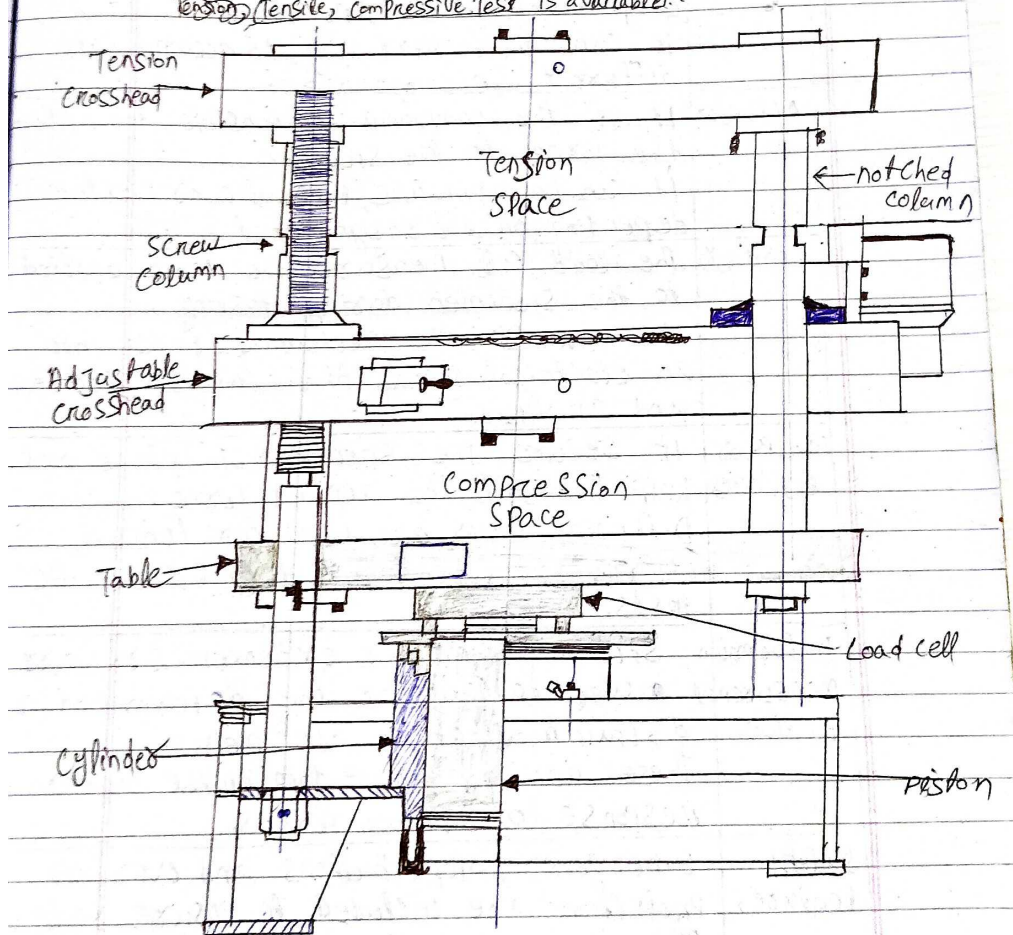


UNIVERSAL TESTING MACHINE

Introduction:-

A Universal Testing machine (UTM) is a versatile piece of equipment used to perform a variety of mechanical tests on materials, components and structures. It typically consists of several key parts and components. Here's an overview of the main parts of a UTM.

~~Tension~~ (Tensile, Compressive Test is available):-



Universal Testing machine

Component	function
Load frame	main structure of the UTM, providing support for the other components. designed to withstand and distribute loads during testing.
Crosshead	movable part that holds and moves the grips or fixtures to apply tension or compression to the specimen. It can move vertically to accommodate different specimen sizes.
Actuator	It is the component responsible for applying the force to the specimen. It can be hydraulic, pneumatic, or electric depending on the design of the UTM.
Load cell	The load cell measures the force applied to the specimen during testing. It converts the mechanical force into an electrical signal that can be recorded and analyzed.
Grips or fixtures	It secure the specimen in place and help transmit the applied force. Different grips are used for tension, compression, bending, and other specific tests.
Deformation measurement device	Strain gauges or extensometers are used to measure the deformation or displacement of the specimen. These devices provide data on the material's response to applied forces.
Safety features	Emergency stop buttons and overload protection are included to ensure safe operation.

control and Data Acquisition system.	The control system manages the testing parameters, such as load and displacement rates. The data acquisition system records and stores data during the test, including force and deformation measurements.
Software	UTM software is used for test setup, data collection, and analysis. It allows users to customize tests parameters and generate test reports.
Display unit	A display unit shows real-time data during the test, including load, displacement, and other relevant parameters.
Power Supply	UTMs require a power supply to operate the various components, such as the actuator and control system.
Base	The base provides stability and supports for the entire UTM.

These are the general components of a universal Testing machine. The specific design and features may vary depending on the manufacturer and the intended applications of the machine.

Tensile Test for mild steel:-

The American Society for Testing and Materials (ASTM) has published guidelines for conducting test. When a material (specimen) is subjected to a load, its stress-strain diagram provides valuable information about its mechanical properties and behaviour under various condition. The stress-strain curve is a graphical representation that illustrates the relationship between stress and strain for a given material.

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Here's an analysis of the stress-strain diagram for mild steel (MS) specimen (solid cylindrical rod, L/D ratio = 4, where L = gauge length of specimen, D = diameter of specimen).

(1) Elastic Region:-

In the initial part of the stress-strain curve, the material undergoes elastic deformation. Stress (σ) is directly proportional to strain (ϵ) according to Hooke's law ($\sigma = E\epsilon$), where E is the Young's modulus. mild ~~steel~~ steel typically exhibits linear behavior in this region.

(2) Yield point:-

The Yield point is the stress at which the material begins to exhibit plastic deformation. For mild steel, this point is well-defined and marks the onset of permanent deformation. Beyond the yield point, the material may no longer return to its original shape after the removal of the load.

(3) Plastic Deformation:-

The stress-strain curve enters the plastic region where further strain results in permanent deformation. The material continues to deform plastically until it reaches the ultimate tensile strength.

(4) Ultimate tensile strength (UTS):-

The UTS is the maximum stress that the material can withstand. At this point, the material experiences its maximum strength before failure.

(5) Necking

After reaching UTS, some materials, including mild steel, undergo necking.

necking is a localized reduction in cross-sectional area, leading to further strain concentration.

(6) Fracture Points:-

The stress-strain curve reaches its fracture point where the material fails. Fracture may occur suddenly or be preceded by a reduction in load-carrying capacity.

(7) Analysis:-

⇒ Elastic Modulus (Young's modulus):- The slope of the elastic region represents the material's stiffness or elastic modulus.

⇒ Yield Strength:- The stress at the yield point indicates the maximum stress a material can withstand without permanent deformation.

⇒ Ultimate Tensile Strength (UTS):- The UTS represents the maximum stress the material can endure.

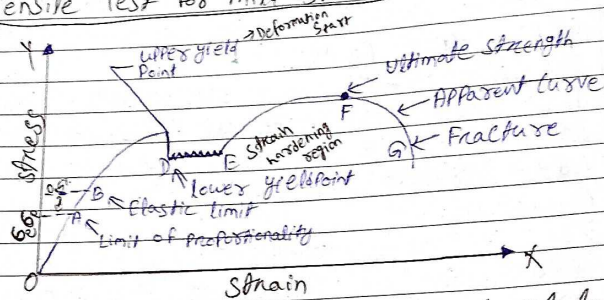
⇒ Ductility:- The elongation and reduction in area after the yield point provide insights into the material's ductility.

⇒ Toughness:- The area under the stress-strain curve represents the material's toughness, indicating its ability to absorb energy.

⇒ Understanding the stress-strain diagram for mild steel is crucial for designing structures and components, ensuring they can withstand expected loads without failure or excessive deformation.

ody

Tensile Test for mild steel



Stress strain curve forms under tensile loading

Chapter 1

Simple Stress and Strain

* mass:- It is an amount of matter that is contained in a body.

⇒ unit is kg.

⇒ It is a scalar quantity.

* Weight:- It is a force with which it is attracted towards the centre of earth.

⇒ unit is Newton. (mass $\times$ acceleration due to gravity)

⇒ It is a vector quantity.

* Strength:- The ability to resist the external force is known as strength.

⇒ It is a mechanical property.

* Stress:- The internal resistance force ($\frac{N}{m^2}$) offered by a body per unit area. (unit = Pascal)

($\frac{F}{A}$) ⇒ stress is denoted as sigma (σ) = $\frac{\text{Force } F}{\text{Area } A}$

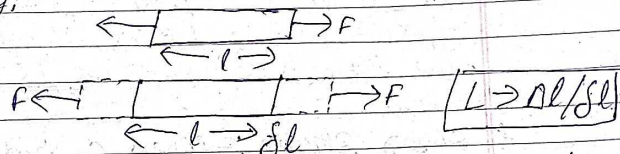
* Strain:- The amount of deformation in the direction of the applied force divided by the original length of the material.

Pascal's law :- Pressure transmit equally in all directions in a body.

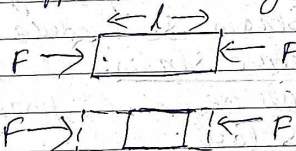
Types of Stress :-

- (i) Tensile Stress
- (ii) Compressive Stress
- (iii) Shear Stress
- (iv) Bending Stress
- (v) Torsion Stress

(1) Tensile Stress :- The stress produces due to two equal and opposite pulling forces acting on a body.



(2) Compressive Stress :- The stress produces due to two equal and opposite pushing forces acting on a body.



⇒ In this case the length may decrease or the body may break down.

(3) Shear Stress :- When the body is subjected to two equal and opposite tangential/surfacial forces so the body may be sheared off hence the stress so produce is called shear stress.

Strain :-

⇒ It is defined as the energy stored in a body due to deformation.

- * It is 4 type :- (i) Tensile Strain
(ii) Compressive Strain
(iii) Shear Strain
(iv) Volumetric Strain.

* Tensile Strain:- The stress produced due to two equal and opposite pulling force acting on the body, hence the strain so produced is known as tensile strain.

* Compressive Strain:-
⇒ The stress produced due to two equal and opposite pushing forces acting on the body, hence the strain so produced is known as compressive strain.

* Shear Strain:- when a body subjected to two equal and opposite tangential/surface forces so body may be sheared off hence the strain so produce is called shear strain.

* Volumetric Strain:- It is the ratio betⁿ change in volume to original value.

Hook's law.

⇒ It states that within elastic limit linear stress is directly proportional to linear strain.

$$\sigma \propto \epsilon \text{ or } \sigma = E \times \epsilon$$

$$E = \frac{\sigma}{\epsilon} = \frac{\text{linear stress } (\sigma)}{\text{linear strain } (\epsilon)}$$

E = Young's modulus
= modulus of elasticity.

Stress along the transverse axis

* Lateral Strain:- When a body is objected to linear strain ϵ is produced in the same direction an opposite kind of strain is produce in every direction that

- opposite kind of strength, is known as lateral strain. mathematically:- $\frac{\delta d}{d}$

$$\text{linear strain} = \frac{\delta l}{l}$$

Poisson's Ratio ^(μ) - It is ratio between lateral strain to linear strain. $\frac{\delta d}{d} = \mu \frac{\delta l}{l}$ $\mu = 0.25 \text{ to } 0.3$

Elastic limit:- It is a limit within which if we apply force and withdraw it body will be returned to its original position or regain its original shape and size.

δl = change in length

$$\frac{\delta l}{l} = \frac{F \cdot l}{A \cdot E} \Rightarrow F = \text{force} \quad A = \text{Area} \quad l = \text{length} \quad E = \text{Young's modulus}$$

Note:-

$$1 \text{ kPa} = 10^3 \text{ Pa} = 10^3 \frac{\text{N}}{\text{mm}^2}$$

$$1 \text{ MPa} = 10^6 \text{ Pa} = 1 \frac{\text{N}}{\text{mm}^2}$$

$$1 \text{ GPa} = 10^9 \text{ Pa} = 10^3 \frac{\text{N}}{\text{mm}^2}$$

a) Data given

Steel bar = 2 m

Area = 150 mm²

$$E = 200 \text{ GPa} = 200 \times 10^3 \frac{\text{N}}{\text{mm}^2}$$

$$F = 15 \text{ kN}$$

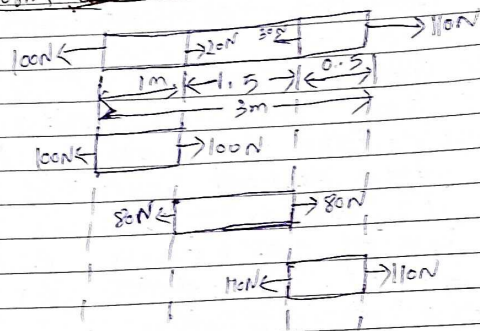
$\delta l = ?$

$$\delta l = \frac{15000 \times 2000}{150 \times 200 \times 10^3} = 1 \text{ mm}$$

Principle of Superposition

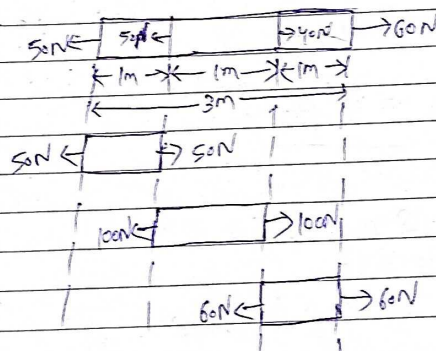
⇒ The process of finding of the resultant deformation when diff. magnitude of forces acting at a different section is known as Principle of Superposition.

