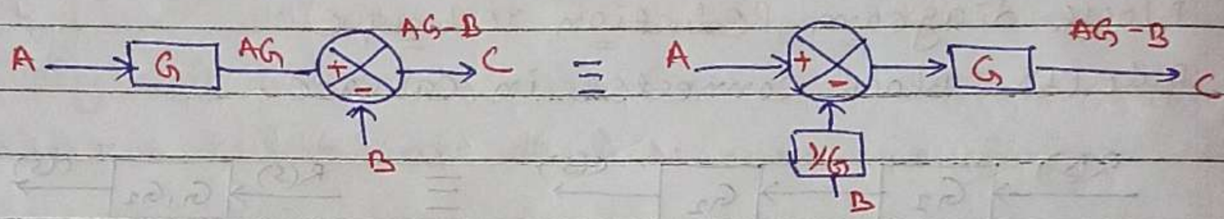
Rule (5) -: Moving a take off point after a block.



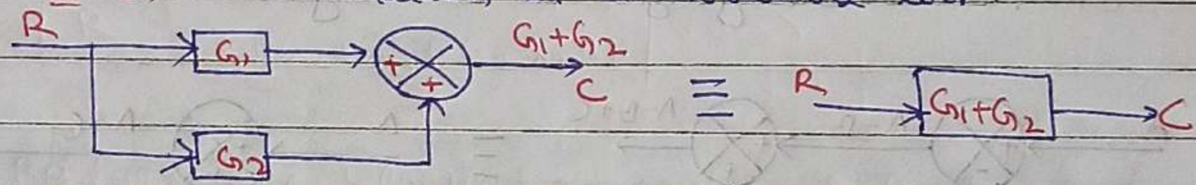
Rule (6) -: Moving a summing point before a block.



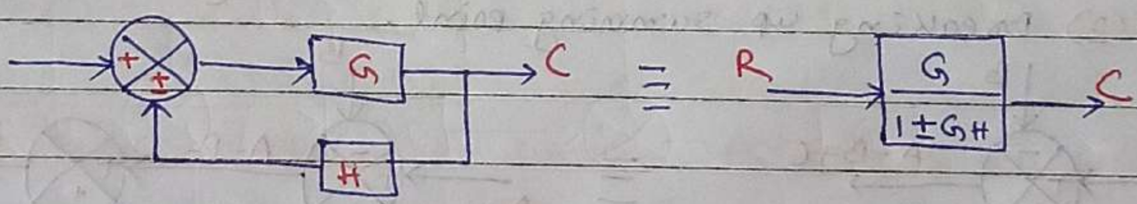
Rule (7) -: Moving a summing point after a block.



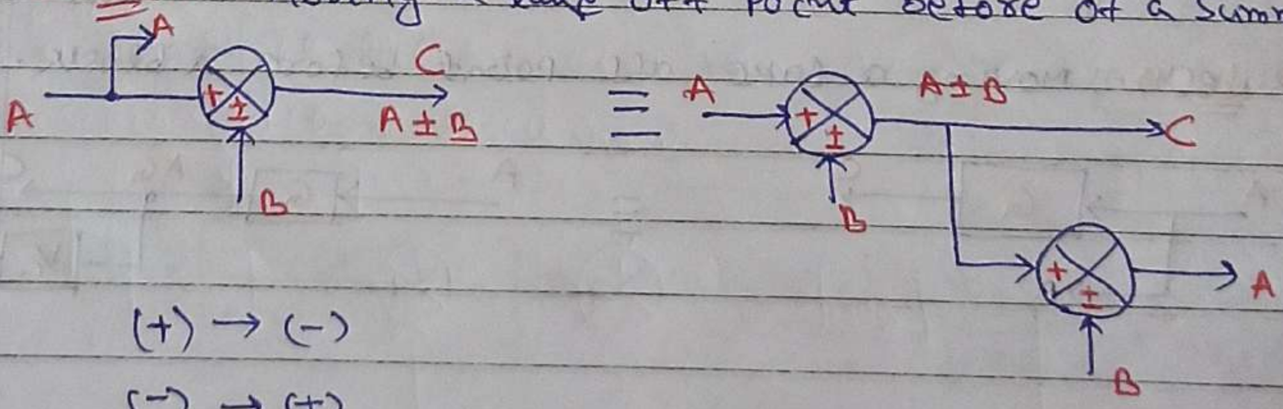
Rule (8) -: Elimination of a forward loop.



Rule (9) -: Elimination of a feedback loop.



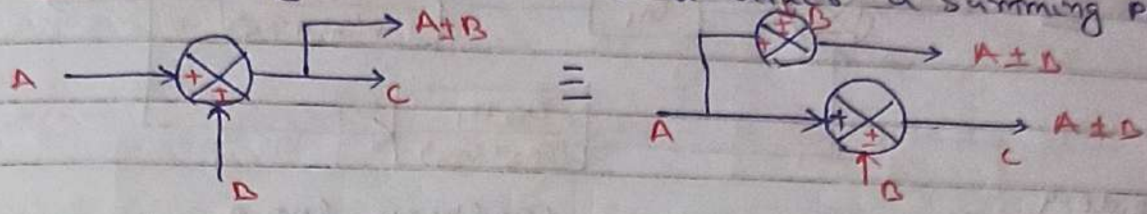
Rule (10) -: Moving a take off point before of a Summing point



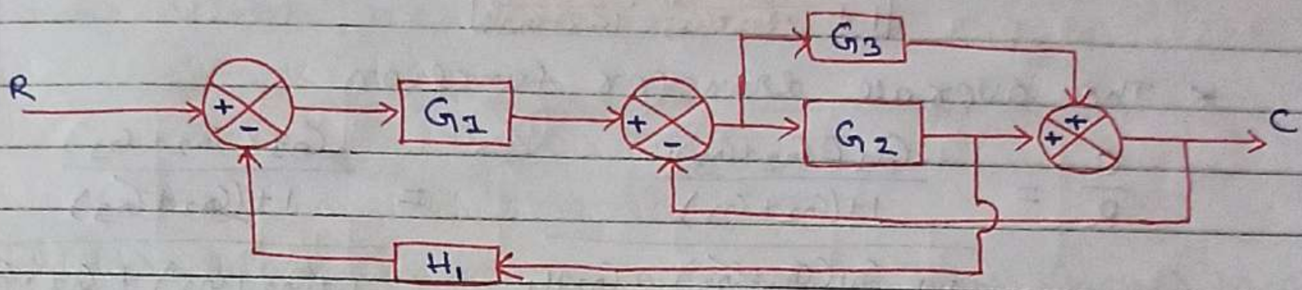
$(+) \rightarrow (-)$

$(-) \rightarrow (+)$

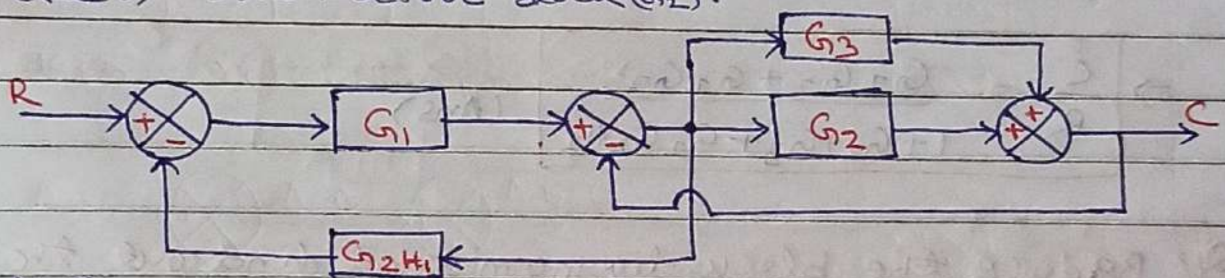
Rule (11) :- Moving a take off point after a summing point.



Q// Determine the transfer function (C/R) from the block diagram.

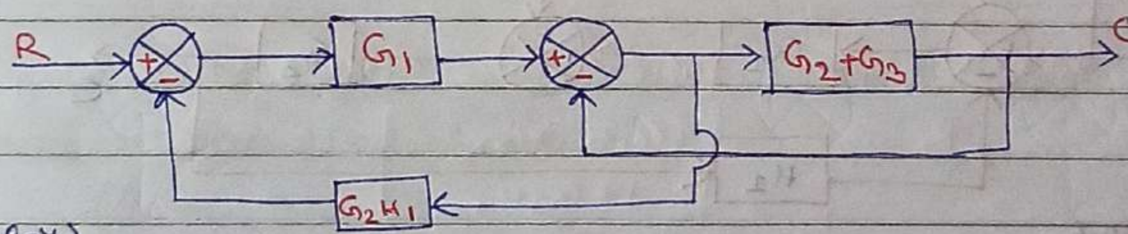


Ans :- Step-1 Shift the take off point after the block (G_2) to a (Rule-5) position before block (G_2).



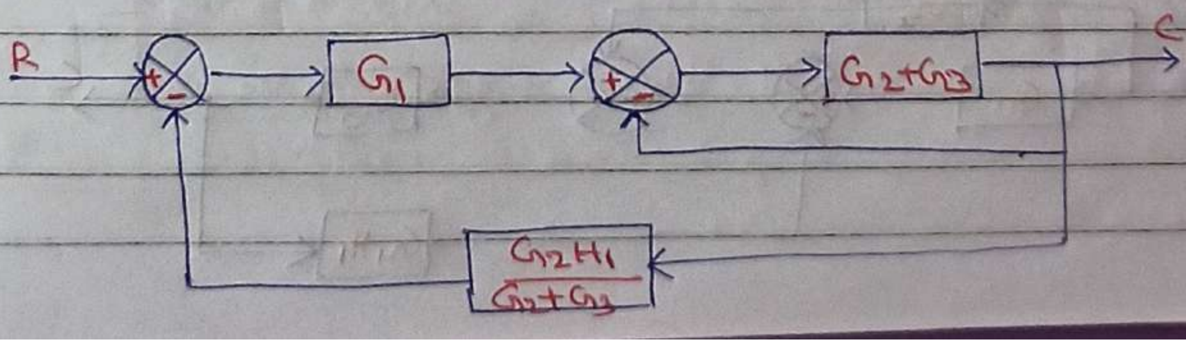
(Rule-8)

Step-2 Elimination of a forward loop (Eliminate the summing point after block (G_2).



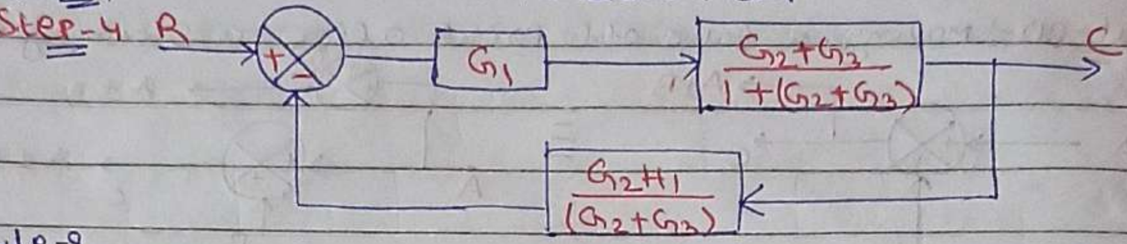
(Rule-4)

Step-3 Shift the take off point before block (G_2+G_3) to a position after block (G_2+G_3).