Uniform Section



$$100 - 20 = 80$$

$$110 - 30 = 80$$



$$50 + 50 = 100$$

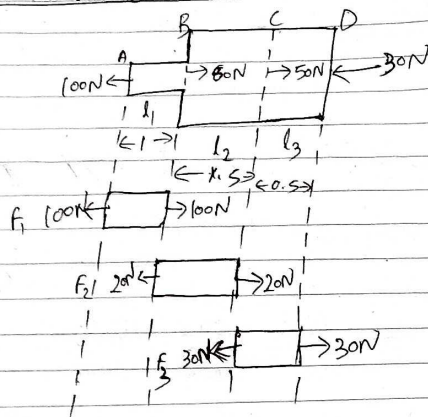
$$60 + 40 = 100$$

$$\delta l = \frac{1}{AE} \left[\pm F_1 l_1 \pm F_2 l_2 \pm F_3 l_3 \right]$$

$$\delta l = \frac{F_1 l_1}{AE} \Rightarrow \delta l = \frac{F_1 l_1}{AE} + \frac{F_2 l_2}{AE} + \frac{F_3 l_3}{AE}$$

$$\Rightarrow \delta l = \frac{1}{AE} \left[\pm \frac{F_1 l_1}{A_1} \pm \frac{F_2 l_2}{A_2} \pm \frac{F_3 l_3}{A_3} \right]$$

Non-Uniform section



$$\delta l = \frac{1}{E} \left[\frac{F_1 l_1}{A_1} + \frac{F_2 l_2}{A_2} - \frac{F_3 l_3}{A_3} \right]$$

Given data :- $E = 200 \text{ GPa}$ $F_1 = 100 \text{ N}$
 $A_1 = 150 \text{ m}^2$ $F_2 = 20 \text{ N}$
 $A_2 = 200 \text{ m}^2$ $F_3 = 30 \text{ N}$
 $A_3 = 200 \text{ m}^2$

$$\Rightarrow \delta l = \frac{1}{200} \left[\frac{100 \times 1}{150} + \frac{20 \times 1.5}{200} - \frac{30 \times 0.5}{200} \right]$$

$$= 3.708 \times 10^{-6}$$

* Types of load

(1) Tensile :- The ~~loads~~ ^{load} act away each other on a body.



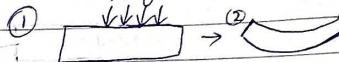
(2) Compressive :- The ~~loads~~ ^{load} act toward each other on a body.



(3) Shear :- The ~~force~~ ^{load} act on body then body is create a angle.



(4) Bending :- The load Apply the body, when body in Bend.



(5) Torsion :- The load Apply the body is Twist Them.



NOTE:-

Deci :- 10^{-1}	Deca :- 10^1	$1 \text{ km} = 10^6 \text{ mm}$
Centi :- 10^{-2}	Hecta :- 10^2	1×10^3
milli :- 10^{-3}	kilo :- 10^3	$10^{-3} \rightarrow 10^2 \rightarrow 10^3 \quad 10^{2+3} = 10^5$
micro :- 10^{-6}	mega :- 10^6	
nano :- 10^{-9}	giga :- 10^9	
pico :- 10^{-12}	Tera :- 10^{12}	
Femto :- 10^{-15}	peta :- 10^{15}	
atto :- 10^{-18}	Exa :- 10^{18}	

* 5 GPa

$\Rightarrow 1 \text{ mm}^2$

$\Rightarrow 5 \times 10^9 \frac{\text{N}}{\text{m}^2}$

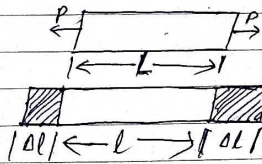
$\Rightarrow \frac{5 \times 10^9 \text{ N}}{10^3 \text{ mm} \times 10^3 \text{ mm}} \Rightarrow \frac{5 \times 10^9 \text{ N}}{10^6 \text{ mm}^2}$

$= 5 \times 10^3 \frac{\text{N}}{\text{mm}^2}$

* Strain :- It is the ratio of change in dimension to the original dimension.

$\Rightarrow$ It is denoted by ϵ (epsilon)

$\Rightarrow \epsilon = \frac{\text{change in length}}{\text{original length}} = \frac{\Delta L}{L}$



Hook's law :- It states that within the elastic limit stress is directly proportional to strain.

mathematically :- $\sigma \propto \epsilon$

$\Rightarrow \sigma = E \epsilon$

Young's modulus :- It is the ratio of stress to strain.

$E = \frac{\sigma}{\epsilon} = \frac{P/A}{\Delta L/L}$

$\Rightarrow E = \frac{PL}{A \Delta L}$

Q. A Steel rod having cross sectional area 10mm^2 is under goes a Pull of 20N . Find the Stress develop.

Ans Stress = $\frac{P}{A}$
 $= \frac{20\text{N}}{10\text{mm}^2} = 2\text{N/mm}^2$

Q. A steel rod having length 10m is subjected to a tensile load of 20kN having cross-sectional area $20\text{mm} \times 20\text{mm}$. Find the elongation take young's modulus $E = 210\text{GPa}$

$1\text{m} = 1000\text{mm}$

Given:- $l = 10\text{m} = 10 \times 10^3$

$T = 20\text{kN} = 20 \times 10^3$

$E = 210\text{GPa}$

$A = 20\text{mm} \times 20\text{mm}$

$\Delta l = ?$

$\Delta l = \frac{Pl}{AE} = \frac{20 \times 10^3 \times 10 \times 10^3}{20 \times 20 \times 210 \times 10^3} = \frac{10^3}{420} = 2.38\text{mm}$

Exercise-3.1

(1) A steel bar 2m long and 150mm^2 cross section is subjected to an axial pull of 15kN . Find the elongation of the bar. Take $E = 200\text{GPa}$

Ans Given:- length = $2\text{m} = 2 \times 10^3\text{mm}$

Area = 150mm^2

Pull = $15\text{kN} = 15 \times 10^3\text{N}$

$E = 200\text{GPa} = 200 \times 10^3\text{Pa}$

$\Delta l = ?$

$\Delta l = \frac{Pl}{AE} = \frac{15 \times 10^3 \times 2 \times 10^3}{150 \times 200 \times 10^3} = \frac{15000 \times 2}{150 \times 200} = \frac{200}{200}$

$= 1.0\text{mm}$
(Ans)

(2) A straight bar of 500mm length has its cross sectional area of 500mm². And the magnitude of the compressive load under which it will decrease its length by 0.2mm. Take E for the bar material as 200GPa.

Sol Given: - $l = 500\text{mm}$
 $A = 500\text{mm}^2$
 $\Delta l = 0.2\text{mm}$
 $E = 200\text{GPa}$
 $P = ?$

$$\Delta l = \frac{Pl}{AE}$$

$$P = \frac{\Delta l \times AE}{l}$$

$$P = \frac{0.2 \times 500 \times 200 \times 10^3}{500}$$

$$\Rightarrow 40,000\text{N} = 40 \times 10^3\text{N}$$

$$P = 40\text{KN (ans)}$$

(3) An alloy bar 1m long and 200mm² in cross section is subjected to a compressive force of 20kN. If the modulus of elasticity for the alloy is 100GPa, find the decrease in length of the bar.

Sol Given: - length (l) = 1m = $1 \times 10^3\text{mm}$

$$\text{Area (A)} = 200\text{mm}^2$$

$$\text{compressive force } P = 20\text{KN} = 20 \times 10^3\text{N}$$

$$E = 100\text{GPa}$$

$$\Delta l = ?$$

$$\Delta l = \frac{P L}{A E} = \frac{20 \times 10^3 \times 1 \times 10^3}{200 \times 100 \times 10^3}$$

$$= \frac{20,000}{20,000} = 1 \text{ mm}$$

$$\Delta l = 1 \text{ mm (Ans)}$$

(4) A hollow cylinder 4m long has outside and inside diameters of 75mm and 60mm respectively. Find the stress and deformation of the cylinder, when it is carrying an axial tensile load of 50 kN. Take $E = 100 \text{ GPa}$.

Given = hollow cylinder length = 4m = $4 \times 10^3 \text{ mm}$

Cylinder diameters = $R_2 = 75$

$R_1 = 60$

$P = 50 \text{ kN} = 50 \times 10^3 \text{ N}$

$E = 100 \text{ GPa}$

A hollow cylinder Area =

$$A = \pi (R_2^2 - R_1^2) h$$

$$= \pi (75^2 - 60^2) \times \frac{\pi}{4} (R_2^2 - R_1^2)$$

$$= \pi (75^2 - 60^2)$$

$$= 1590.43 \text{ mm}^2$$

$$\text{Stress } (\sigma) = \frac{P}{A} = \frac{50 \times 10^3}{1590.43} = 31.43 \text{ MPa}$$

$$\Delta l = \frac{P L}{A E} = \frac{50 \times 10^3 \times 4 \times 10^3}{1590.43 \times 100 \times 10^3}$$

$$= \frac{50,000 \times 4}{1590.43 \times 100}$$

$$= 1.257 \text{ mm}$$

(Ans)

(5) A hollow cast iron column has internal diameter of 200 mm. What should be the external diameter of the column, so that it can carry a load of 1.6 MN without the stress exceeding 90 MPa.

Ans Given = internal diameter (d) = 200 mm

$$\text{Load (P)} = 1.6 \text{ MN} = \cancel{1.6 \times 10^6 \text{ N}} \\ 1.6 \times 10^4 \text{ N}$$
$$1.6 \times 10^4 \text{ N}$$

$$\alpha = \frac{P}{A}$$

$$A = \frac{P}{r} = \frac{100 \times 100}{90} = 111.1111$$

$$A = \pi(R^2 - r^2)$$

R^2 A

$$R^2 - \pi^2 = \frac{A}{\pi}$$

$$\Rightarrow R^2 = \frac{A}{\pi} + r^2$$

$$d = 200 \text{ mm}$$

$$\delta = 100 \text{ mm}$$

$$\Rightarrow R^2 = \frac{17,777.78}{3.141} + R^2$$

$$= 5659.91 + (100)^2$$

2 5689.91 + 10,000

$$R^2 = 15659.91$$

$$\Rightarrow R = \sqrt{15659.9}$$

$$\approx 125.13 \text{ mm}$$

External Diameter = $2R = 2 \times 25.13$

$\Rightarrow 250.26 \text{ mm}$ (Ans)

(6) A brass rod 1.5 m long and 20 mm diameter was found to deform 1.9 mm under a tensile load of 40 kN. calculate the modulus of elasticity of the Rod.

Given length (L) = 1.5 m = 1.5×10^3 = 1500
 load (P) = 40 kN = 40×10^3 N
 ΔL (deformation) (mm) = 1.9 mm
 $A = \pi \cdot (r)^2 = \pi \cdot (10)^2 = 3.14 \times (10)^2$
 $= 314.1$

$E = ?$

$\Delta L = \frac{PL}{AE}$

$\Rightarrow \Delta L \cdot E = \frac{PL}{A}$

$\Rightarrow E = \frac{PL}{A \cdot \Delta L} = \frac{40 \times 10^3 \times 1.5 \times 10^3}{314.1 \times 1.9}$
 $= 100,537.87 \text{ MPa}$
 $= 100.54 \text{ GPa (Ans)}$

(7) A steel wire of 80 m length and 1 mm² cross-sectional area is freely hanging from a tower. what will be its elongation due to its self weight? Take specific weight of the steel as 78.6 kN/m³ and modulus of elasticity as 200 GPa.

Ans Given

length (L) = 80 m ~~80000 mm~~

Area (A) = 1 mm² = 0.000001 m²

Specific weight (P) = 78.6 kN/m³

(modulus elasticity) E = 200 GPa = 200,000 MPa

~~g = acceleration due to gravity = 9.81 m/s²~~

(8)

$$\Delta l = \frac{W \times l^2 \times A}{2E}$$

$$W = \rho \times g \times l \times A$$

$$= 78.6 \times 9.81 \times 80 \times 6.000001$$

$$= 615.07 N$$

$$\Delta l = \frac{W \times l^2 \times A}{2E}$$

$$\delta l = \frac{Wl}{2AE}$$

$$\Rightarrow \delta l = \frac{Wl}{2E}$$

$$\Delta l = \frac{Pl}{AE} = \frac{78.6 \times 80}{1 \times 10^{-6} \times 200 \times 10^3} = 0.055$$

$$\Delta l = \frac{Wl}{2AE}$$

$$\Rightarrow W = \frac{\Delta l \cdot 2AE}{l} = \frac{0.055 \times 2 \times 1 \times 10^{-6} \times 200 \times 10^3}{80}$$

$$= 0.055 \times 2 \times 1 \times 10^{-6} \times 200 \times 10^3$$

$$= 2.75 \times 10^{-4} = \frac{11}{40,000}$$

$$= 0.000275$$

(8) A Steel wire of 1 mm diameter is freely hanging under its own weight. If the extension of the wire should not exceed 2.5 mm, what should be its maximum length? Take E for the wire material as 200 GPa and its specific weight as 78.5 kN/m³.

Sol Given

$$\text{diameter } (d) = 1 \text{ mm}$$

$$l = ? \text{ max length}$$

$$\Delta l = 2.5 \text{ mm}$$

$$\text{Specific weight} = 78.5 \text{ kN/m}^3 = 78.5 \times 10^3 \text{ N/m}^3$$

$$E = 200 \text{ GPa} = 200 \times 10^9 \text{ Pa}$$

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$$A = \pi r^2 = \pi \times 0.5^2 = \pi \times 0.25 = 0.785 \text{ mm}^2$$

$$\Delta l = \frac{wl^2}{2E}$$

$$2.5 = \frac{2 \times 0.785 \times 200 \times 10^3 \times l^2}{2 \times 10^5}$$

$$2.5 = 1.57 \times 10^{-10} l^2$$

$$l^2 = \frac{2.5}{1.57 \times 10^{-10}} = 1.59 \times 10^{10} \text{ mm}^2$$

$$l = \sqrt{1.59 \times 10^{10}} = 126.1 \text{ mm}$$

$$l = 112.87 \text{ mm}$$

* Law of Superposition

when multiple loads act on an elastic material
The total strain is the sum of the individual strains caused by each load acting separately.

Assignment 28-08-24

① Simple Stress and Strain :- (first class 11-07-2024 assignment)

UTM :- Universal testing machine

⇒ Change in dimension to the original dimension

* $\frac{\text{change in length (}\Delta l\text{)}}{\text{actual length (}l\text{)}} = \text{strain}$ (upsilon (ϵ))

* Stress = sigma (σ)

* Longitudinal axis

Longitudinal Strength length of the body of the direction of applied load

* The ratio of lateral strain to longitudinal strain = $\left(\frac{1}{m}\right) = \mu$

Value of $m = 3.4$

~~Poisson's~~ Poisson's Ratio :- (μ) = 0.25 - 0.3

Strength of material other name :- mechanics of solid
mechanics of material

UTM :- Stress, strength graph

⇒ Elastic lim

⇒ Proportional lim

⇒ under ^{yield} ~~point~~ point

⇒ lower lim yielding point

⇒ ultimate stress

⇒ fracture point

② * assumption of mechanics:-

- ⇒ Studies the objects under various types of forces and motion
- ⇒ Deals with the fundamental principles of motion, force, energy, and momentum.
- ⇒ Deals with the fundamental principles of motion, force, energy and momentum.
- ⇒ covers topics like kinematics, dynamics, statics, etc.
- ⇒ Focuses on how objects move and respond to forces.

* Strength of materials assumption

- ⇒ Principles of mechanics to analyze the internal forces and stresses within materials.
- ⇒ Examines how materials deform, fail or withstand external loads.
- ⇒ covers topics like Stress, Strain, elasticity, Plasticity and failure criteria.
- ⇒ It focuses on the internal behavior of materials and analyzing material behavior and failure of materials under various loads.

③

difference b/w Pressure and Stress

Pressure	Stress
Pressure represent intensity of external forces acting at a point.	It is represent internal resisting forces develop at a point.
⇒ It is always acts normal to the surface.	⇒ It is act either normal or parallel to the surface
⇒ magnitude of Pressure at a point in all direction remain same.	⇒ magnitude of Stress at a point in all the direction are unequal.

⑨ H

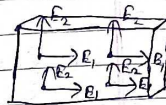
⑤

⑤

④ Homogeneous material :-

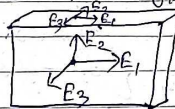
It is defined as a material which have same elastic properties in all points at given direction.

⇒ It is independent of points.



Isotropic material :-

It is defined as a material which have same elastic properties in all direction at given point.



Example-1 Problem

- ⑤ A mild steel rod 20mm diameter is subjected to an axial pull of 50kN. Determine the tensile stress induced in the rod and the elongation if the unloaded length is 5m.

$E = 210 \text{ GN/m}^2$

Given $d = 20 \text{ mm}$ $l = 5 \text{ m}$

$P = 50 \text{ kN}$

$\frac{\text{N}}{\text{m}^2} = \text{Pascal}$

$\frac{\text{N}}{\text{mm}^2} = \text{mega Pascal}$

Area of cross-section of the Rod $A = \pi d^2$

$A = \pi (20)^2 \times 10^{-6}$
 $= 314 \times 10^{-6} \text{ m}^2$

Stress $= \sigma = \frac{P}{A} = \frac{50 \times 10^3}{314 \times 10^{-6}} = 159.155 \text{ N/mm}^2$

Elongation $\Delta l = \frac{Pl}{AE} = \frac{50 \times 10^3 \times 5 \times 10^3}{314 \times 10^{-6} \times 210 \times 10^9} = 3.789 \text{ mm}$

→ $A = \frac{\pi d^2}{4}$
 $= \frac{\pi (20)^2}{4} = 100\pi$
 314.17

Example 1.2

A short hollow cast iron cylinder of wall thickness 10mm is to carry a compressive load of 600kN. Determine the outside diameter of the cylinder if the ultimate crushing stress for the material is 540 MN/m². Use a factor of safety of 6.

Sol Let d_o be the outside diameter of the cylinder in mm. Then area of cross section of the cylinder is,

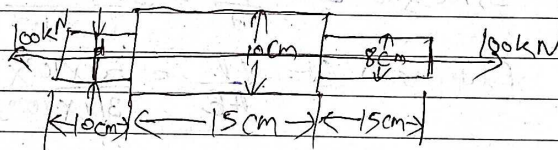
$$A = \frac{\pi}{4} \{ d_o^2 - (d_o - 20)^2 \} \times 10^{-6}$$
$$= \pi (d_o - 10) \times 10^{-5} \text{ m}^2$$

$$\text{Safe load} = \pi (d_o - 10) \times 10^{-5} \times \frac{540}{6} \times 10^6$$
$$= 900 \times \pi (d_o - 10)$$
$$= 600 \times 10^3$$

$$d_o = 222.2 \text{ mm}$$

Example 1.3

A round bar as shown in figure is subjected to an axial tensile load of 100 kN. What must be the diameter 'd' if the stress there is to be 100 MN/m²? Find also the total elongation. $E = 200 \text{ GPa}$.



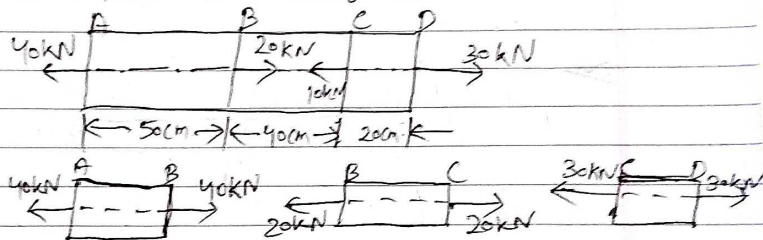
$$\text{stress, } \sigma = \frac{P}{\left(\frac{\pi}{4}\right) d^2}$$

$$100 \times 10^6 = \frac{100 \times 10^3}{\left(\frac{\pi}{4}\right) d^2}$$

$$\text{diameter, } d = \sqrt{\frac{4}{\pi \times 10^3}} = 0.03568 \text{ m} = 35.68 \text{ mm}$$

$$\begin{aligned} \text{Total elongation, } \Delta l &= \frac{P}{E} \left[\frac{l_1}{A_1} + \frac{l_2}{A_2} + \frac{l_3}{A_3} \right] \\ &= \frac{100 \times 10^3}{200 \times 10^9} \left[\frac{0.10}{10^{-3}} + \frac{0.15}{\frac{\pi}{4} \times (100)^2 \times 10^{-6}} + \frac{0.15}{\frac{\pi}{4} \times 10^{-6}} \right] \\ &= \frac{10^{-4}}{2} [1 + 0.191 + 0.299] = \frac{1.490 \times 10^{-4}}{2} \\ &= 0.0745 \text{ mm} \end{aligned}$$

Example-14:- A steel bar 25mm diameter is loaded as fig. Determine the stresses in each part and the total elongation $E = 210 \text{ GPa}$



$$\text{Sol } A = \frac{\pi d^2}{4} = \frac{\pi \times (25)^2 \times 10^{-6}}{4} = 490.87 \times 10^{-6} \text{ m}^2$$

$$\sigma_{AB} = \frac{40 \times 10^3}{490.87 \times 10^{-6}} = 81.488 \text{ MN/m}^2$$

$$\sigma_{BC} = \frac{20 \times 10^3}{490.87 \times 10^{-6}} = 40.744 \text{ MN/m}^2$$

$$\sigma_{CD} = \frac{30 \times 10^3}{490.87 \times 10^{-6}} = 61.116 \text{ MN/m}^2$$

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$$\text{Total elongation, } \Delta l = \frac{1}{AE} \sum p_i l_i$$

$$= \frac{10^3}{490.87 \times 10^{-6} \times 210 \times 10^9} [40 \times 0.5 + 20 \times 0.4 + 30 \times 0.2]$$

$$= \frac{10^3 \times 34 \times 10^3}{490.87 \times 10^{-6} \times 210 \times 10^9} = 0.3298 \text{ mm}$$

Example 1.5

A steel bar consists of two parts AB and BC having areas of cross-section of 4 cm^2 and 5 cm^2 respectively. It is rigidly fixed at end A and end C is at a distance of 1 m from the other rigid horizontal support. A load of 100 kN is applied vertically downward at B. Determine the reactions produced by the rigid horizontal support and the stress in the parts AB and BC of the bar. $E = 200 \text{ GPa}$.

Sol

$$\Delta_{AB} = \frac{100 \times 10^3 \times 1.25 \times 10^3}{4 \times 10^{-4} \times 200 \times 10^9} = 1.5625 \text{ mm}$$

Principle of

Load: Load is defined as external force or couple which a member is subjected during its ~~restored~~ functionality.

Gradual load value to sudden impact load value is double.



cross sectional

$$d_i + 2t = d_o$$

$$d_i = d_o - 2t$$

$$d_i = d_o - 2 \cdot 10$$

$$d_i = d_o - 20$$

$$A = \frac{\pi}{4} (d_o^2 - d_i^2)$$

$$= \frac{\pi}{4} \{ d_o + d_o - 20 \} (d_o - d_o + 20)$$

$$\Rightarrow \frac{\pi}{4} \{ (2d_o - 20) \times 20 \}$$

$$= \pi (10d_o - 100)$$

$$\Rightarrow \frac{\sigma_u}{F.O.S} = \frac{P}{A}$$

$$\Rightarrow \frac{540}{6} = \frac{600 \times 10^3}{\pi (10d_o - 100)}$$

$$\Rightarrow 90 \times \pi (10d_o - 100) = 600 \times 10^3$$

$$\Rightarrow 90 \times \pi \cdot 10 (d_o - 10) = 600 \times 10^3$$

$$\Rightarrow 90 \pi (d_o - 10) = 600 \times 10^3$$

$$\Rightarrow 9 \pi (d_o - 10) = 600 \times 10$$

$$\Rightarrow d_o = \frac{6000}{9\pi} + 10$$

$$= 222.20 \text{ mm}$$

$$d_i = 202.20$$

$$\Rightarrow \frac{540}{\cancel{\text{mm}}} = \frac{600 \times 10^3}{\pi(10d_0 - 100)}$$

Approximately = 4

Allowable Stress $\approx$ Ultimate Stress

$$\Rightarrow 540 \times \pi(10d_0 - 100) = 600 \times 10^3$$

$$\Rightarrow 540 \times 10\pi(d_0 - 10) = 600 \times 10^3$$

$$\Rightarrow 5400 \times \pi(d_0 - 10) = 600 \times 10^3$$

$$= 54 \times \pi(d_0 - 10) = 6000$$

$$\Rightarrow d_0 = \frac{6000}{54\pi} + 10$$

$$= 45.36$$

$$\Rightarrow d_0 = \frac{45.36}{25.36}$$

~~Allowable Stress~~ = $\frac{\text{Ultimate Stress}}{\text{F.O.S}}$

Assumptions made while deriving some eqn:-

- (1) material is assumed to be Homogeneous and Isotropic.
- (2) material obeys Hooke's law
(Induced Stress and Strain are within the elastic region)
- (3) member is assumed to be Prismatic
(i.e. uniform cross-sectional dimensions throughout the length of member.)
- (4) load is assumed as static load.
- (5) self weight of the member is neglected
- (6) member is assumed to be under static extl cond.