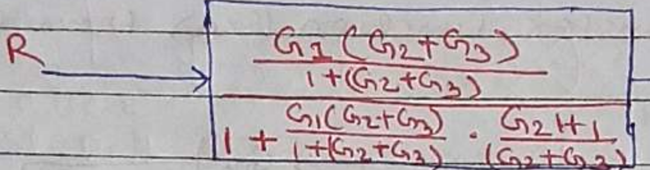
Rule-9 Elimination of a feedback loop.

Step-4



Rule-9

Step-5 Elimination of a feedback loop

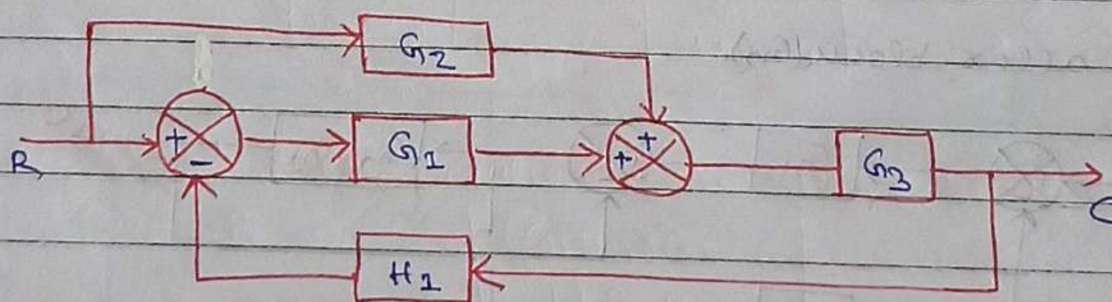


* The overall transfer function

$$\frac{C}{R} = \frac{\dfrac{G_1(G_2+G_3)}{1+(G_2+G_3)}}{1+\dfrac{G_1(\cancel{G_2+G_3})}{1+(G_2+G_3)}\cdot\dfrac{G_2H_1}{(\cancel{G_2+G_3})}} = \frac{\dfrac{G_1(G_2+G_3)}{\cancel{1+(G_2+G_3)}}}{\dfrac{1+(G_2+G_3)+G_1\cdot G_2H_1}{\cancel{1+(G_2+G_3)}}}$$

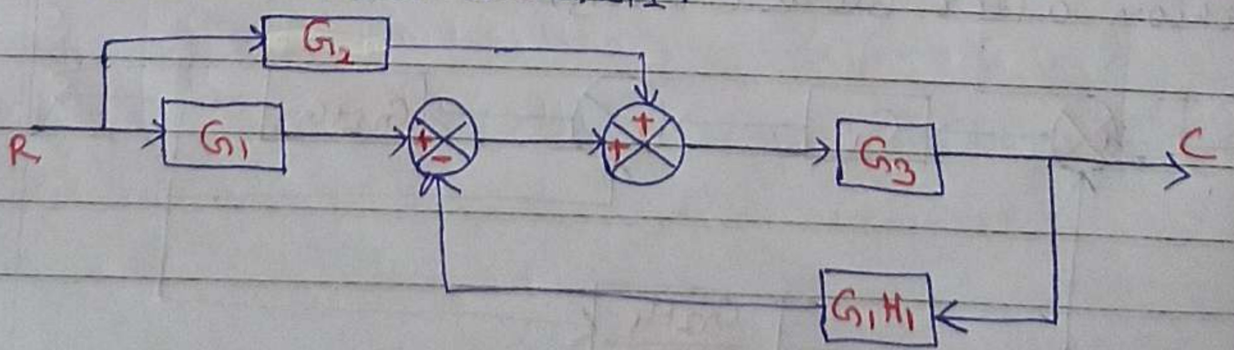
$$\Rightarrow \boxed{\frac{C}{R} = \frac{G_1G_2+G_1G_3}{1+G_2+G_3+G_1G_2H_1}} \text{ (Ans)}$$

Q/ Reduce the block diagram and find out the overall transfer function (C/R).



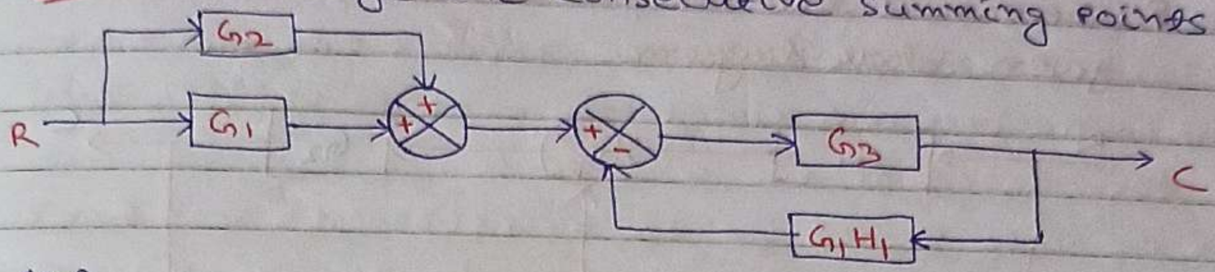
Rule-6

Ans :- step-1 Shift the summing point before block G_1 to a position after block G_1.



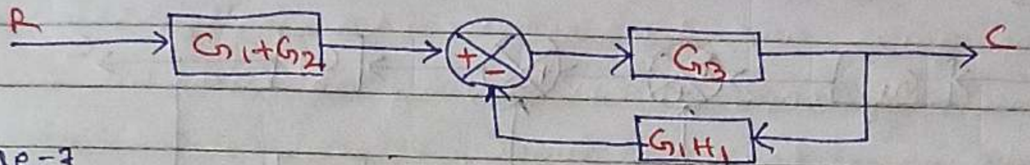
Rule-2

Step-2 Interchange the consecutive summing points.



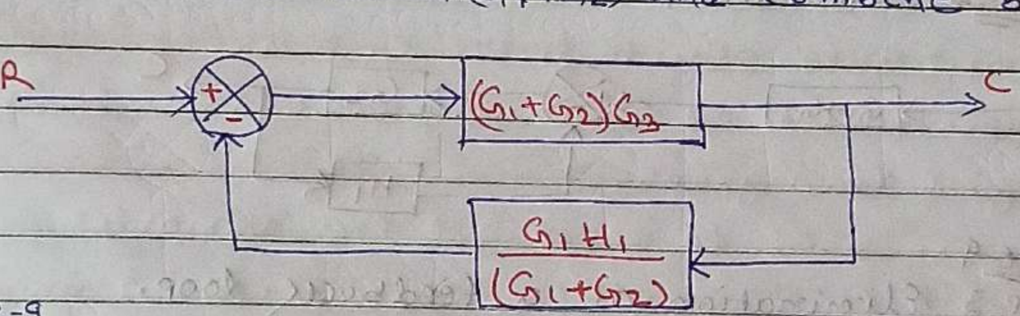
Rule-8

Step-3 Elimination of a forward loop. (Eliminate the summing point after block G_1.



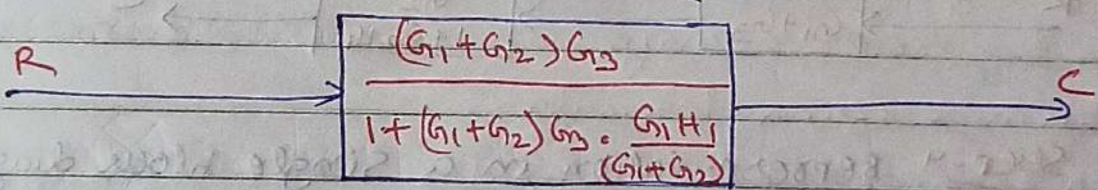
Rule-7

Step-4 Shift the summing point after block (G_1+G_2) to a position before block (G_1+G_2) and combine blocks (G_1+G_2) & G_3.



Rule-9

Step-5 Elimination of a feedback loop.

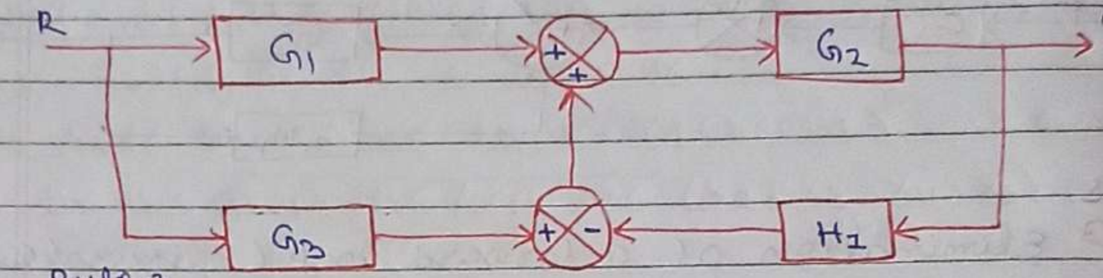


* The overall transfer function

$$\frac{C}{R} = \frac{(G_1+G_2)G_3}{1+\cancel{(G_1+G_2)}G_3 \cdot \frac{G_1H_1}{\cancel{(G_1+G_2)}}}$$

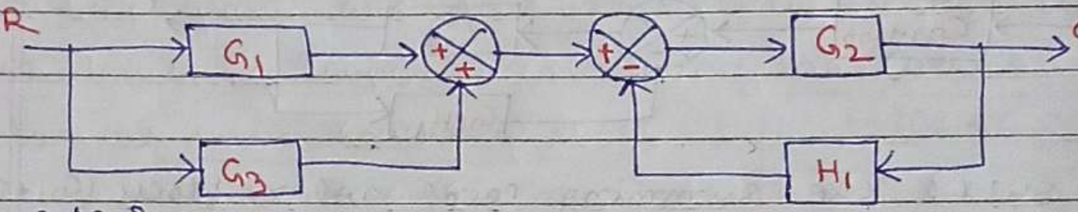
$$\Rightarrow \boxed{\frac{C}{R} = \frac{G_1G_3+G_2G_3}{1+G_1G_3H_1}} \text{ (Ans)}$$

Q/ Determine the overall transfer function of the given block diagram.



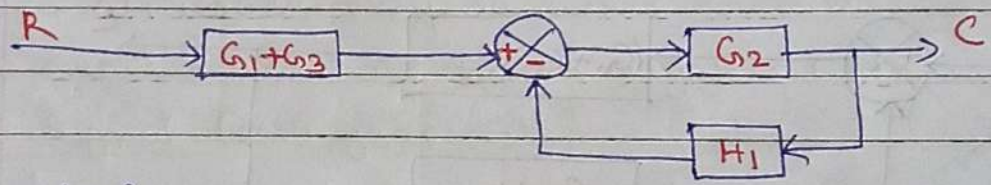
Rule-2

Ans -: Step-1 Interchanging of summing points.