⇒ Factor of Safety is used to get safe dimension of the component in presence of unreality of above mention assumption.

Homogeneous:-

A material is said to be homogeneous when it exhibits same elastic properties at any point in a given direction (i.e. elastic properties are independent of point).

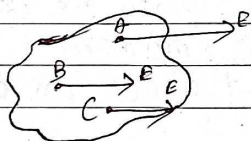
Isotropic:-

A material is said to be isotropic when it exhibits same elastic properties in any direction at a given point (i.e. elastic properties are - Independent of direction).

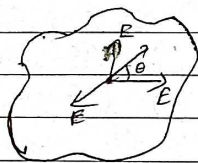
A material is said to be both homogeneous and isotropic when it exhibits same elastic properties in any direction and at any given point. Every homogeneous material need not be a isotropic material and vice-versa, but few materials are said to be both homogeneous and isotropic.

Anisotropic material

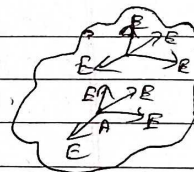
A material is said to be anisotropic, when it exhibits direction dependent properties at a given point.



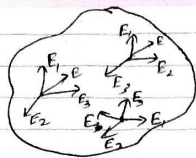
Homogeneous material



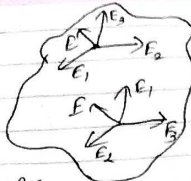
Isotropic material



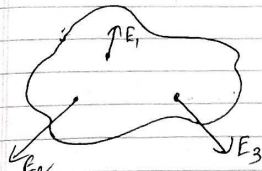
Homogeneous and isotropic



Homogeneous and Anisotropic



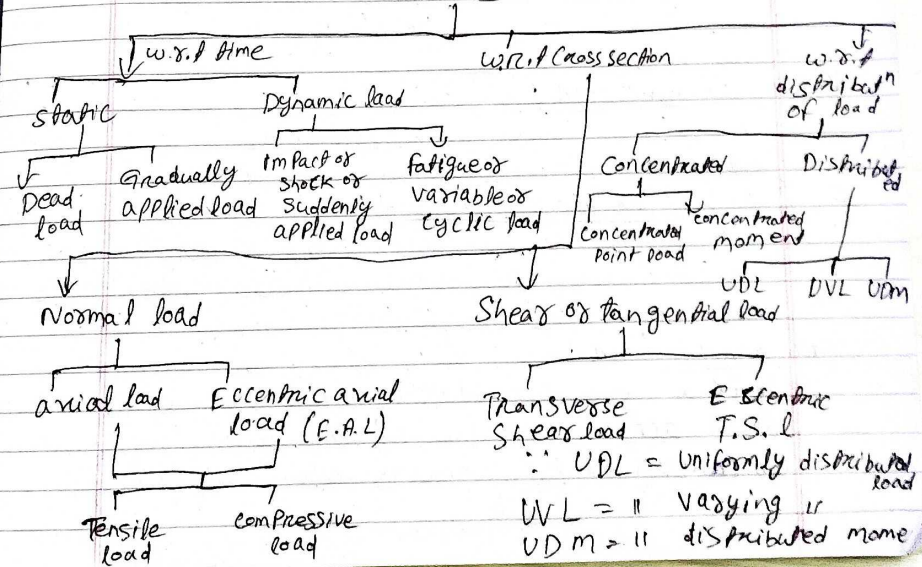
Non homogeneous and isotropic material



Can't say $\Rightarrow$ may be homogeneous or non homogeneous

$\Rightarrow$ may be isotropic or anisotropic

LOADS



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* (i) Static load :- It is a type of load that remains constant or don't change with time.
⇒ constant magnitude and direction.

(a) Dead load :- It is a type of static load.
⇒ It is a constant or permanent load.
⇒ It doesn't move and unchanging weight.

(b) Gradually applied load :-
It is a type of load that is applied slowly and incrementally over a period of time.

* (ii) dynamic load :- It is a load that varies in magnitude, direction or force and weight that changes rapidly.

(a) Impact or suddenly applied load :- ^{extreme}
It is ^{load which} applied suddenly and with great force.
or It is applied short time.

(b) Fatigue or cyclic load :-
It is a load which causes repeated stress cycles leading to material degradation and potential failure over time.

* (iii) Normal load :-

It is also known as perpendicular load. It acts at a 90° angle to the surface of an object.

* (iv) Shear or tangential load :-

an object or structure to deform by sliding or rotating along a plane parallel to the force. are known as shear loads.

(a) axial load :-

It is a load which is compressive or tensile force that acts along the central axis of an object or causing it to compress, stretch or deform.

(b) eccentric^{axial} load :- It is type of axial load which an offset distance from the central axis on an object or structure.

(c) transverse shear load :-
It is a force applied at a right angle to the surface or perpendicular to its longitudinal axis.

* (a) Concentrated Point load :- It is a type of load which applied at a single, specific point on a material or object.

(b) Concentrated moment :-
It is also known as point moment, is a force that causes a rotation or twisting effect on a material at a specific point.

* Concentrated load :- It is a force applied to a very small area of a structural or material.

* Uniformly distributed load :-
It is a type of load that is spread evenly across a surface or area.

(b) Uniformly varying load :-
It is a type of load that is increases or decreases uniformly over a length or area.

(c) Uniformly distributing moment :-
It is a type of load which a rotational force to be applied evenly across a length or area.

Assignment

- ① An elastic rod 25 mm in diameter, 200 mm long extends by 0.25 mm under a tensile load of 40 kN. Find the intensity of stress the strain and the elastic modulus for the material of the rod.

Sol Given $d = 25 \text{ mm}$

length $= l = 200 \text{ mm}$

Tension $= P = 40 \text{ kN}$

$$\text{Area of section } A = \frac{\pi \times 25^2}{4} = 490.87 \text{ mm}^2$$

$$\text{Intensity of stress} = f = \frac{P}{A} = \frac{40 \times 1000}{490.87} \text{ N/mm}^2 = 81.49 \text{ N/mm}^2$$

$$\text{Strain} = e = \frac{\delta}{L} = \frac{0.25}{200} = 0.00125$$

$$\text{Elastic modulus} = E = \frac{f}{e} = \frac{81.49}{0.00125} = 65192 \text{ N/mm}^2$$

$$= 0.652 \times 10^5 \text{ N/mm}^2$$

- ② A cast iron column has an external diameter of 300 mm and ~~25 mm~~ ^{25 mm} thick. Find the safe compressive load on the column with a factor of safety of 5 if the crushing strength of the material is 550 N/mm².

Given:- Area of the column section =

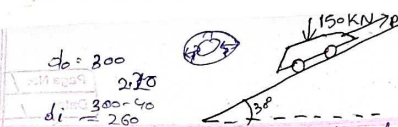
$$A = \frac{\pi}{4} (300^2 - 260^2) = 17592.78 \text{ mm}^2$$

$$\text{Safe stress in compression} = \frac{550}{5} = 110 \text{ N/mm}^2$$

$$\text{Safe compressive load} = 110 \times 17592.78 = 1935205.8 \text{ N}$$

$$= 1935.2058 \text{ kN}$$

- ③ A vehicle weighing 150 kN is slowly pulled up a smooth 30° plane by a 25 mm diameter cable. Find the tensile stress in the cable.



Let P = pull in the cable to just drag the vehicle up the slope $P = 150 \sin 30^\circ = 75 \text{ kN}$.

Tensile stress in the cable $= P = \frac{75 \times 10^3}{\frac{\pi}{4} (25)^2} = 152.8 \frac{\text{N}}{\text{mm}^2}$

- (4) A steel rod is 2.25 m long and must not stretch more than 1.5 mm when a tensile load of 12 kN is applied to it. Find (a) the smallest diameter for the rod to be used and (b) the corresponding tensile stress caused by the load. Take $E = 2 \times 10^5 \frac{\text{N}}{\text{mm}^2}$

Sol Given = d = minimum diameter

Tensile stress in the rod

$$\delta = \frac{PL}{AE} \therefore A = \frac{PL}{E \Delta L} = \frac{12 \times 10^3 \times 2250}{2 \times 10^5 \times 1.50}$$

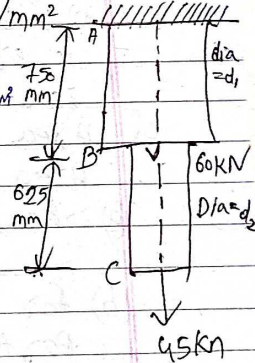
$$A = \frac{\pi d^2}{4} = 90 \therefore d = 10.705 \text{ mm}$$

$$\Rightarrow F = \frac{P}{A} = \frac{12 \times 10^3}{90} = 133.33 \frac{\text{N}}{\text{mm}^2}$$

- (5) Two solid cylindrical rods AB and BC are welded at B as shown in Fig 1.8. Determine the smallest allowable diameters d_1 and d_2 of AB and BC for an allowable tensile stress of 150 N/mm^2

Sol Tension in the rod $= P = 60 + 45 = 105 \text{ kN}$
minimum area required $= A_1 = \frac{105 \times 10^3}{150} = 700 \text{ mm}^2$

Rod AB $\rightarrow \frac{\pi d_1^2}{4} = 700$
 $\Rightarrow d_1 = 29.85 \text{ mm}$



Red BC :- Tension in the rod = $P_2 = 45 \text{ kN}$
 Minimum area required = $A_2 = \frac{45 \times 10^3}{150} = 300 \text{ mm}^2$

$$\Rightarrow \frac{\pi d_2^2}{4} = 300$$

$$\Rightarrow d_2 = 19.54 \text{ mm}$$

- (6) A 30mm diameter steel rod when subjected to an axial tensile force was subjected to a strain of 0.60×10^{-3} . Find the tensile force that caused the above strain. Take $E = 200 \text{ kN/mm}^2$

Sol Stress = Strain $\times$ Young's Modulus
 $= 0.60 \times 10^{-3} \times 200 = 0.12 \text{ kN/mm}^2$

Tensile force = Stress $\times$ area of the rod
 $= 0.12 \times \frac{\pi}{4} \times 30^2 = 84.82 \text{ kN}$

- (7) Fig. 1.9 shows a vertical stepped post ABC whose upper and lower parts are of diameters 30mm and 50mm respectively. A vertical load $W_1 = 12 \text{ kN}$ is applied at the top. An additional load W_2 is uniformly applied around the shelf at B.

(a) Find the normal stress in the upper parts of the post.

(b) If it is desired that the stress in the upper and lower parts should be the same, find the load W_2 .

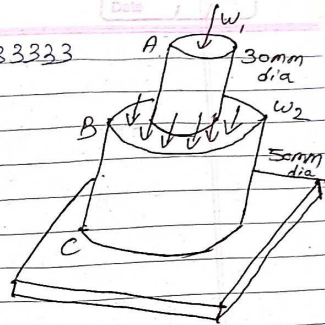
Sol Area of the upper part = $\frac{\pi}{4} \times 30^2 = 225\pi \text{ mm}^2$
 Area of the lower part = $\frac{\pi}{4} \times 50^2 = 625\pi \text{ mm}^2$

(a) Stress in the upper part = $\frac{12000}{225\pi} = 16.98 \text{ N/mm}^2$

(b) for the stresses in the upper and lower parts to be equal $\frac{12000}{225\pi} = \frac{W_2 + 12000}{625\pi}$

$$\Rightarrow w_2 + 12000 = \frac{625}{225} \times 12000 = 33333$$

$$\Rightarrow w_2 = 21333 \text{ N}$$



- (8) a pressurized cylinder of 250 mm internal diameter provided with a sealing cover plate fastened with 12 mm diameter steel bolts. The pressure inside the cylinder is 2 N/mm². If the allowable tensile stress in the bolts is 75 N/mm². Find the number of bolts required to fasten the cover plate.

Sol Total upward force on the cover plate

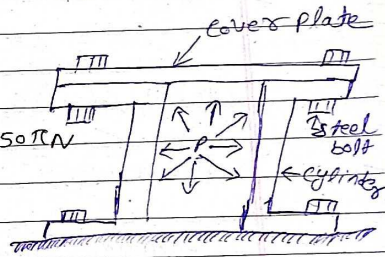
$$\Rightarrow P = \frac{p \pi D^2}{4}$$

$$= \frac{2 \times \pi \times 250^2}{4} = 31250 \pi \text{ N}$$

$$\text{Safe tension per bolt} = \frac{F_t \pi d^2}{4}$$

$$= \frac{75 \times \pi \times 12^2}{4} = 2700 \pi \text{ N}$$

$$\text{Number of bolts required} = \frac{31250 \pi}{2700 \pi} = 11.6 \text{ - say 12 bolts}$$



- (9) The external and internal diameters of an aluminium tube serving as a compression member are 30 mm and 25 mm respectively. The yield stress for aluminium is $\sigma_y = 270 \text{ N/mm}^2$ and the ultimate stress is $\sigma_u = 310 \text{ N/mm}^2$.

Find the allowable compressive force if the factors of safety with respect to the yield stress and the ultimate stress are 4 and 5 respectively.

Sol Safe stress based on yield stress = $\frac{270}{4} = 67.5 \text{ N/mm}^2$

Safe stress based on ultimate stress = $\frac{310}{5} = 62 \text{ N/mm}^2$

$\therefore$ Safe stress to be adopted = 62 N/mm^2

$\therefore$ Safe compressive force = $\frac{\pi}{4} (30^2 - 25^2) \times 62 \text{ N}$
 $= 13391 \text{ N}$

10 Find the maximum and minimum stresses produced in the stepped bar shown in fig 1.11 due to an axially applied compressive load of 12 kN.

Sol Area of the upper part

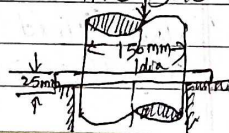
$$A_1 = \frac{\pi}{4} (12)^2 = 113.09 \text{ mm}^2$$

Area of the lower part $A_2 = \frac{\pi}{4} (25)^2 = 490.87 \text{ mm}^2$

maximum stress $f_{\max} = \frac{P}{A_1} = \frac{12 \times 10^3}{113.09} = 106.10 \text{ N/mm}^2$

minimum stress $f_{\min} = \frac{P}{A_2} = \frac{12 \times 10^3}{490.87} = 24.45 \text{ N/mm}^2$

(11) A 150 mm diameter shaft has a 20 mm thick surrounding collar and is subjected to an axial compressive load of 600 kN and is supported as shown in fig. 1.12. Find the compressive stress induced in the shaft. Find also the shear stress induced in the collar.



Sol- Area of the shaft section

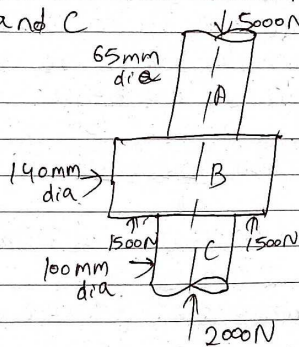
$$A = \frac{\pi}{4} (150)^2 = 17671.32 \text{ mm}^2$$

Compressive stress in the shaft $= \frac{600 \times 10^3}{17671.32} = 33.95 \text{ N/mm}^2$

Shear area of the collar $= \pi \times 150 \times 25 = 11780.88 \text{ mm}^2$

Shear stress in the collar $= \frac{600 \times 10^3}{11780.88} = 50.93 \text{ N/mm}^2$

(15) The ~~anchor~~ ^{thrust bearing} shaft is subjected to the loads shown in fig. Determine the average normal stress developed in cross section through A, B and C



Average normal stress on section A $= \frac{5000}{\frac{\pi \times 65^2}{4}} = 1.507 \text{ N/mm}^2$

Average normal stress on section B $= \frac{5000}{\frac{\pi \times 140^2}{4}} = 0.325 \text{ N/mm}^2$

Average normal stress on section C $= \frac{3000}{\frac{\pi \times 100^2}{4}} = 0.255 \text{ N/mm}^2$

(16) The index block has the dimensions shown in fig. If the material fails when the average normal stress is 0.80 N/mm^2 determine the largest centrally applied vertical load P it can carry

Sol Area of the block

$$\Rightarrow A = 150 \times 350 - 2[(100 \times 25) + (75 \times 100)]$$
$$= 82500 \text{ mm}^2$$

Average normal stress $= 0.80 \text{ N/mm}^2$ at failure

Largest Centrally Applied Load $= 0.80 \times 82500$

$$\Rightarrow 26000 \text{ N} = 26 \text{ kN}$$

- (17) A pull of 7.5 kN has to be applied to a 40m long steel wire. Determine the smallest diameter wire that can be used if the tensile stress shall not exceed 150 N/mm^2 and the extension of the wire shall not exceed 20mm. Take $E = 2 \times 10^5 \text{ N/mm}^2$

Sol Limiting the tensile stress to 150 N/mm^2 ,
area of wire required $A = \frac{7.5 \times 10^3}{150} = 50 \text{ mm}^2$

Extension consideration $\frac{PL}{AE} = \Delta l$

$$A = \frac{PL}{E \Delta l} = \frac{7.5 \times 10^3 \times 40 \times 10^3}{2 \times 10^5 \times 20} = 75 \text{ mm}^2$$

Area of wire required $\geq A$ = greater of the values determined, $A = \frac{\pi d^2}{4} \geq 75$

$$\therefore d = \sqrt{\frac{300}{\pi}} = 9.772 \text{ mm}$$

- 23 A steel rod 25mm in diameter and 2 metre long is subjected to an axial pull of 45 kN. Find (i) the intensity of stress (ii) the strain and (iii) elongation. Take $E = 2 \times 10^5 \text{ N/mm}^2$

Sol Diameter of rod $= 25 \text{ mm}$

Length of rod $= 2 \text{ metre}$

Pull $= P = 45 \text{ kN} = 45000 \text{ N}$

$$\text{Area of Rod} = \frac{\pi}{4} (25)^2 = 490.87 \text{ mm}^2$$

$$(i) \text{ Intensity of Stress} = F = \frac{P}{A} = \frac{45000}{490.87} = 91.67 \text{ N/mm}^2$$

$$(ii) \text{ Strain} = e = \frac{F}{E} = \frac{91.67}{2 \times 10^5} = 0.00045835$$

$$(iii) \text{ Elongation} = \Delta l = \text{Strain} \times \text{original length} \\ = 0.00045835 \times 2000 = 0.9167 \text{ mm}$$

(24) A copper Rod 3mm in diameter when subjected to a pull of 495 N extends by 0.07 mm over a gauge length of 100 mm. Calculate the Young's modulus for copper.

$$\text{Sol Area of the Rod} = A = \frac{\pi d^2}{4} = \frac{\pi \times 3^2}{4} \text{ mm}^2 = 7.07 \text{ mm}^2$$

$$\text{Intensity of Stress} = \frac{P}{A} = \frac{495}{7.07} = 70.01 \text{ N/mm}^2$$

$$\text{Strain} = e = \frac{0.07}{100} = 0.0007$$

$$\text{Young's modulus} = E = \frac{F}{e} = \frac{70.01}{0.0007} = 1 \times 10^5 \text{ N/mm}^2$$

(25) A wooden tie is 75 mm wide, 150 mm deep and 1.50 metre long. It is subjected to an axial pull of 45000 N. The stretch of the member is found to be 0.6380 mm. Find the Young's modulus for the material.

$$\text{Sol Area of the tie} = A = 75 \times 150 \text{ mm}^2$$

$$\text{Pull} = P = 45000 \text{ N}$$

$$\text{Stress} = \sigma = \frac{P}{A} = \frac{45000}{75 \times 150} = 4 \text{ N/mm}^2$$

$$\text{Strain} = e = \frac{\text{Change in length}}{\text{original length}} = \frac{0.6380}{1.5 \times 1000} = 0.0004253$$

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$$\therefore \text{Young's modulus} = E = \frac{F}{\epsilon} = \frac{4}{0.004253} = 9405.1 \text{ N/mm}^2$$

(26) A brass rod is subjected to a pull of 2.70 kN. If the permissible stress in tensile is 140 N/mm^2 and the extension for the rod shall not exceed 1.5 mm. find

(a) Smallest diameter of the rod,

(b) The greatest length of the rod, Take $E = 1.05 \times 10^5 \text{ N/mm}^2$

Sol Minimum sectional area of the rod $= A = \frac{2.7 \times 10^3}{140}$
 $= 19.286 \text{ mm}^2$

Given data

Pull $= 2.7 \times 10^3$
 $\sigma = 140 \text{ N/mm}^2$
 $\epsilon = 1.5 \text{ mm}$
 $E = 1.05 \times 10^5 \text{ N/mm}^2$

If d is the smallest diameter of the rod,

$$A = \frac{\pi d^2}{4} = 19.286 \therefore d = 4.955 \text{ mm}$$

$$\text{Extension of the rod} \frac{\Delta L}{L} = \frac{PL}{AE} = 1.5$$

$$\text{Extension of the rod} = l = \frac{AE \Delta L}{P} = \frac{\pi \times 1.5^2 \times 1.05 \times 10^5}{2700} = 103 \text{ mm}$$

(27) A cast iron tube has to support a compressive load of 7.25 kN. If the maximum allowable change in length is 0.025 Percent, determine the maximum compressive stress in the tube and the minimum wall thickness if the outer diameter of the tube is 50 mm. Take $E = 6.7 \times 10^4 \text{ N/mm}^2$

Sol $P = 7.25 \text{ kN} = 7250 \text{ N}$

$\Delta L \times 100 = 0.0025$

Compressive stress $= \sigma = 6.7 \times 10^4 \times 0.00025 = 16.75 \text{ N/mm}^2$

Area of the tube section $= A = \frac{P}{\sigma} = \frac{7250}{16.75} = 432.84 \text{ mm}^2$

outer diameter $d_1 = 50 \text{ mm}$, inner diameter d_2

$$\frac{\pi}{4} (50^2 - d_2^2) = 432.84 \therefore 50^2 - d_2^2 = 551.11 \text{ mm}^2$$

$$d_2 = 44.15 \text{ mm}$$

$$\text{Thickness of tube} = t = \frac{d_1 - d_2}{2} = \frac{50 - 44.15}{2} = 2.925 \text{ mm}$$

28. A load of 4000 N has to be raised at the end of a steel wire. If the unit stress in the wire must not exceed 80 N/mm², what is the minimum diameter required? What will be the extension of 3.50 metre length of wire? Take $E = 2 \times 10^5 \text{ N/mm}^2$.

Sol. Load on the wire = $W = 4000 \text{ N}$

$$\text{Area required} = A = \frac{W}{\sigma} = \frac{4000}{80} = 50 \text{ mm}^2$$

Let the diameter of the wire be $d \text{ mm}$

$$\frac{\pi d^2}{4} = 50$$

$$d = \sqrt{\frac{4 \times 50}{\pi}} = 7.979 \text{ mm}$$

Extension:

$$\Delta l = \text{Strain} \times \text{original length}$$

$$= \frac{\sigma}{E} l = \frac{80}{2 \times 10^5} \times 3500 = 1.40 \text{ mm}$$

(29) A wooden tie is 60 mm wide, 120 mm deep and 1.5 metres long. It is subjected to an axial pull of 30 kN. The stretch of the member is found to be 0.625 mm. Find the Young's modulus for the tie material.

Sol. Area of the tie = $A = 60 \times 120 = 7200 \text{ mm}^2$

$$P = 30 \text{ kN} = 30,000 \text{ N}$$

$$\text{Stress} = \sigma = \frac{P}{A} = \frac{30,000}{7200} = 4.167 \text{ N/mm}^2$$

$$\text{Strain} \Rightarrow \epsilon = \frac{\text{Change in length}}{\text{original length}} = \frac{0.625}{1.5 \times 1000} = 4.167 \times 10^{-4}$$

$$\text{Young's modulus} = E = \frac{\sigma}{\epsilon} = \frac{4.167}{4.167 \times 10^{-4}} = 10000 \text{ N/mm}^2$$

(30) A 20mm diameter brass rod was subjected to a tensile load of 40kN. The extension of the rod was found to be 254 divisions in the 200mm extensionmeter. If each division is equal to 0.001mm, find the elastic modulus of brass.