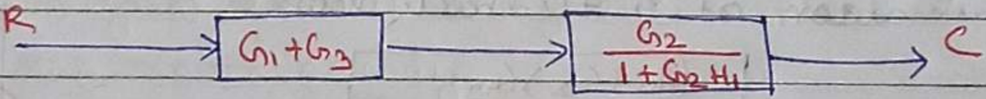
Rule-8

Step-2 Elimination of a forward loop

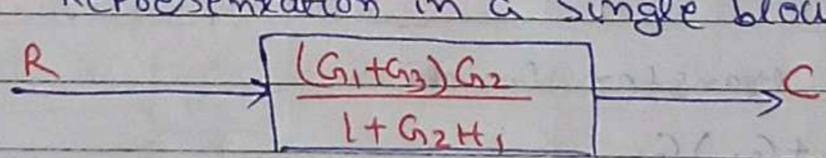


Rule-9

Step-3 Elimination of a feedback loop.



Step-4 Representation in a single block diagram.



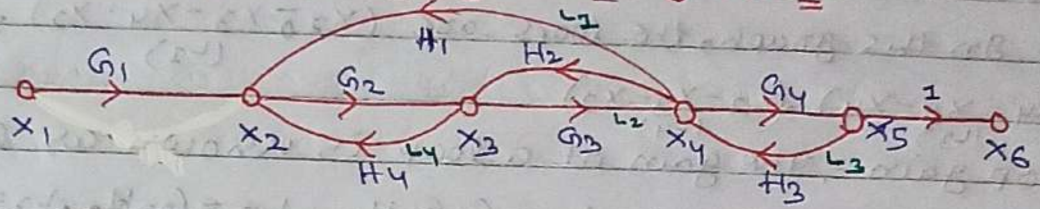
* The overall transfer function

$$\boxed{\frac{C}{R} = \frac{(G_1+G_3)G_2}{1+G_2H_1} = \frac{G_1G_2+G_2G_3}{1+G_2H_1}}$$ (Ans)

Signal flow Graphs

Signal flow graphs -: It is defined as a graphical representation of the control system which was given by "Mason". It is also an equivalent representation of the block diagram.

Terms used in signal flow graphs -:



1. **Node -:** Node is a point which represents a variable or a signal
   Ex-: $x_1, x_2, x_3, x_4, x_5, x_6$ are the nodes in the above graph.
2. **Input node (source) -:** It is a node having outgoing branches.
   Ex-: x_1 is the input node.
3. **output node (sink) -:** It is a node having incoming branches.
   Ex-: x_6 is the output node.
4. **Mixed node -:** It is a node having both incoming and outgoing branches. Ex-: x_2, x_3, x_4, x_5
5. **Branch -:** Branch is a line which joint two nodes.
   → In this Graph, there are eight branches.
   → The arrow on the branch represent the signal flow.
6. **Transmittance -:** The gain of a branch is known as Transmittance
   Ex-: $G_1, G_2, G_3, G_4, H_1, H_2, H_3, H_4$ are the transmittance.
7. **Path -:** It is a collection of continuos branches from input side to the output side along one direction.
   → No node should be traversed (Repeat) more than once.
   Ex-: $(x_1 - x_2 - x_3 - x_4 - x_5 - x_6)$ is the path of the graph.
8. **Path gain -:** It is the multiplication of gain of individual branches along a path.
9. **Forward path -:** It is a collection of continuous branches from input to output only in forward direction.
   Ex-: $(x_1 - x_2 - x_3 - x_4 - x_5 - x_6)$ = forward path.

10. Forward path gain -: The gain of a forward path is known as forward path gain. Ex-: Forward path gain $= G_1G_2G_3G_4$

11. Loop -: Loop is a closed path which originates and terminated at the same node.

→ No node should be traversed (repeat) more than once.

Ex-: In this graph, the loops are $\underset{(L_1)}{(x_2 - x_3 - x_4 - x_2)}$, $\underset{(L_2)}{(x_3 - x_4 - x_3)}$, $\underset{(L_3)}{(x_4 - x_5 - x_4)}$, $\underset{(L_4)}{(x_2 - x_3 - x_2)}$

12. Loop gain -: The gain of a loop is known as loop gain.

Ex-: Loop gains are $L_1 = G_2G_3H_1$, $L_2 = G_3H_2$, $L_3 = G_4H_3$, $L_4 = G_2H_4$

13. Non touching loops -: Loops having no common node is known as non-touching loops.

Ex-: L_3 and L_4 are the non touching loops.

Properties of Signal flow graph -:

1. It is applicable only to linear systems.
2. The expression must be in algebaric from to draw the signal flow graph.
3. Nodes are used to represent the variables.
4. The signal flow graph of a system is not unique.
5. The signal travels along the directed lines but not in the reverse way.

Mason's Gain formula -:

* To findout overall transfer function using signal flow graph, Mason's gain formula is used.

* T.F. $\boxed{M = \dfrac{\sum_{K=1}^{N} P_K \Delta_K}{\Delta}}$

where P_K = Gain of K^{th} forward path in the signal flow graph

Δ = Determinant of the graph.

* Δ = 1 - (Sum of individual loop gain) + Sum of gain product of two non touching loops) - (Sum of gain product of three non touching loops) + ____

Δ_k = Determinant of the graph which is formed by loops not touching the kth forward path.

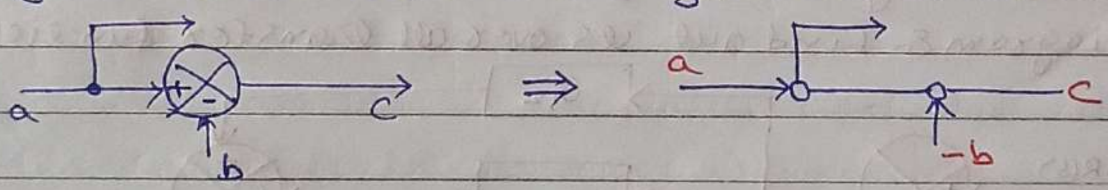
* Δ_k = 1 - [Sum of gain of all loops which are not touching the kth forward path.

Procedure for construction of signal flow graph-:

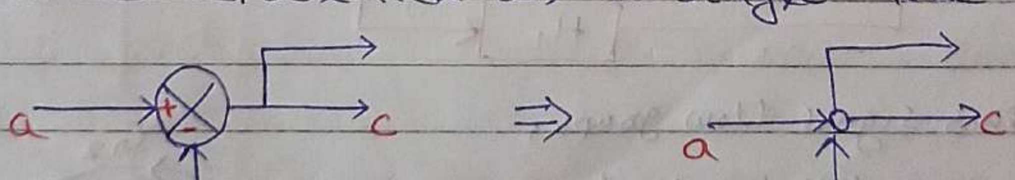
1. Determine the system Equation in an algebraic form.
2. Represent each variable in the system by a node.
3. Connect the nodes by proper branches and write their transmittances.
4. Use Mason's Gain formula to findout the overall transfer function.

Conversion from block diagram to signal flow graph-:

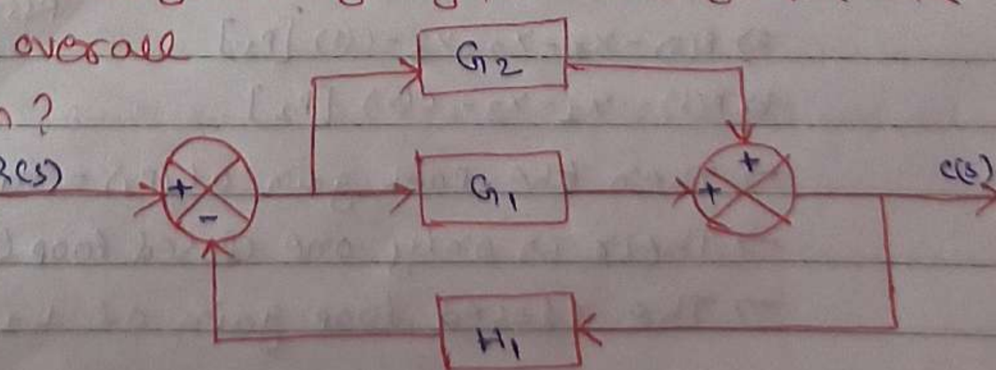
1. Represent the output and input with separate nodes.
2. Take the take off point & summing point as a node.
3. If the take off point is before the summing point then they are represented as two separate nodes.



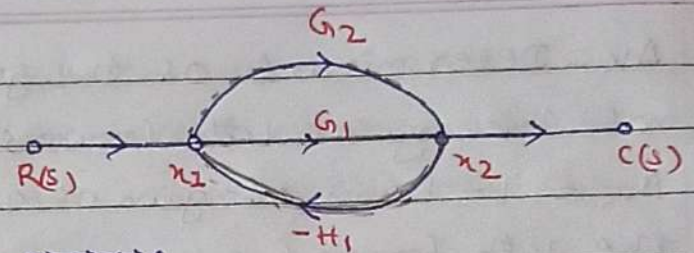
4. If the take off point is after a summing point then both are represented as a single node.



Q// Represent the block diagram by signal flow graph and determine the overall transfer function?

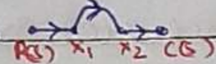


Ans-: * In this signal flow graph, there are two forward paths.



a) $R(S) - x_1 - x_2 - C(S)$ (P_1)

b) $R(S) - x_1 - x_2 - C(S)$ (P_2)



* Then the path gain of $(P_1) = G_1$ & path gain of $(P_2) = G_2$
* The two closed loops are L_1 & L_2.
* The closed loop gain of $(L_1) = -G_1H_1$ & gain of $(L_2) = -G_2H_1$

→ As both the loops L_1 & L_2 are touching both forward paths. The two path factors are $\Delta_1 = 1$ & $\Delta_2 = 1$

→ The graph determinant is $\Delta = 1 - (L_1 + L_2)$

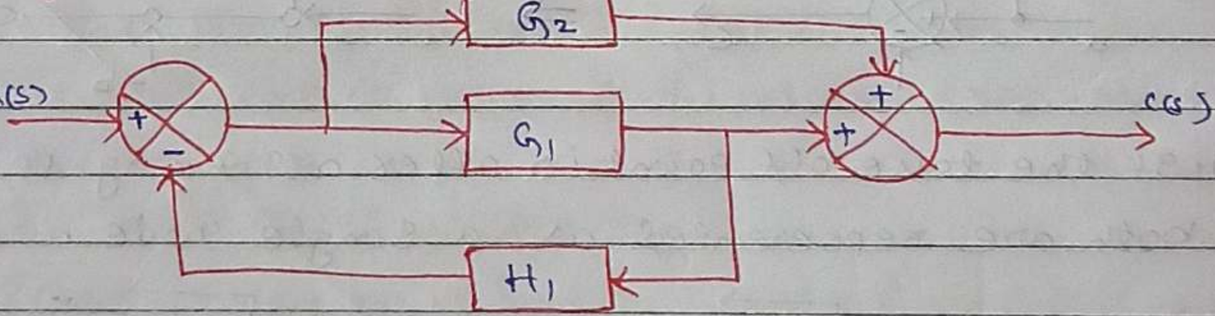
$$\Rightarrow \Delta = 1 - (-G_1H_1 - G_2H_2) = 1 + G_1H_1 + G_2H_2$$

→ Applying Mason's gain formula -:

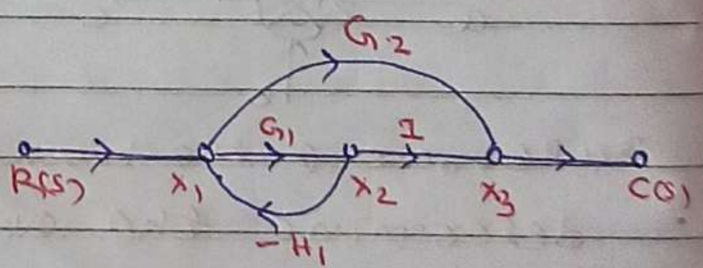
$$G(S) = \frac{C(S)}{R(S)} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta} = \frac{G_1 \cdot 1 + G_2 \cdot 1}{1 + G_1H_1 + G_2H_2} = \frac{G_1 + G_2}{1 + G_1H_1 + G_2H_2}$$

(Ans)

Q11 Draw a signal flow graph for the given block diagram & find out its over all transfer function ?