Sol. Area of the rod $= A = \pi (d/2)^2 \text{ mm}^2 = 314.1568 \text{ mm}^2$

Pull $= P = 40 \text{ kN} = 40,000 \text{ N}$

$$\text{Stress} = \sigma = \frac{P}{A} = \frac{40,000}{314.1568} = 127.325 \text{ N/mm}^2$$

Length of the specimen $= l = 200 \text{ mm}$

Extension $= \Delta l = 254 \times 0.001 = 0.254 \text{ mm}$

$$\text{Strain} = \epsilon = \frac{\Delta l}{l} = \frac{0.254}{200} = 0.00127$$

$$\text{Young's modulus } E = \frac{\sigma}{\epsilon} = \frac{127.325 \text{ N/mm}^2}{0.00127} = 1.0025 \times 10^5 \text{ N/mm}^2$$

(31) The following data refer to a mild steel specimen tested in a laboratory

(i) Diameter of the specimen $= 25 \text{ mm}$.

(ii) Length of the specimen $= 300 \text{ mm}$.

(iii) Extension under a load of 15kN $= 0.45 \text{ mm}$.

(iv) Load at yield point $= 127.65 \text{ kN}$

(v) maximum load $= 208.60 \text{ kN}$

(vi) Length of the specimen after failure $= 375 \text{ mm}$.

(vii) Neck diameter $= 17.75 \text{ mm}$.

Determine

(i) Young's modulus (ii) Yield point (iii) ultimate stress

(iv) Percentage of elongation (v) Percentage reduction

in area (vi) Stress adopting a factor of safety of 2

Sol Area of the Specimen $= A = \frac{\pi \times 25^2}{4} = 490.87 \text{ mm}^2$

at a load of 15 kN, stress intensity $= \sigma = \frac{15 \times 10^3}{490.87} = 30.56 \text{ N/mm}^2$

Strain corresponding to this load $= \frac{0.045}{300} = 1.5 \times 10^{-4}$

Young's modulus $= E = \frac{30.56}{1.5 \times 10^{-4}} = 2.037 \times 10^5 \text{ N/mm}^2$

Stress at Yield Point $= \frac{127.65 \times 10^3}{490.87} = 260.05 \text{ N/mm}^2$

Ultimate Stress $= \frac{208.60 \times 10^3}{490.87} = 424.96 \text{ N/mm}^2$

Percentage elongation $= \frac{375 - 300}{300} \times 100 = 25\%$

Percentage reduction in area $= \frac{d^2 - d_2^2}{d^2} \times 100$

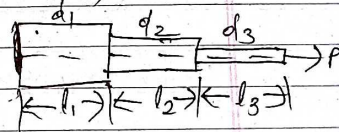
Safe Stress $= \frac{\text{Yield Stress}}{\text{factor of Safety}} = \frac{260}{2.5} = 104 \text{ N/mm}^2$
 $= 49.59\%$

(4) A bar consists of three lengths l_1, l_2, l_3 and of d_1, d_2 , and d_3 and is subjected to an axial pull P . Show that the extension of the rod is given by $\Delta l = \frac{4P}{\pi E} \left(\frac{l_1}{d_1^2} + \frac{l_2}{d_2^2} + \frac{l_3}{d_3^2} \right)$

Sol Stress in 3 parts of the bar

$\sigma_1 = \frac{P}{\frac{\pi}{4} d_1^2} = \frac{4P}{\pi d_1^2}$

$\sigma_2 = \frac{P}{\frac{\pi}{4} d_2^2} = \frac{4P}{\pi d_2^2}$



$$\sigma_3 = \frac{P}{\frac{\pi}{4} d_3^2} = \frac{4P}{\pi d_3^2}$$

Extensions of the three parts are

$$dl_1 = \frac{\sigma_1 l_1}{E} = \frac{4Pl_1}{\pi d_1^2 E}$$

$$\Rightarrow dl_2 = \frac{4Pl_2}{\pi d_2^2 E}$$

$$\Rightarrow dl_3 = \frac{4Pl_3}{\pi d_3^2 E}$$

$$\text{Total extension} = dl = dl_1 + dl_2 + dl_3$$

$$= \frac{4Pl_1}{\pi d_1^2 E} + \frac{4Pl_2}{\pi d_2^2 E} + \frac{4Pl_3}{\pi d_3^2 E}$$

$$dl = \frac{4P}{\pi E} \left(\frac{l_1}{d_1^2} + \frac{l_2}{d_2^2} + \frac{l_3}{d_3^2} \right)$$

(Q9) A straight bar 450mm long is 20mm in diameter for the first 250mm length and 10mm dia, for the remaining length. If the bar is subjected to an axial pull of 10 kN find the extension of the bar. Take $E = 2 \times 10^5 \text{ N/mm}^2$

Sol Area of the first part $= A_1 = \frac{\pi}{4} \times 20^2 = 314.16 \text{ mm}^2$

Area of the second part $= A_2 = \frac{\pi}{4} \times 10^2 = 78.54 \text{ mm}^2$

Stress in the first part $= \sigma_1 = \frac{10 \times 10^3}{314.16} = 31.83 \frac{\text{N}}{\text{mm}^2}$

Stress in the second part $= \sigma_2 = \frac{10 \times 10^3}{78.54} = 127.32 \frac{\text{N}}{\text{mm}^2}$

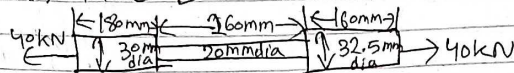
Total extension of the bar $= \Delta l = \frac{\sigma_1 l_1}{E} + \frac{\sigma_2 l_2}{E}$

$$= \frac{1}{E} (\sigma_1 l_1 + \sigma_2 l_2)$$

$$= \frac{1}{2 \times 10^5} (31.83 \times 250 + 127.32 \times 200) \text{ mm}$$

$$= 0.167 \text{ mm}$$

- (46) a bar consisting of three lengths. Find the stresses in the three parts and the total extension of the bar for an axial pull of 40 kN. Take $E = 2 \times 10^5 \text{ N/mm}^2$.



Sol Load $P = 40 \text{ kN} = 40,000 \text{ N}$

$$\text{Intensity of stress in the part AB} = \sigma_1 = \frac{P}{A_1} = \frac{40000}{\frac{\pi}{4} (30)^2} = 56.58 \text{ N/mm}^2$$

$$\text{Intensity of stress in the part BC} = \sigma_2 = \frac{P}{A_2} = \frac{40000}{\frac{\pi}{4} (20)^2} = 127.4 \text{ N/mm}^2$$

$$\text{Intensity of stress in the part CD} = \sigma_3 = \frac{P}{A_3} = \frac{40000}{\frac{\pi}{4} (32.5)^2} = 48.21 \text{ N/mm}^2$$

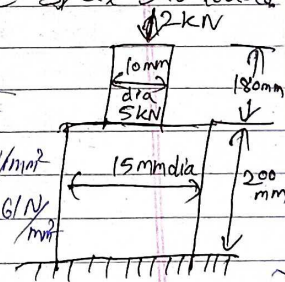
$$\begin{aligned} \text{Total Extension } \Delta l &= \Delta l_1 + \Delta l_2 + \Delta l_3 \\ &= \frac{\sigma_1}{E} l_1 + \frac{\sigma_2}{E} l_2 + \frac{\sigma_3}{E} l_3 = \frac{1}{E} (\sigma_1 l_1 + \sigma_2 l_2 + \sigma_3 l_3) \\ &= \frac{1}{2 \times 10^5} (56.58 \times 180 + 127.4 \times 260 + 48.21 \times 160) \text{ mm} \\ &= 0.255 \text{ mm} \end{aligned}$$

- 47 Find the decrease in length of the steel bar loaded. Take $E = 2 \times 10^5 \text{ N/mm}^2$.

Sol Stress in the upper part $P_1 = \frac{2000}{\frac{\pi}{4} \times 10^2} = 25.46 \text{ N/mm}^2$

Stress in the lower part $P_2 = \frac{7000}{\frac{\pi}{4} \times 15^2} = 39.61 \text{ N/mm}^2$

Decrease in length of the bar $= \Delta l = \frac{P_1}{E_1} l_1 + \frac{P_2}{E_2} l_2 = \frac{1}{E} (P_1 l_1 + P_2 l_2)$



$$= \frac{1}{2 \times 10^5} (25.96 \times 180 + 36.61 \times 200)$$

$$= 0.0625 \text{ mm}$$

4.8/20) Decrease in length of the aluminium rod

$$\Delta l = \frac{P_1 l_1}{A_1 E_1} + \frac{P_2 l_2}{A_2 E_2}$$

$$= \frac{125 \times 10^3 \times 300}{\frac{\pi}{4} (60)^2 \times 7 \times 10^4} + \frac{125 \times 10^3 \times 450}{\frac{\pi}{4} (60)^2 \times 7 \times 10^4}$$

$$= 0.7105 \text{ mm}$$

Now consider the Steel rod.

Let A = Area of the rod

for the condition that the decrease in length to be

the same as that of the aluminium rod

$$\Delta l = \frac{Pl}{AE_1} = 0.7105$$

$$A = \frac{Pl}{E \Delta l} = \frac{125 \times 10^3 \times 750}{2 \times 10^5 \times 0.7105} = 659.747 \text{ mm}^2$$

For the condition the stress in the Steel rod to be 160 N/mm^2

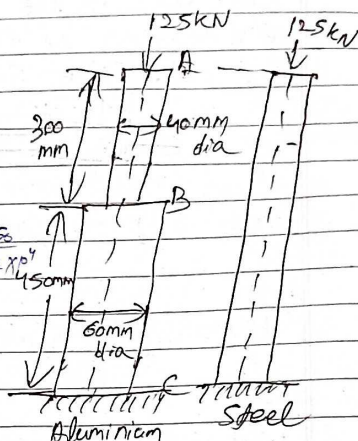
$$A = \frac{125 \times 10^3}{160} = 781.25 \text{ mm}^2$$

Area of the Steel rod required = greater of the values determined

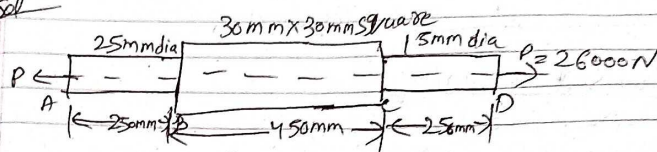
$$A = 781.25 \text{ mm}^2$$

$$\frac{\pi d^2}{4} = 781.25$$

$$d = 31.54 \text{ mm}$$



49. Sol



$$\text{Pull} = 26000\text{ N}$$

$$E = 2 \times 10^5\text{ N/mm}^2$$

$$\text{Part AB: } \sigma = \frac{26000}{\frac{\pi}{4}(25)^2} = 52.97\text{ N/mm}^2$$

$$\text{Extension} = \frac{52.97}{2 \times 10^5} \times 250 = 0.0662\text{ mm}$$

$$\text{Part BC: } \sigma = \frac{26000}{30 \times 30} = 28.89\text{ N/mm}^2$$

$$\text{Extension} = \frac{28.89}{2 \times 10^5} \times 450 = 0.0650\text{ mm}$$

$$\text{Part CD: } \sigma = \frac{26000}{\frac{\pi}{4}(15)^2} = 147.13\text{ N/mm}^2$$

$$\text{Extension} = \frac{147.13}{2 \times 10^5} \times 250 = 0.1839\text{ mm}$$

$$\text{Total Extension} = 0.0662 + 0.0650 + 0.1839 = 0.315\text{ mm}$$

Assignment
whether stress is vector or non vector
what is tensor quantity

E and C

Principle of Superposition

Tensor quantity:

Those physical quantity which require magnitude, direction and a plane in which it acts is known as tensor quantity.

* whether stress is vector or non and why?

⇒ Stress is a tensor quantity, which is more complex than a vector or scalar.

Why not a vector: -

because stress has multiple components acting in different direction but vector making to a ^{represent} single direction.

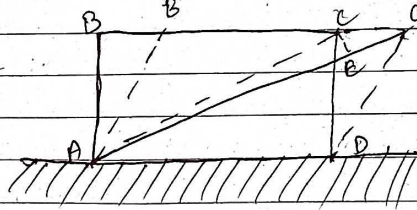
Why not a scalar: -

because stress has both magnitude and direction but scalar quantity only magnitude not ~~any~~ represent direction.

Why it is tensor

because it is describe multiple components direction dependence or linear relationship between stress component.

consider a cube a unit size acted upon by a simple shear stress



E and c

From fig-1

CE is perpendicular to diagonal AC'
increase in length of diagonal AC = EC' = CC' cos 45°
But angle CC'E is equal to angle BCA = 45°
Therefore strain along AC

$$\begin{aligned}
 &= \frac{EC'}{AC} \\
 &= \frac{CC' \cos 45^\circ}{AC} \\
 &= \frac{CC' \cos 45^\circ}{\frac{DC}{\cos 45^\circ}} \\
 &= \frac{CC' \cos^2 45^\circ = CC' \left(\frac{1}{\sqrt{2}}\right)^2}{DC} \\
 &= \frac{1}{2} \frac{CC'}{DC}
 \end{aligned}$$

$$\text{Strain} = E = \frac{1}{2} \phi \quad \dots (i)$$

Linear strain of diagonal AC = strain produced by tensile force along AC + strain produced by compressive force along BD

$$\begin{aligned}
 e &= \underbrace{\frac{T}{E}}_{\text{shear stress}} \rightarrow \text{strain} + \frac{T}{mE} \\
 e &= \frac{T}{E} \left(1 + \frac{1}{m}\right) \quad \dots (ii)
 \end{aligned}$$

from equation (i) and (ii)

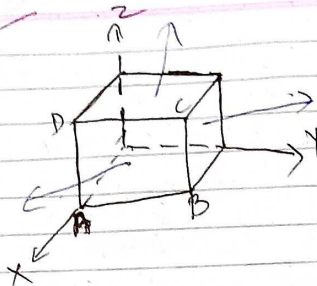
$$\frac{T}{E} \left(1 + \frac{1}{m}\right) = \frac{1}{2} \phi = \frac{1}{2} \frac{T}{C}$$

$$\Rightarrow \frac{T}{E} \left(1 + \frac{1}{m}\right) = \frac{1}{2} \frac{T}{C}$$

$$\Rightarrow E = 2C \left(1 + \frac{1}{m}\right)$$

$$\begin{aligned}
 &\because C = \frac{T}{\phi} \\
 &\Rightarrow \frac{1}{2} \phi = \frac{1}{2} \frac{T}{C}
 \end{aligned}$$

Example



Volume of cube l^3
 $\delta V = \frac{\delta V}{\delta l} = 3l^2$

$\Rightarrow \delta V = 3l^2 \delta l$

Volumetric strain $\frac{\delta V}{V} = \frac{3l^2 \delta l}{l^3}$
 $= \frac{3\delta l}{l}$

$\Rightarrow \frac{\delta V}{V} = \frac{3\delta l}{l} \dots (i)$

consider the effect of mutual perpendicular stresses on side DC

Strain of DC due to stress on side DC

DC due to stress along X $= \frac{\sigma}{E}$

" " " " " " Y $= -\frac{\sigma}{mE}$

" " " " " " Z $= -\frac{\sigma}{mE}$

Total Strain of DC $= \frac{\sigma}{E} - \frac{\sigma}{mE} - \frac{\sigma}{mE}$

$$\frac{\delta l}{l} = \frac{\sigma}{E} \left(1 - \frac{2}{m}\right) \dots (i)$$

from eqn (i) and (ii)

$$\frac{\delta l}{l} = \frac{3\sigma}{E} \left(1 - \frac{2}{m}\right)$$

$$E l = 3 \frac{\sigma}{E} \left(1 - \frac{2}{m}\right)$$

$$= \frac{E l}{\sigma} = \frac{3}{E} \left(1 - \frac{2}{m}\right)$$

But $\frac{E l}{\sigma}$ is a reciprocal of modulus k

$$\Rightarrow E = 3k \left(1 - \frac{2}{m}\right) \quad \left(\frac{1}{k} = \frac{3}{E} \left(1 - \frac{2}{m}\right) \right)$$

$$\Rightarrow E = 2C \left(1 + \frac{1}{m}\right)$$

$$\Rightarrow E = 3k \left(1 - \frac{2}{m}\right)$$

$$\Rightarrow E = 2C \left(1 + \frac{1}{m}\right)$$

$$\frac{E}{2C} = \left(1 + \frac{1}{m}\right) \quad (i)$$

The relation between E and

k is given by

$$E = 3k \left(1 - \frac{2}{m}\right)$$

$$\Rightarrow E = 3k \left(1 - \frac{2}{m}\right) = 2C \left(1 + \frac{1}{m}\right)$$

$$\Rightarrow E = 3k \left(1 - \frac{2}{m}\right) = 2C \left(1 + \frac{1}{m}\right)$$

$$\frac{E}{3k} = \left(1 - \frac{2}{m}\right) \dots (ii)$$

From equation (i) and (ii)

$$\frac{E}{3k} + \frac{E}{3k} = 1 - \frac{2}{m}$$

$$\frac{E}{3k} = 1 - \frac{2}{m}$$

$$= \left(1 + \frac{1}{m}\right) + \left(1 - \frac{2}{m}\right)$$

$$\frac{E}{2C} = 1 + \frac{1}{m}$$

$$= \frac{m+1}{m} + \frac{m-2}{m}$$

$$\frac{E}{3k} + \frac{E}{2C} = 2 - \frac{1}{m}$$

$$= \frac{m}{m}$$

$$\Rightarrow E = 3k \left(1 - \frac{2}{m}\right) = 2C \left(1 + \frac{1}{m}\right)$$

~~E and k~~

$$\Rightarrow E = 3k \left(1 - \frac{2}{m}\right) = 2C \left(1 + \frac{1}{m}\right)$$

$$\Rightarrow 3k - \frac{6k}{m} = 2C + \frac{2C}{m}$$

$$\Rightarrow 3k - 2C = \frac{6k}{m} + \frac{2C}{m}$$

$$\Rightarrow 3k - 2C = \frac{1}{m} (6k + 2C)$$

$$\Rightarrow \frac{1}{m} = \frac{3k - 2C}{6k + 2C} \quad \dots (iv)$$

~~E and k~~

relation betⁿ E and k

$$\Rightarrow E = 2C \left(1 + \frac{1}{m}\right)$$

$$E \Rightarrow 2C + \frac{2C}{m}$$

$$\Rightarrow \frac{2C}{m} = 2C - E$$

$$\frac{1}{m} = \frac{2C - E}{2C} \quad \dots (v)$$

from eqn (iv) and (v)

$$\frac{1}{m} = \frac{3k - 2C}{6k + 2C} = \frac{2C - E}{2C} \Rightarrow \left(1 - \frac{E}{2C}\right)$$

$$= \frac{(3k - 2C) \cdot 2C}{(6k + 2C)(2C)} \quad \dots$$

$$\Rightarrow \frac{E}{2C} = 1 + \frac{3k - 2C}{6k + 2C}$$

$$\Rightarrow \frac{E}{2C} = \frac{6k + 2C + 3k - 2C}{6k + 2C} = \frac{9k}{6k + 2C}$$

$$\Rightarrow \frac{F}{A} = \frac{\sigma}{\sigma + 2\epsilon}$$

$$\Rightarrow E = \frac{\sigma \times A}{\sigma + 2\epsilon} = \frac{\sigma (A/C)}{\sigma (3K/C)}$$

True

$$\Rightarrow E = \frac{\sigma}{\sigma + 2\epsilon}$$

$$\Rightarrow E = \frac{3K}{3K + 2C}$$

Stress and Strain Hooke's Law and elastic constant

Elastic constant

⇒ Elastic materials obey's Hooke's law.

⇒ Stress is directly proportional to the strain

⇒ The const of proportionality is represents by elastic constant.

⇒ Depending upon the nature of strain the three elastic constant are.

⇒ modulus of elasticity

⇒ modulus of rigidity

⇒ Bulk modulus.

⇒ modulus of elasticity

within the elastic limit the ratio of linear stress to the linear strain is known as Young's modulus or modulus of elasticity.

⇒ it is denoted by E

$$\Rightarrow E = \frac{\text{Stress}}{\text{Strain}} = \frac{F/A}{\Delta L/L} = \frac{\sigma}{\epsilon} = \frac{PL}{A\Delta L}$$

~~Chapter~~ Thin cylindrical shell

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⇒ Unit = ~~10~~ N/mm^2 or kN/m^2

⇒ The bar is stretched by an amount L under the action of the force F .

* Modulus of rigidity

⇒ The ratio of shear stress to the corresponding shear strain within elastic limit

⇒ It is denoted by N or G or C

⇒ We shall denote modulus of rigidity by (G)

⇒ Shear stress is denoted by τ or T or ϕ

$$\Rightarrow \text{Modulus of rigidity } (G) = \frac{\tau}{\phi}$$

⇒ Units = N/mm^2 or kN/mm^2 or kN/m^2

⇒ The type of stress induced in the bar is shear stress.

⇒ The shear modulus is the ratio of the shear stress to the shear strain.

* Bulk modulus

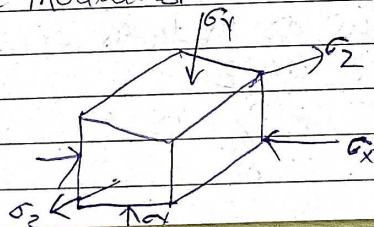
⇒ When a body is subjected to equal direct stress along three mutually perpendicular directions. The ratio between direct stress to corresponding volumetric strain.

⇒ The ratio of direct stress to the corresponding volumetric strain is called as bulk modulus.

⇒ It is denoted by K

$$\Rightarrow \text{Bulk modulus } K = \frac{\sigma}{\epsilon_v} = \frac{\text{Direct Stress}}{\text{Volumetric strain}}$$

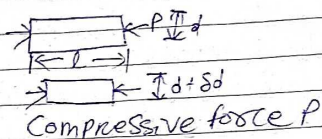
⇒ The compressibility is the inverse of the bulk modulus.



Lateral strain

⇒ When a bar is subjected to longitudinal strain, there is a change in lateral dimension also called lateral strain.

⇒ It is the ratio betn the change in lateral dimensions to the original lateral dimension

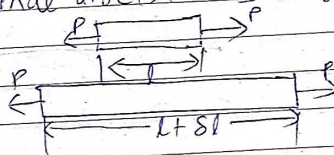


$$\text{Lateral strain} = \frac{\text{Change in diameter } (\delta d)}{\text{original diameter } (d)}$$

$$\frac{\text{Change in lateral dimension}}{\text{Change in lateral dimension}}$$

Longitudinal strain

⇒ The deformation change in length per unit length in longitudinal direction is longitudinal strain. $\epsilon = \frac{\delta l}{l}$



Units = no units for longitudinal strain

Poisson's ratio

⇒ It is the ratio of lateral strain to the longitudinal strain

⇒ It is denoted by $\frac{1}{m}$ or μ

$$\frac{1}{m} = \frac{\text{Lateral strain } (\delta d/d)}{\text{Longitudinal strain } (\delta l/l)}$$

units = no unit

value = elastic materials usually lies between 0.25 to 0.33

Factor of Safety:-

⇒ It is defined as the ratio of the ultimate stress to the working stress.

$$F.S = \frac{\sigma_u}{\sigma_s}$$

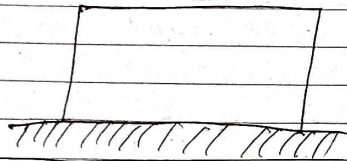
⇒ Factor of safety is decided by designers by experience and based on reliability required.

⇒ Working stress is also known as design stress.

⇒ F.O.S is the ratio between ultimate stress to design stress.

⇒ For a ductile material, after passing the ultimate strength, the material thins and stretches at a lower stress level before breaking.

Relation between Young's modulus E and Rigidity modulus G



Unit-2

Thin cylindrical and spherical shell under

Introduction

⇒ The fluids are stored under pressure in pressure vessels or shells and transmitted from one place to the other through pipes.

⇒ Example of thin shell

⇒ LPG cylinder

⇒ Perfume and spray bottle

⇒ Steam ~~boiler~~ boilers

⇒ Air compressor cylinder

⇒ Air brake master cylinder

⇒ Hydraulic cylinders.

⇒ Spherical vessels are also used for storing gas under pressure.

⇒ When it is a gas or vapour, the pressure is constant in all directions within the vessel

⇒ In case of liquid, the pressure is lowest at the top and increases due to self-weight when depth is very high.

Thin cylindrical shells

⇒ A cylindrical vessel or shell is classified as thin or thick depending upon the thickness of the plate in relation to the internal diameter of the cylinder.

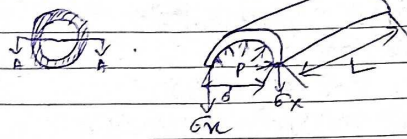
⇒ The ratio of $\frac{t}{D} = \frac{1}{20}$ can be considered suitable line of demarcation between thin and thick cylinder.