Ans -: → In this signal flow graph, there are two forward paths (P_1) & (P_2)



a) $R(S) - x_1 - x_2 - x_3 - C(S)$ $[P_1]$

b) $R(S) - x_1 - x_3 - C(S)$ $[P_2]$

→ Then the path gain of $(P_1) = G_1$ and path gain of $(P_2) = G_2$

→ Their is only one closed loop (L_1) $(x_1 - x_2 - x_1)$

→ The closed loop gain of $L_1 = -G_1H_1$

→ As the loop L_1 touches both forward paths, the two path factors are $\Delta_1 = 1$ and $\Delta_2 = 1$

→ The graph determinant is $\Delta = 1-(L_1) = 1-(G_1H_1) = 1+G_1H_1$

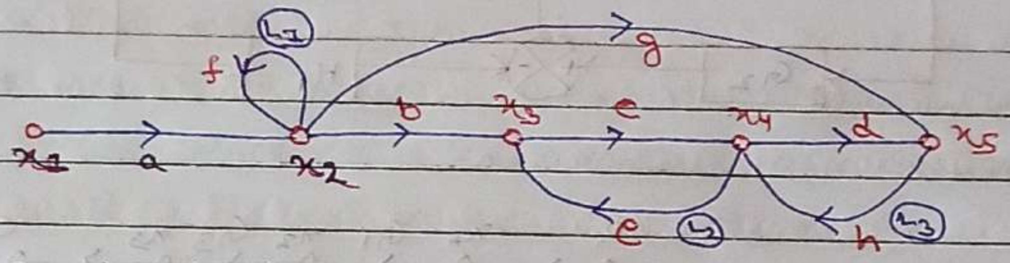
→ Applying Mason's gain formula-:

$$G(s) = \frac{C(s)}{R(s)} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta} = \frac{G_1 \cdot 1 + G_2 \cdot 1}{1+G_1H_1} = \frac{G_1+G_2}{1+G_1H_1} \text{ (Ans)}$$

Q// Represent the following set of Eq^{n}s by a signal flow graph and determine the overall gain relating x_5 and x_1.

$$x_2 = ax_1 + fx_2 ; \; x_3 = bx_2 + ex_4 ; \; x_4 = cx_3 + hx_5 ; \; x_5 = dx_4 + gx_2$$

Sol^m-: The variables x_1, x_2, x_3, x_4 and x_5 are represented by nodes



* In this signal flow graph, there are two forward paths.
$(P_1) = x_1 - x_2 - x_3 - x_4 - x_5$ & $(P_2) = x_1 - x_2 - x_5$

* The path gain of $(P_1) = abcd$ and path gain of $(P_2) = ag$

* There are three loops. (L_1, L_2 & L_3)

* The loops gains are $L_1 = f$; $L_2 = ce$; $L_3 = dh$

* Since two pairs of non-touching loops are their. so loop gain of non-touching loops are $L_1L_2 = fce$ & $L_1L_3 = fdh$.

* Here all the loops touch the forward path (P_1), so path factor for (P_1) is $\Delta_1 = 1$

→ But the loop $L_2 = ce$ is not touching the forward path (P_2). So path factor for (P_2) is $\Delta_2 = 1 - L_2 = 1 - ce$.

→ Now the graph determinant is

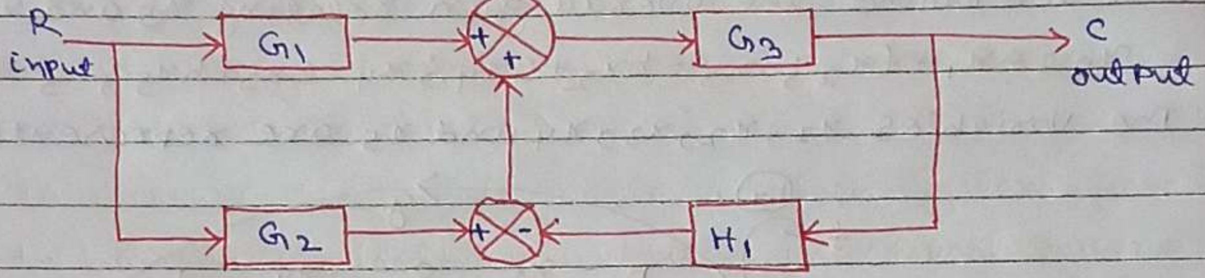
$$\Delta = 1-(L_1+L_2+L_3)+(L_1L_2+L_1L_3)$$
$$= 1-(f+ce+dh)+(fce+fdh)$$
$$= 1-f-ce-dh+fce+fdh$$

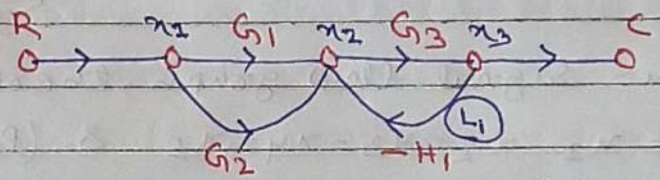
Applying Mason's gain formula

$$G(s) = \frac{x_5}{x_1} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta} = \frac{abcd \cdot 1 + ag \cdot (1-ce)}{1-f-ce-dh+fce+fdh}$$

$$= \frac{abcd + ag - agce}{1-f-ce-dh+fce+fdh} \quad \text{(Ans)}$$

Q1) Draw the signal flow graph of the given block diagram & determine the transfer function?





* Two forward paths. $(P_1) = R - x_1 - x_2 - x_3 - C$ & $(P_2) = R - x_1 - x_2 - x_3 - C$

* forward path gain of $(P_1) = G_1G_3$ & path gain of $(P_2) = G_2G_3$
* The only loop is L_1 & its loop gain $L_1 = -G_3H_1$
* The closed loop touches both the forward paths. Therefore, path factors are $\Delta_1 = 1$ and $\Delta_2 = 1$
* The graph determinant is

$$\Delta = 1-(L_1) = 1-(-G_3H_1) = 1 + G_3H_1$$

* Applying Mason's gain formula

$$G(s) = \frac{C}{R} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta} = \frac{G_1G_3 \cdot 1 + G_2G_3 \cdot 1}{1+G_3H_1}$$

$$= \frac{G_1G_3 + G_2G_3}{1+G_3H_1} \quad \text{(Ans)}$$

CP-5

Time Response Analysis

Time Response -: It is the variation of output with respect to time.

→ In order to obtain the output with respect to time, a system requires input.

→ Input function is known as excitation function.

Standard Test Signals -:

* Ref. to Ch-1 for step signal, Ramp signal, parabolic signal and impulse signal.

Order of a System -:

* The highest power in the characteristics Equation is denoted as the order of a system.

Ex-: $G(s)\,H(s) = \dfrac{K(S+5)}{S(S+1)(S+2)}$

→ The characteristic eqn can be written as

$1 + G(s)\,H(s) = 0$

$\Rightarrow 1 + \dfrac{K(S+5)}{S(S+1)(S+2)} = 0 \Rightarrow S(S+1)(S+2) + K(S+5) = 0$

$\Rightarrow (S^2+S)(S+2) + K(S+5) = 0$

$\Rightarrow S^3 + 2S^2 + S^2 + 2S + KS + 5K = 0$

$\Rightarrow S^3 + 3S^2 + S(2+K) + 5K = 0$

→ The highest power of above characteristic equation is 3. So it is a third order system.

Type of a System -:

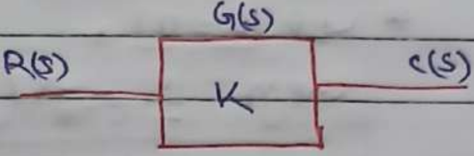
* The no. of poles at origin indicates the type of System.

Ex-: $G(s)\,H(s) = \dfrac{K}{S^4(S+5)}$

→ Since 4 poles at the origin. It is a type 4 System.

Time Response of a zero order System -:

* The transfer function of ~~type~~ zero order system



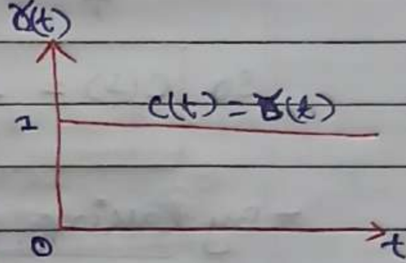
(Block diagram of zero order system)

$$G(S) = \frac{C(S)}{R(S)} \quad (\because G(S) = K)$$

$$\Rightarrow K = \frac{C(S)}{R(S)} \Rightarrow \boxed{C(S) = K\,R(S)} \quad -(1)$$

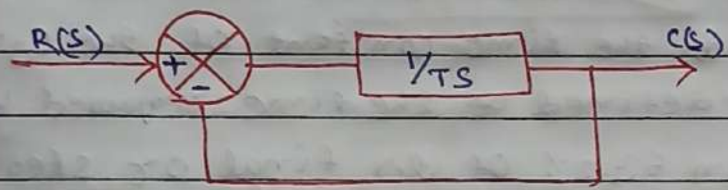
→ Taking inverse laplace transform of Eqn (1)

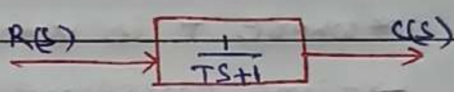
$$\boxed{C(t) = K\,r(t)} \quad -(2)$$



→ For unit step input $K = 1$, So $\boxed{C(t) = r(t)}$

Time Response of 1st order system -:





→ The overall transfer function for 1st order system is

$$= \frac{G(S)}{1+G(S)} = \frac{1/TS}{1+1/TS} = \frac{\cancel{1/TS}}{TS+1/\cancel{TS}} = \frac{1}{1+TS} \quad -(1)$$

where T = time constant.

a) Response to unit step input -:

* For a unit step input $\boxed{R(t) = u(t)}$ — (2)

Taking laplace transform of Eqn (2), $\boxed{R(S) = \frac{1}{S}}$ — (3)

→ As we know transfer function $\frac{C(S)}{R(S)} = \frac{1}{1+TS}$ (for 1st order system)

$$\Rightarrow C(S) = \frac{R(S)}{1+TS} = \frac{1}{S}\left[\frac{1}{1+TS}\right]$$

$$\Rightarrow C(S) = \frac{1/T}{S\left(\frac{1}{T}+S\right)} \quad \text{(Divide T on both numerator and denominator)}$$

$$\Rightarrow C(S) = \frac{A}{S} + \frac{B}{\left(S+\frac{1}{T}\right)} \quad \text{(Using partial fraction)}$$

→ Comparing numerator ⇒ $\frac{1}{T} = A\left(S+\frac{1}{T}\right) + B(S)$

* Let $\boxed{S=0}$,

then $\frac{1}{T} = A\left(0+\frac{1}{T}\right) + B\cancel{.0}^{\,0}$ ⇒ $\frac{1}{T} = \frac{A}{T}$ ⇒ $\boxed{A = 1}$

* Let $\boxed{S = -\frac{1}{T}}$

then $\frac{1}{T} = A\left(-\cancel{\frac{1}{T}}+\cancel{\frac{1}{T}}\right) + B\left(-\frac{1}{T}\right)$ ⇒ $\frac{1}{T} = \frac{-B}{T}$ ⇒ $\boxed{B=-1}$

So $\boxed{C(S) = \frac{1}{S} - \frac{1}{S+\frac{1}{T}}}$ —(4) (By putting the value of A & B)

* By taking inverse Laplace transform of eqn (4)

⇒ $\boxed{c(t) = 1 - e^{-t/T}}$ — (5)

Case (1) -: If time $\boxed{(t) = T}$ (Time constant)

then $c(t) = 1 - e^{-T/T} = 1 - e^{-1} = 0.632$ or 63.2%

* where T is known as the time constant of the system.

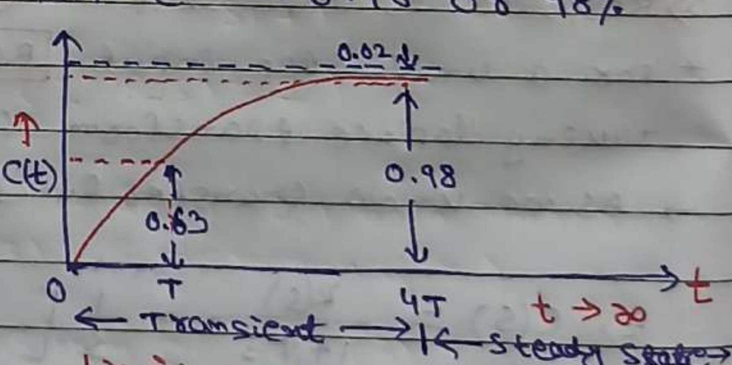
→ Time constant is defined as the time required by the signal to attain 63.2% of its final or steady state value.

Case (2) -: If $\boxed{t = 2T}$

then $c(t) = 1 - e^{-2T/T} = 1 - e^{-2} = 0.864$ or 86.4%

Case (3) -: If $t = 4T$ (Settling time (ts))

then $c(t) = 1 - e^{-4T/T} = 1 - e^{-4} = 0.98$ or 98%



Unit step response of first order system

* By taking derivative of eqn (5)

$$\left.\frac{d\,c(t)}{dt}\right|_{t=0} = \left.\frac{1}{T}e^{-t/T}\right|_{t=0} = \frac{1}{T}$$

Where T = Time constant.

* Now for error response of the system

$$\boxed{e(t) = r(t) - c(t) = 1 - (1 - e^{-t/\tau}) = e^{-t/\tau}} \quad -(6)$$

* For steady state error

$$\boxed{e_{ss} = \lim_{t \to \infty} e(t) = \lim_{t \to \infty} e^{-t/\tau} = 0} \quad -(7)$$

b) Response to the unit Ramp input -:

→ For unit Ramp input $\boxed{r(t) = t}$ — (1)

* Taking laplace transform of eqn (1), $\boxed{R(s) = \frac{1}{s^2}}$ —(2)

* As we know for 1st order control system $\frac{C(s)}{R(s)} = \frac{1}{1+Ts}$

then $C(s) = \frac{R(s)}{1+Ts} = \frac{1}{s^2}\left[\frac{1}{1+Ts}\right]$ $(\because R(s) = 1/s^2)$

$\Rightarrow C(s) = \frac{1/T}{s^2(s+\frac{1}{T})}$ (∵ Divide 'T' on both numerator & denominator)

$$\Rightarrow \boxed{C(s) = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+\frac{1}{T}}} \quad \text{(By using partial fraction)} \quad -(3)$$

→ By comparing numerator ⇒ $\boxed{\frac{1}{T} = As\left(s+\frac{1}{T}\right) + B\left(s+\frac{1}{T}\right) + Cs^2}$ —(4)

* put $\boxed{s=0}$,

then $\frac{1}{T} = A \cdot 0 \cancel{\left(0+\frac{1}{T}\right)} + B\left(0+\frac{1}{T}\right) + \cancel{C \cdot 0^2}$

$\Rightarrow \frac{1}{T} = \frac{B}{T} \Rightarrow \boxed{B = 1}$

* Put $\boxed{s = -1/T}$,

then $\frac{1}{T} = A \cdot \frac{-1}{T}\cancel{\left(\frac{-1}{T} + \frac{1}{T}\right)} + \cancel{B\left(\frac{-1}{T} + \frac{1}{T}\right)} + C \cdot \left(\frac{-1}{T}\right)^2$

$\Rightarrow \frac{1}{\cancel{T}} = \frac{C}{T^{\cancel{2}}} \Rightarrow \boxed{C = T}$

Now comparing the coefficients of s^2 on both sides of eqn (4)

$\Rightarrow 0 = As^2 + Cs^2 \Rightarrow A + C = 0 \Rightarrow A + T = 0 \Rightarrow \boxed{A = -T}$

* Put the value of A, B & C in eqn (3)

$$\Rightarrow \boxed{C(s) = \frac{-T}{s} + \frac{1}{s^2} + \frac{T}{s+\frac{1}{T}}} \quad -(5)$$

Taking inverse laplace transform function of Eqⁿ(5)

$c(t) = -T + t + Te^{-t/T}$

$\Rightarrow \boxed{c(t) = t - T(1 - e^{-t/T})}$ — (6)

* The error signal is $e(t) = r(t) - c(t)$

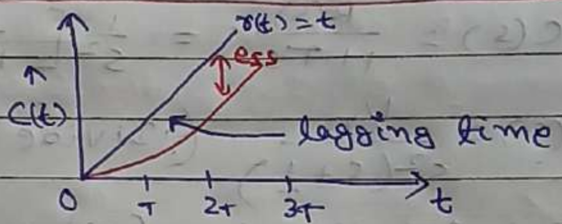
$\Rightarrow e(t) = t - [t - T(1 - e^{-t/T})]$

$\Rightarrow e(t) = \cancel{t} - \cancel{t} + T(1 - e^{-t/T})$

$\Rightarrow \boxed{e(t) = T(1 - e^{-t/T})}$ — (7)

* Now the steady state error is

$$\boxed{e_{ss} = \lim_{t\to\infty} e(t) = \lim_{t\to\infty} T(1 - e^{-t/T}) = T} \quad -(8)$$



(Unit ramp response of 1st order system)

c) Unit impulse Response of a 1st order System →

* For unit impulse input $\boxed{r(t) = \delta(t)}$ — (1)

* Taking laplace transform of Eqⁿ(1), $\boxed{R(s) = 1}$ — (2)

→ As we know, the 1st order transfer function is

$$\frac{C(s)}{R(s)} = \frac{1}{sT+1} \Rightarrow C(s) = \frac{R(s)}{sT+1}$$

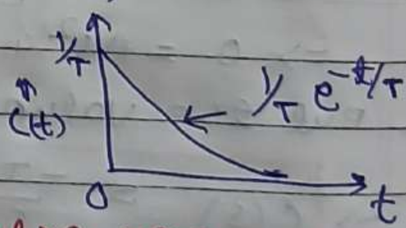
$$\Rightarrow C(s) = \frac{1}{sT+1} \quad (\because R(s) = 1)$$

$$\Rightarrow \boxed{C(s) = \frac{1/T}{\left(s + \frac{1}{T}\right)}} \quad -(3)$$

(Divide 'T' on both numerator & denominator)

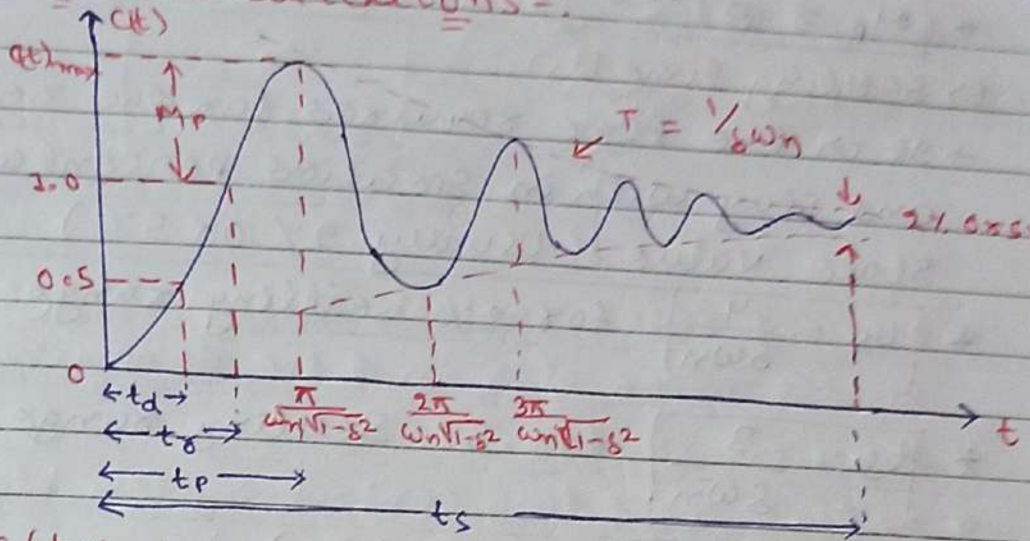
By taking laplace transform of Eqⁿ (3)

$$\Rightarrow \boxed{c(t) = \frac{1}{T} e^{-t/T}} \quad -(4)$$



(Unit impulse response of 1st order system)

Time Response Specifications -:



a) Delay time (t_d) -:

→ It is the time required for the response or output to reach from 0 to 50% of the steady state value at the first instant.

$$\boxed{t_d = \frac{1+0.7\delta}{\omega_n}}$$

where δ = Damping factor / Damping ratio

ω_n = undamped natural frequency.

b) Rise time (t_r) -:

→ It is defined as the time taken for the response to go from 0 to 100% of the steady state value at the 1st instant.

$$\boxed{t_r = \frac{\pi - \phi}{\omega_d}} \text{ or } \boxed{t_r = \frac{\pi - \phi}{\omega_n\sqrt{1-\delta^2}}}, \quad \boxed{\phi = \tan^{-1}\frac{\sqrt{1-\delta^2}}{\delta}}$$

$\boxed{\omega_d = \omega_n\sqrt{1+\delta^2}}$ = Damped frequency.

c) Peak time (t_p) -:

→ It is the time required for the response to reach the 1st peak overshoot.

$$\boxed{t_p = \frac{\pi}{\omega_d}} \text{ or } \boxed{t_p = \frac{\pi}{\omega_n\sqrt{1-\delta^2}}}$$

d) Peak overshoot or Maximum overshoot (M_p) -:

→ It is the maximum deviation of the instantaneous output above the steady state value.

→ % peak overshoot $= \frac{C(t_p) - C(\infty)}{C(\infty)} \times 100\%$

→ This is normally expressed in percentage.

* $\boxed{M_p = e^{\frac{-\pi\delta}{\sqrt{1-\delta^2}}}}$ & $\boxed{\% M_p = e^{\frac{-\pi\delta}{\sqrt{1-\delta^2}}} \times 100}$

e> settling time (t_s):-

→ It is the time required for the response to reach and stay with in specified percentage of steady state value. (Usually 2% or 5%)

* $\boxed{t_s = \frac{4}{\delta\omega_n}}$ for 2% settling range

* $\boxed{t_s = \frac{3}{\delta\omega_n}}$ for 5% settling range

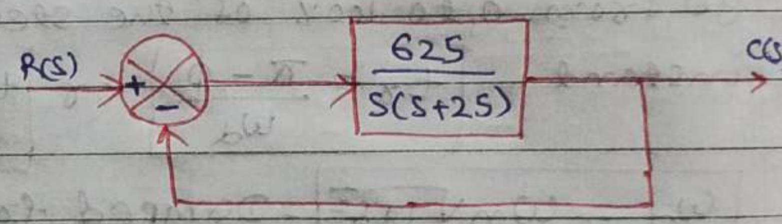
f> Steady state error (e_{ss})-:

→ It is defined as the error between the actual output and desired output as 't' tends to infinity. ($t \to \infty$)

$$\boxed{e_{ss} = \lim_{t\to\infty} e(t) = \lim_{t\to\infty} [r(t) - c(t)]}$$

Q// A second order system is given below of unit step input.

Determine



a> Natural frequency

b> Damping ratio

c> Damped frequency d> Peak time

e> percentage of overshoot f> 5% settling time.

g> Rise time.

Ans-: $G(s) = \frac{625}{S(S+25)}$, $H(S) = 1$

* The closed loop transfer function of the system is

$$\frac{C(S)}{R(S)} = \frac{G(S)}{1+G(S)H(S)} = \frac{\frac{625}{S(S+25)}}{1+\frac{625}{S(S+25)}} = \frac{\frac{625}{S(S+25)}}{\frac{S(S+25)+625}{S(S+25)}}$$

$$= \frac{625}{S^2+25S+625}$$

* Characteristic equation is $s^2+25s+625=0$ —(1)
* But the standard characteristic Eqn is $s^2+2\delta\omega_n s+\omega_n^2=0$ —(2)

a) Natural frequency (ω_n) :-

$$\omega_n^2 = 625 \Rightarrow \omega_n = \sqrt{625} = 25 \text{ rad/sec}$$

b) Damping ratio (δ) :-

$$2\delta\omega_n = 25 \Rightarrow \delta = \frac{25}{2\omega_n} = \frac{\cancel{25}}{2\times\cancel{25}} = \frac{1}{2} = 0.5$$

c) Damped frequency (ω_d) :-

$$\omega_d = \omega_n\sqrt{1-\delta^2} = 25\sqrt{1-0.5^2} = 21.65 \text{ rad/sec}$$

d) Peak time (t_p) :-

$$t_p = \frac{\pi}{\omega_n\sqrt{1-\delta^2}}$$

$$\Rightarrow t_p = \frac{\pi}{\omega_d} = \frac{\pi}{21.65} = 0.145 \text{ sec.} \quad (\because \omega_d = \omega_n\sqrt{1-\delta^2})$$

e) Percentage of overshoot ($\%M_p$) $= e^{\frac{\pi\delta}{\sqrt{1-\delta^2}}} \times 100$

$$= e^{\frac{\pi\times 0.5}{\sqrt{1-0.5^2}}}\times 100 = 0.163\times 100 = 16.3\%$$

f) 5% settling time (t_s) $= \frac{3}{\delta\omega_n} = \frac{3}{0.5\times 25} = 0.24$ sec.

g) Rise time (t_r)

$$t_r = \frac{\pi-\phi}{\omega_n\sqrt{1-\delta^2}}$$

But $\phi = \tan^{-1}\left(\frac{\sqrt{1-\delta^2}}{\delta}\right)$

$$\Rightarrow t_r = \frac{\pi-\phi}{\omega_d}$$

$$\Rightarrow \phi = \tan^{-1}\left(\frac{\sqrt{1-0.5^2}}{0.5}\right) = 60^\circ$$

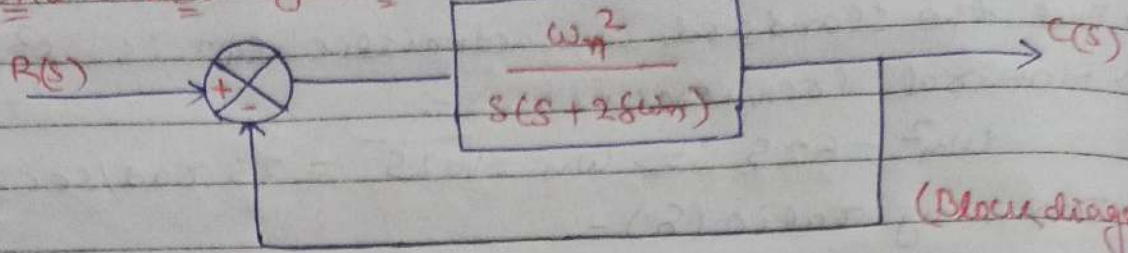
$$\Rightarrow t_r = \frac{\pi - 1.047}{21.65}$$

Convert it into radian

$$\phi = \Rightarrow 60^\circ \times \frac{\pi}{180} = 1.047$$

$$\Rightarrow t_r = 0.096 \text{ sec}$$

second order system :-



(Block diagram)

* A second order control system is one in which the highest power of 's' in the denominator of transfer function is two.
* In this closed loop control system block diagram has $G(S) = \frac{\omega_n^2}{S(S+2\delta\omega_n)}$, $H(S) = 1$

where $G(S)$ = forward path transfer function
$H(S)$ = feedback transfer function
ω_n = undamped natural frequency
δ = damping factor/ damping ratio.

* Now, the closed loop transfer function is

$$\frac{C(S)}{R(S)} = \frac{G(S)}{1+G(S)H(S)}$$

$$= \frac{\frac{\omega_n^2}{S(S+2\delta\omega_n)}}{1+\frac{\omega_n^2}{S(S+2\delta\omega_n)}} = \frac{\frac{\omega_n^2}{S(S+2\delta\omega_n)}}{\frac{S(S+2\delta\omega_n)+\omega_n^2}{S(S+2\delta\omega_n)}}$$

$$\Rightarrow \boxed{\frac{C(S)}{R(S)} = \frac{\omega_n^2}{S^2+2\delta\omega_n S+\omega_n^2}}$$

* So the characteristic equation for 2nd order system is $\boxed{S^2+2\delta\omega_n S+\omega_n^2 = 0}$

Time Response-:

* Time response of a control system is defined as the variation of output with respect to time, when an input signal is applied to the system.
* The time response of a control system consists of two parts. a> transient response b> steady state response.

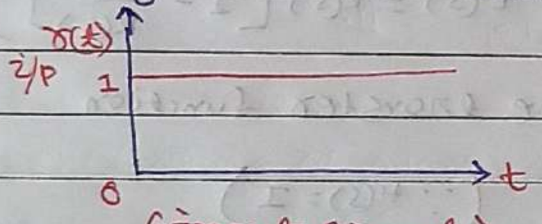
a> Transient response-:

→ The time response of a control system for a very short time is called transient response.

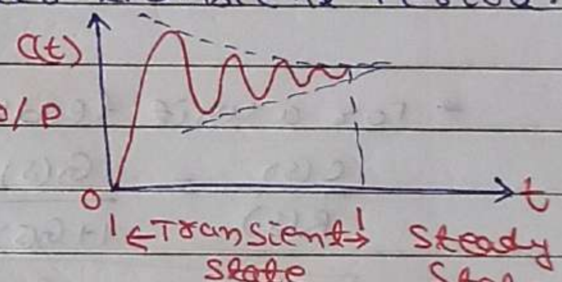
b> Steady state response-:

→ The time response of a control system is a state of the output, when time approaches to infinity. ($t \to \infty$)

→ Steady state is achieved after transient period.



(input signal)



(Time response)

Steady state response-:

→ when an input signal is applied to a system, it gives output response.

→ It is a time response and consist of two parts namely transient state and steady state.

$$\boxed{c(t) = c_t(t) + c_{ss}(t)}$$

where, $c(t)$ = total time response

$c_t(t)$ = transient response

$c_{ss}(t)$ = Steady state response

Steady state error-:

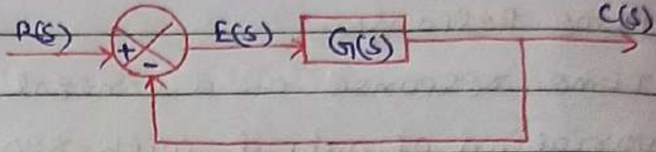
→ It is defined as the variation between the steady state output and the input.

→ The steady state error represents the accuracy of the control systems.

→ If the variation is small, the system is more accurate.

→ The error is zero for an ideal system.

* The error signal is defined as the difference between the input & output signal.



(Closed loop control system)

* $\boxed{E(S) = R(S) - C(S)}$ or $\boxed{E(S) = R(S)\left[1 - \frac{C(S)}{R(S)}\right]}$ —(1)

* The steady state error can be obtained by using final value theorem, which is given by

$$e_{ss} = \lim_{t \to \infty} = \lim_{S \to 0} S\,E(S)$$

$$\Rightarrow \boxed{e_{ss} = \lim_{S \to 0} S\,R(S)\left[1 - \frac{C(S)}{R(S)}\right]} \quad \text{—— (2)}$$

$$\left(\because E(S) = R(S)\left[1 - \frac{C(S)}{R(S)}\right]\right)$$

* For a unit feedback, the transfer function

$$\frac{C(S)}{R(S)} = \frac{G(S)}{1 + G(S)H(S)} \qquad (\because H(S) = 1)$$

$$\Rightarrow \boxed{\frac{C(S)}{R(S)} = \frac{G(S)}{1 + G(S)}} \quad \text{—— (3)}$$

put eq^n(3) in eq^n (2), then

$$e_{ss} = \lim_{S \to 0} S\,R(S)\left[1 - \frac{G(S)}{1+G(S)}\right]$$

$$\Rightarrow e_{ss} = \lim_{S \to 0} S\,R(S)\left[\frac{1 + \cancel{G(S)} - \cancel{G(S)}}{1+G(S)}\right]$$

$$\Rightarrow e_{ss} = \lim_{S \to 0} S\,R(S)\left[\frac{1}{1+G(S)}\right]$$

$$\Rightarrow \boxed{e_{ss} = \lim_{S \to 0} \frac{S\,R(S)}{1+G(S)}} \quad \text{—(4)}$$

→ From the above expression, it is cleared that steady state error depends on input to the system R(S) & open loop transfer function G(S).H(S).

* The steady state error can be obtained by using two methods.

i) Error constant method ii) Generalised error series method.

Error constant method -:

→ It can be expressed by two types of methods.

a) Static error coefficient b) Dynamic error coefficient

Static error coefficient -:

→ Static error coefficient depends on open loop transfer function $G(s)H(s)$.

→ For evaluating steady state error, the input function is specified as either unit step (displacement) or unit ramp (velocity) or unit parabolic (acceleration).

a) Static positional error coefficient -: (K_p)

* Static positional error coefficient (K_p) is associated with unit step input applied to a closed-loop control system.

* Let $\boxed{r(t) = u(t)}$ —(1)

Taking laplace transform of eqn (1)

$$\boxed{R(s) = 1/s} \quad -(2)$$

* Steady state error, $e_{ss} = \lim_{s \to 0} s \cdot \frac{R(s)}{1+G(s)H(s)}$

$$\Rightarrow e_{ss} = \lim_{s \to 0} s \cdot \frac{\frac{1}{s}}{1+G(s)H(s)} = \frac{1}{1+\lim_{s \to 0} G(s)H(s)}$$

put $\boxed{K_p = \lim_{s \to 0} G(s)H(s)}$, K_p is called as static positional error coefficient, Therefore

$$\boxed{e_{ss} = \frac{1}{1+K_p}} \quad -(3)$$

b) Static velocity error coefficient -: (K_v)

* Static velocity error coefficient is associated with unit ramp input applied to a closed-loop control system.

* Let $\boxed{r(t) = t}$ — (1)

Taking laplace transform of Eqn (1)

$\boxed{R(s) = \frac{1}{s^2}}$ — (2)

* The steady state error, $e_{ss} = \lim_{s \to 0} s \cdot \frac{R(s)}{1+G(s)H(s)}$

$\Rightarrow e_{ss} = \lim_{s \to 0} s \cdot \frac{\frac{1}{s^2}}{1+G(s)H(s)}$ $\Rightarrow e_{ss} = \lim_{s \to 0} \frac{1}{s(1+G(s)H(s))}$

$\Rightarrow e_{ss} = \lim_{s \to 0} \frac{1}{s+sG(s)H(s)} = \lim_{s \to 0} \frac{1}{sG(s)H(s)}$

* Put $\boxed{K_v = \lim_{s \to 0} s\, G(s)H(s)}$, K_v is called static velocity error coefficient.

Then, $\boxed{e_{ss} = \frac{1}{K_v}}$ — (3)

c) Static acceleration error coefficient :- (K_a)

→ Static acceleration error coefficient is associated with unit parabolic input applied to a closed loop control system.

* Let $\boxed{r(t) = \frac{t^2}{2}}$ — (1)

Taking laplace transform of Eqn (1)

$\boxed{R(s) = \frac{1}{\cancel{2}} \cdot \frac{2!}{s^{2+1}} = \frac{1}{s^3}}$ — (2)

* The steady state error, $e_{ss} = \lim_{s \to 0} s \cdot \frac{R(s)}{1+G(s)H(s)}$

$\Rightarrow e_{ss} = \lim_{s \to 0} s \cdot \frac{\frac{1}{s^{\cancel{3}2}}}{1+G(s)H(s)}$ $\Rightarrow e_{ss} = \lim_{s \to 0} \frac{1}{s^2[1+G(s)H(s)]}$

$\Rightarrow e_{ss} = \lim_{s \to 0} \frac{1}{s^2 + s^2G(s)H(s)} = \lim_{s \to 0} \frac{1}{s^2G(s)H(s)}$

* Put $\boxed{K_a = \lim_{s\to 0} s^2 G(s) H(s)}$, K_a is called static acceleration error coefficient.

Then, $\boxed{e_{ss} = \frac{1}{K_a}}$ — (3)

Open-loop transfer function :-

* The product of the forward path transfer function and the feedback path transfer function of a control system is known as open-loop transfer function.

$$\boxed{G(s)H(s) = \frac{K(1+sT_a)(1+sT_b)\text{-------}}{s^N(1+sT_1)(1+sT_2)\text{-------}}}$$

* where K = forward path gain.

N = The no. of poles at the origin.

* For the above Eqn, poles are $-\frac{1}{T_1}, -\frac{1}{T_2}$ ------

& zeros are $-\frac{1}{T_a}, -\frac{1}{T_b}$ ------

Type of transfer functions and steady state error :-

1a) Type '0' system with unit step input :-

* For a type '0' system, the open-loop transfer function is given by $\boxed{G(s)H(s) = \frac{K(1+sT_a)(1+sT_b)\text{-------}}{(1+sT_1)(1+sT_2)\text{-----}}}$

then, $K_p = \lim_{s\to 0} G(s)H(s)$

$$\Rightarrow K_p = \lim_{s\to 0} \frac{K(1+sT_a)(1+sT_b)\text{------}}{(1+sT_1)(1+sT_2)\text{-----}}$$

$$\Rightarrow \boxed{K_p = K}$$

* The steady state error $\boxed{e_{ss} = \frac{1}{1+K_p} = \frac{1}{1+K}}$

b) Type '0' system with unit ramp input :-

* For a type '0' system, the open-loop transfer function is given by $\boxed{G(s)H(s) = \frac{K(1+sT_a)(1+sT_b)\text{-------}}{(1+sT_1)(1+sT_2)\text{-------}}}$

then, $K_v = \lim_{S \to 0} S\, G(S)\, H(S)$

$$\Rightarrow K_v = \lim_{S \to 0} S \cdot \frac{K(1+ST_a)(1+ST_b)\text{-----}}{(1+ST_1)(1+ST_2)\text{-----}}$$

$$\Rightarrow \boxed{K_v = 0}$$

* Now, the steady state error, $\boxed{e_{ss} = \frac{1}{K_v} = \frac{1}{0} = \infty}$

c) Type '0' system with unit parabolic input-:

→ For a type '0' system, the open-loop transfer function is given by, $\boxed{G(S)\,H(S) = \frac{K(1+ST_a)(1+ST_b)\text{----}}{(1+ST_1)(1+ST_2)\text{----}}}$

Then, $K_a = \lim_{S \to 0} S^2\, G(S)\, H(S)$

$$\Rightarrow K_a = \lim_{S \to 0} S^2 \cdot \frac{K(1+ST_a)(1+ST_b)\text{-----}}{(1+ST_1)(1+ST_2)\text{-----}}$$

$$\Rightarrow \boxed{K_a = 0}$$

* The steady state error, $\boxed{e_{ss} = \frac{1}{K_a} = \frac{1}{0} = \infty}$

2a) Type '1' system with unit step input-:

→ For a type '1' system, the open-loop transfer function is given by, $\boxed{G(S)\,H(S) = \frac{K(1+ST_a)(1+ST_b)\text{----}}{S(1+ST_1)(1+ST_2)\text{-----}}}$

then $K_p = \lim_{S \to 0} G(S)\, H(S)$

$$\Rightarrow K_p = \lim_{S \to 0} \frac{K(1+ST_a)(1+ST_b)\text{-----}}{S(1+ST_1)(1+ST_2)\text{-----}}$$

$$\Rightarrow \boxed{K_p = \infty}$$

* The steady state error,

$$\boxed{e_{ss} = \frac{1}{1+K_p} = \frac{1}{1+\infty} = 0}$$

b) Type '1' system with unit ramp input -:

* For a type '1' system, the open-loop transfer function is given by $\boxed{G(s)\,H(s) = \frac{K(1+sT_a)(1+sT_b)\text{------}}{s(1+sT_1)(1+sT_2)\text{------}}}$

then, $K_v = \lim_{s \to 0} s\,G(s).H(s)$

$$\Rightarrow K_v = \lim_{s \to 0} \cancel{s} \cdot \frac{K(1+sT_a)(1+sT_b)\text{-----}}{\cancel{s}(1+sT_1)(1+sT_2)\text{-----}}$$

$$\Rightarrow \boxed{K_v = K}$$

* The steady state error, $\boxed{e_{ss} = \frac{1}{K_v} = \frac{1}{K}}$

c) Type '1' system with unit parabolic input -:

* For a type '1' system, the open-loop transfer function is given by $\boxed{G(s)\,H(s) = \frac{K(1+sT_a)(1+sT_b)\text{-----}}{s(1+sT_1)(1+sT_2)\text{-----}}}$

then, $K_a = \lim_{s \to 0} s^2\,G(s).H(s)$

$$\Rightarrow K_a = \lim_{s \to 0} s^{\cancel{2}} \frac{K(1+sT_a)(1+sT_b)\text{-----}}{\cancel{s}(1+sT_1)(1+sT_2)\text{-----}}$$

$$\Rightarrow \boxed{K_a = 0}$$

* The steady state error, $\boxed{e_{ss} = \frac{1}{K_a} = \frac{1}{0} = \infty}$

3a) Type '2' system with unit step input -:

→ For a type '2' system, the open-loop transfer function is given by $\boxed{G(s)\,H(s) = \frac{K(1+sT_a)(1+sT_b)\text{-----}}{s^2(1+sT_1)(1+sT_2)\text{-----}}}$

then, $K_p = \lim_{s \to 0} G(s)\,H(s)$

$$\Rightarrow K_p = \lim_{s \to 0} \frac{K(1+sT_a)(1+sT_b)\text{-----}}{s^2(1+sT_1)(1+sT_2)\text{------}}$$

$$\Rightarrow \boxed{K_p = \infty}$$

* The steady state error, $\boxed{e_{ss} = \frac{1}{1+K_p} = \frac{1}{1+\infty} = 0}$

b) Type '2' system with unit ramp input:-

→ For a type '2' system, the open-loop transfer function is given by $\boxed{G(s)H(s) = \frac{K(1+sT_a)(1+sT_b)\text{-----}}{s^2(1+sT_1)(1+sT_2)\text{-----}}}$

Then, $K_v = \lim_{s \to 0} s\, G(s)H(s)$

$$\Rightarrow K_v = \lim_{s \to 0} s \cdot \frac{K(1+sT_a)(1+sT_b)\text{-----}}{s^2(1+sT_1)(1+sT_2)\text{-----}}$$

$$\Rightarrow \boxed{K_v = \infty}$$

* The steady state error, $\boxed{e_{ss} = \frac{1}{K_v} = \frac{1}{\infty} = 0}$

c) Type '2' system with unit parabolic input:-

* For a type '2' system, the open-loop transfer function is given by $\boxed{G(s)H(s) = \frac{K(1+sT_a)(1+sT_b)\text{---}}{s^2(1+sT_1)(1+sT_2)\text{----}}}$

Then, $K_a = \lim_{s \to 0} s^2\, G(s) \cdot H(s)$

$$\Rightarrow K_a = \lim_{s \to 0} s^2 \cdot \frac{K(1+sT_a)(1+sT_b)\text{----}}{s^2(1+sT_1)(1+sT_2)\text{----}}$$

$$\Rightarrow \boxed{K_a = K}$$

* The steady state error,

$$\boxed{e_{ss} = \frac{1}{K_a} = \frac{1}{K}}$$

Q1) Determine position, velocity and acceleration error constant of a unity feedback system having forward path transfer function given by $G(s) = \frac{20}{s(s+5)(s+10)}$. Find steady state error.

Ans-: It is a unity feedback system. So $H(s) = 1$

* For open loop transfer function $= G(s)H(s) = \frac{20}{s(s+5)(s+10)}$

a) Position error constant $(K_p) = \lim_{s\to 0} G(s)H(s)$

$$\Rightarrow K_p = \lim_{s\to 0} \frac{20}{s(s+5)(s+10)} = \frac{20}{0} = \infty$$

→ steady state error $(e_{ss}) = \frac{1}{1+K_p} = \frac{1}{1+\infty} = \frac{1}{\infty} = 0$

b) velocity error constant $(K_v) = \lim_{s\to 0} s\, G(s)H(s)$

$$\Rightarrow K_v = \lim_{s\to 0} \cancel{s} \cdot \frac{20}{\cancel{s}(s+5)(s+10)}$$

$$\Rightarrow K_v = \frac{2\cancel{0}}{5\times 1\cancel{0}} = \frac{2}{5} = 0.4$$

→ steady state error $(e_{ss}) = \frac{1}{K_v} = \frac{1}{0.4} = 2.5$

c) Acceleration error constant $(K_a) = \lim_{s\to 0} s^2 G(s)H(s)$

$$\Rightarrow K_a = \lim_{s\to 0} s^2 \frac{20}{\cancel{s}(s+5)(s+10)} = 0$$

→ steady state error $(e_{ss}) = \frac{1}{K_a} = \frac{1}{0} = \infty$

Q. Find the position, velocity and acceleration error constant of a feedback control system whose open loop transfer function is $\frac{20(S+3)}{S^2(S^2+8S+100)}$ and also find the steady state error.

Ans-: This is a type '2' system.

$$G(S)\,H(S) = \frac{20(S+3)}{S^2(S^2+8S+100)}$$

a) Position error constant $(K_P) = \lim_{S\to 0} G(S)\,H(S)$

$$\Rightarrow K_P = \lim_{S\to 0} \frac{20(S+3)}{S^2(S^2+8S+100)} = \infty$$

→ The steady state error $(e_{ss}) = \frac{1}{1+K_P} = \frac{1}{1+\infty} = 0$

b) velocity error constant $(K_V) = \lim_{S\to 0} S \cdot G(S)\,H(S)$

$$\Rightarrow K_V = \lim_{S\to 0} \cancel{S} \cdot \frac{20(S+3)}{S^{\cancel{2}}(S^2+8S+100)} = \infty$$

→ The steady state error $(e_{ss})_V = \frac{1}{K_V} = \frac{1}{\infty} = 0$

c) Acceleration error constant $(K_a) = \lim_{S\to 0} S^2 \cdot G(S)\,H(S)$

$$\Rightarrow K_a = \lim_{S\to 0} \cancel{S^2} \cdot \frac{20(S+3)}{\cancel{S^2}(S^2+8S+100)}$$

$$\Rightarrow K_a = \frac{\cancel{20} \times 3}{\cancel{100}\,5} = \frac{3}{5} = 0.6$$

→ The steady state error $(e_{ss}) = \frac{1}{K_a} = \frac{1}{0.6} = 1.66$

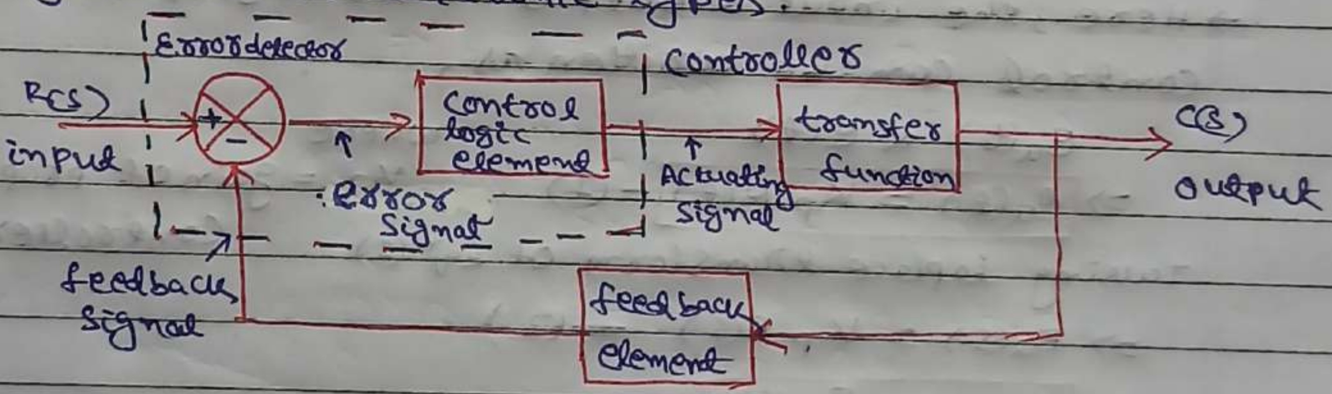
Automatic controller & control action :-

→ An automatic control system is used to maintain its output within its desirable limits by means of automatic controller.

→ The automatic controller determines the value of controlled variables by compared the actual value to the desired value (reference input), determine the deviation and produces a control signal that will reduce the deviation to zero or as small as possible value.

→ The method by which the automatic controller produces the control signal is called control action.

→ The controllers can be electrical, electromechanical hydraulic or electronic types.



Uses of controllers :-

→ It improve the steady state accuracy by reducing the steady state errors.

→ As the steady state accuracy improves, the stability also improves.

→ By the help of controllers, maximum overshoot signal of a system can be controlled.

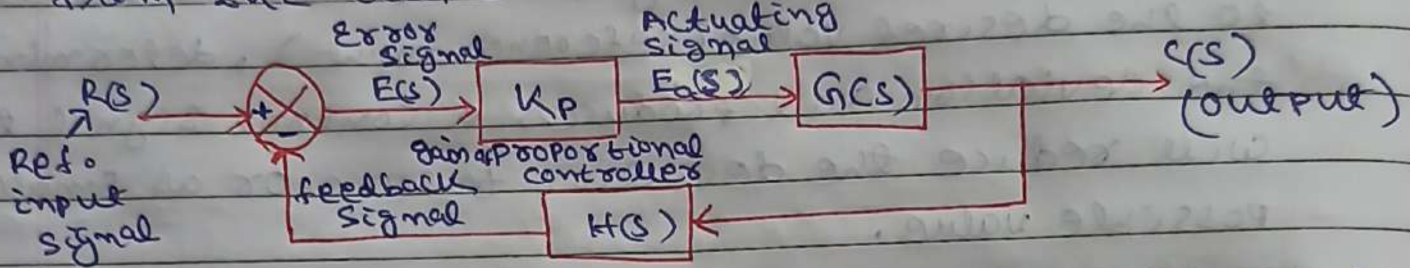
→ It also help in reducing the noise signals produced in the system.

→ Slow response can be made faster by the controllers.

Proportional Control

→ In proportional control, the actuating signal $[e_a(t)]$ is proportional to the error signal $[e(t)]$

→ The error signal is the difference between the reference input signal and the feedback signal obtained from the output.



→ In this block diagram, the actuating signal is proportional to the error signal. Therefore, the system is called proportional control system.

→ Here, the input & output relationship of proportional control can be written as,

$$e_a(t) \propto e(t)$$

$$\Rightarrow \boxed{e_a(t) = K_p\, e(t)} \quad -(1)$$

where K_p = gain of proportional controller

Taking laplace transform of Eq^n (1)

$$\Rightarrow E_a(s) = K_p\, E(s)$$

$$\Rightarrow \boxed{K_p = \frac{E_a(s)}{E(s)}} \quad -(2)$$

Advantages -

→ It reduces the steady state error.

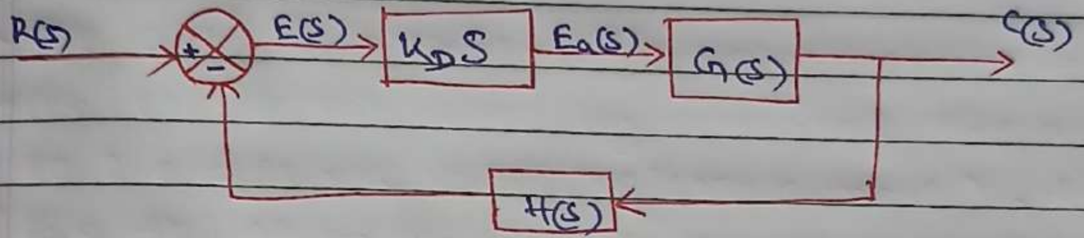
→ It makes the response of the system faster by increasing the forward path gain.

Disadvantages -

→ Max^m overshoot is increased.

Derivative control -:

→ In this type of control action, the actuating signal is proportional to the derivative of error signal.



* Mathematically we can written as

$$e_a(t) \propto \frac{d}{dt} e(t)$$

$$\Rightarrow \boxed{e_a(t) = K_d \frac{d}{dt} e(t)} \quad -(1)$$

where K_D = gain of derivative control

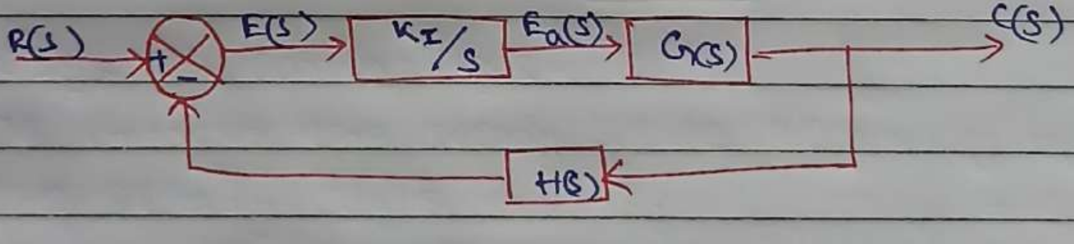
Taking laplace transform of Eqn(1)

$$\Rightarrow E_a(S) = K_D S E(S)$$

$$\Rightarrow \boxed{K_D = \frac{E_a(S)}{S E(S)}} \quad -(2)$$

Integral control -:

→ In this type of control action, the actuating signal is proportional to the integration of error signal.



* Mathematically we can written as

$$e_a(t) \propto \int e(t) \cdot dt$$

$$\Rightarrow \boxed{e_a(t) = K_i \int e(t) \cdot dt} \quad -(1)$$

where K_I = gain of integral control

Taking laplace transform of Eqn(1)

$$\Rightarrow E_a(S) = \frac{K_I}{S} E(S)$$

$$\Rightarrow \boxed{K_I = \frac{S E_a(S)}{E(S)}} \quad -(2)$$