⇒ Thin cylinders are used to operate at pressure upto 30 MN/m^2 or more for high pressure such as 2.50 MN/m^2 or more thick cylinders are used.

- ⇒ The types of stresses are developed
- ⇒ Hoop or Circumferential stresses (along the circumference)
 - ⇒ Longitudinal stress (along the height of the cylinder)

Hoop stresses

- ⇒ It acts in a tangential direction to the circumference of shell



Let d = Internal diameter of the cylinder

t = thickness of the cylinder

p = Internal pressure (gauge) in the cylinder

σ_x = circumferential or hoop stress

$$\text{Bursting force} = \text{pressure} \times \text{projected area} \\ = p(dL)$$

$$\text{Resisting stress} = \sigma_x$$

$$\text{Area} = 2(tL)$$

$$= 2t(\sigma_x)$$

Equating the bursting force to the resisting strength, $pdk = 2kt\sigma_x$

$$\Rightarrow pd = 2t\sigma_x$$

$$\Rightarrow \sigma_x = \frac{pd}{2t}$$

⇒ σ_x is tensile in nature, when there is internal pressure and compression in nature.

Longitudinal Stresses



Let σ_l = Longitudinal stress produced in the shell
Load at the end = Circumference $\times$ thickness

$$\sigma_l = P \left(\frac{\pi}{4} d^2 \right) = \pi d t$$

$$\sigma_l = \frac{P d}{4 t}$$

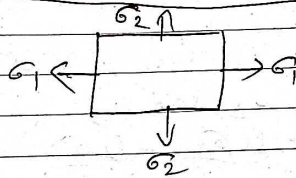
$\therefore$ The Circumferential stress or hoop stress (σ_c)
is twice as great as the longitudinal stress (σ_l)

Hoop Strain

$\Rightarrow$ Circumference depends on the diameter of the shell, so it is the ratio of change in diameter to the original diameter of the shell

$$\text{Hoop strain} = \frac{\delta d}{d}$$

Relationship between stress and strain



The strain in the direction of Y will be longitudinal strain and will be equal to σ_2 / E where
as the strain in the direction of X will be lateral strain and will be equal to $-\mu \times \frac{\sigma_2}{E}$

Let ϵ_x = Total strain in X direction

ϵ_y = Total strain in Y direction

Now total strain in the ~~xy~~ direction of x due to stresses σ_1 and σ_2

$$= \frac{\sigma_1}{E} - \mu \frac{\sigma_2}{E}$$

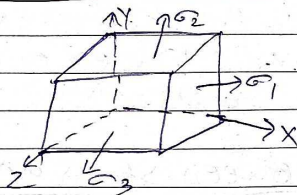
⇒ similarly total strain in the direction of y due to stresses σ_1 and σ_2

$$= \frac{\sigma_2}{E} - \mu \frac{\sigma_1}{E}$$

$$\epsilon_1 = \frac{\sigma_1}{E} - \mu \frac{\sigma_2}{E}$$

$$\epsilon_2 = \frac{\sigma_2}{E} - \mu \frac{\sigma_1}{E}$$

Relationship between stress and strain in three dimensional



Total strain in the direction of x due to stresses σ_1 , σ_2 and σ_3

$$\epsilon_1 = \frac{\sigma_1}{E} - \mu \frac{\sigma_2}{E} - \mu \frac{\sigma_3}{E}$$

Similarly total strains in the direction of y due to stresses σ_1 , σ_2 and σ_3

$$\epsilon_2 = \frac{\sigma_2}{E} - \mu \frac{\sigma_1}{E} - \mu \frac{\sigma_3}{E}$$

and total strains in the direction of z due to stresses σ_1 , σ_2 and σ_3

$$\epsilon_3 = \frac{\sigma_3}{E} - \mu \frac{\sigma_1}{E} - \mu \frac{\sigma_2}{E}$$

Let ϵ_1, ϵ_2 and ϵ_3 are total strains in the direction of x, y and z respectively then

$$\epsilon_1 = \frac{\sigma_1}{E} - \mu \frac{\sigma_2}{E} - \mu \frac{\sigma_3}{E}$$

$$\epsilon_2 = \frac{\sigma_2}{E} - \mu \frac{\sigma_1}{E} - \mu \frac{\sigma_3}{E}$$

$$\epsilon_3 = \frac{\sigma_3}{E} - \mu \frac{\sigma_1}{E} - \mu \frac{\sigma_2}{E}$$

Example

A cylindrical boiler is 2.5 m in diameter and 20 mm in thickness and it carries steam at a pressure of 1.0 N/mm². Find the longitudinal and hoop stress in the shell.

Given $d = 2.5 \text{ m} = 2500 \text{ mm}$

$$t = 20 \text{ mm}$$

$$p = 1.0 \text{ N/mm}^2$$

$$\frac{pd}{4t} = ? \quad \frac{pd}{2t} = ?$$

$$\frac{pd}{4t} = \frac{1.0 \times 2500}{2 \times 20} = 31.25 \text{ N/mm}^2$$

$$\frac{pd}{2t} = \frac{1.0 \times 2500}{2 \times 20} = 62.50 \text{ N/mm}^2$$

$$\therefore \text{Hoop stress} = 62.50 \text{ N/mm}^2$$

$$\text{Longitudinal stress} = 31.25 \text{ N/mm}^2$$

Change in length

The change in length of the cylinder may be determined from the longitudinal strain.

$$\text{Longitudinal strain} = \frac{1}{E} [\sigma_2 - \nu \sigma_1]$$

$$\begin{aligned} \text{Change in length} &= \text{longitudinal strain} \times \text{original length} \\ &= \frac{1}{E} [\sigma_2 - \nu \sigma_1] L = \frac{pd}{4te} [1 - 2\nu] L \end{aligned}$$

Change in diameter

The change in diameter may be determined from the strain on a diameter.

$$\text{i.e.} = \text{Diametral strain} = \frac{\text{Change in diameter}}{\text{original diameter}}$$

The stress acting around a circumference is the hoop or circumferential stress σ_H giving rise to the circumferential strain ϵ_H .

$$\begin{aligned} \text{Change in circumference} &= \text{strain} \times \text{original circumference} \\ &= \epsilon_H \times \pi d \end{aligned}$$

$$\begin{aligned} \text{New circumference} &= \pi d + \pi d \epsilon_H \\ &= \pi d (1 + \epsilon_H) \end{aligned}$$

But this is the circumference of a circle of diameter $d(1 + \epsilon_H)$

$$\therefore \text{New diameter} = d(1 + \epsilon_H)$$

$$\therefore \text{Change in diameter} = d \epsilon_H$$

$$\text{diametral strain } \epsilon_D = \frac{d \epsilon_H}{d} = \epsilon_H$$

$$\begin{aligned} d \epsilon_H &= \frac{d}{E} [\sigma_H - \nu \sigma_L] \\ &= \frac{Pd^2}{4tE} (2 - \nu) \end{aligned}$$

Change in volume

Change in volume = Volumetric strain $\times$ original volume
Volumetric strain = Sum of three mutually perpendicular direct strains.

$$\Delta = \epsilon_L + 2\epsilon_H$$

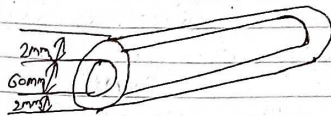
$$= \frac{1}{E} [\sigma_L - \nu \sigma_H] + 2 \frac{1}{E} [\sigma_H - \nu \sigma_L]$$

$$= \frac{1}{E} [\sigma_L + 2\sigma_H - \nu(\sigma_H + 2\sigma_L)]$$

$$\Rightarrow \frac{Pd}{4tE} [1 + 2 - \nu(2 + 2)] = \frac{Pd}{4tE} [5 - 4\nu]$$

Change in ^{internal} Volume = $\frac{Pd}{4tE} [5-4\nu] V$

Example-1 Calculate the bursting pressure for a cold drawn - seamless steel tubing of 60mm inside diameter with 2mm wall thickness. The ultimate strength of steel is 380 MN/m².



Given. $d = 60 \text{ mm}$

$t = 2 \text{ mm}$

Ultimate stress $\sigma_u = 380 \text{ MN/m}^2$

$P = ?$

$$\sigma_u = \frac{Pd}{2t}$$

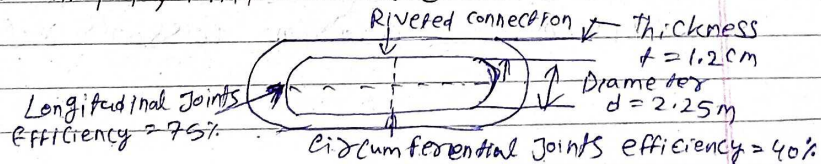
$$380 \times 10^6 = \frac{P \times 0.06}{2 \times 0.02}$$

$$P = \frac{380 \times 10^6 \times 2 \times 0.02}{0.06}$$

$$= 25.33 \times 10^6 \text{ N/mm}^2$$

$$\Rightarrow 25.33 \text{ MPa}$$

Example-2 Q. A air cylinder is 2.25m in diameter made of plates 1.2cm thick. The efficiencies of the longitudinal and circumferential joints are respectively 75% and 40%. If the tensile stress in the plate is to be limited to 120 MN/m², Find the maximum safe air pressure.



Given $d = 2.25\text{m}$

$t = 1.2\text{cm} = 0.012\text{m}$

$\sigma_w = 120\text{MN/m}^2 = 120\text{N/mm}^2$

Efficiency of the longitudinal joints = 75%

Efficiency of the circumferential joints = 40%

Max^m safe air pressure = ?

Considering circumferential stress as 120MN/m^2

$$\sigma_w = \sigma_x = \frac{Pd}{2th}$$

$$\Rightarrow 120 = \frac{P \times 2250}{2 \times 12 \times 0.75}$$

$$P = \frac{120 \times 2 \times 12 \times 0.75}{2250}$$

$$= 0.96\text{N/mm}^2$$

Considering longitudinal stress as 120MN/m^2

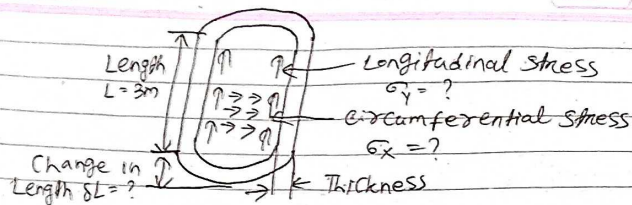
$$\sigma_w = \sigma_y = \frac{Pd}{4th_c}$$

$$\Rightarrow 120 = \frac{P \times 2250}{4 \times 12 \times 0.4}$$

$$P = \frac{120 \times 4 \times 12 \times 0.4}{2250}$$

$$= 1.029\text{N/mm}^2$$

Example-3 A cylindrical shell 3m long which is closed at the ends has an internal diameter of 1m and a wall thickness of 15mm. Calculate the circumferential and longitudinal stresses induced and also change in the dimension of the shell if it is subjected to an internal pressure of 1.5MN/m^2 . $E = 200\text{GN/m}^2$, Poisson's ratio = 0.3



Given

$$L = 3m$$

$$d = 1m$$

$$t = 15mm$$

$$P = 1.5 MN/m^2$$

$$E = 200 GN/m^2$$

$$\frac{L}{m} = 0.3$$

Circumferential Stress = ?

Longitudinal Stress = ?

Change in dimension = ?

① Circumferential Stress (σ_x)

$$\sigma_x = \frac{Pd}{2t}$$

$$= \frac{1.5 \times 10^6 \times 1}{2 \times 15 \times 10^{-3}}$$

$$= 50 \times 10^6 N/m^2$$

$$\sigma_y = \frac{Pd}{4t}$$

$$= \frac{1.5 \times 10^6 \times 1}{4 \times 15 \times 10^{-3}} = 25 \times 10^6 N/m^2$$

$$\text{Change in diameter } \Delta d = \epsilon_x = \frac{1}{E} \left(\sigma_x - \frac{\sigma_y}{m} \right)$$

$$= \frac{\sigma_x}{E} \left(1 - \frac{1}{2m} \right)$$

$$= \frac{50 \times 10^6}{200 \times 10^9} \left(1 - \frac{1}{2 \times 0.3} \right)$$

$$= 0.0002125$$

$$E_x = \frac{\delta d}{d}$$

$$\delta d = E_x \times d = 0.0002125 \times 1 = 0.2125 \times 10^{-3}$$

$$= 0.2125 \times 10^{-3} \text{ mm/increase}$$

Change in length (δL)

$$E_L = \frac{1}{E} \left(\sigma_y - \frac{\sigma_x}{m} \right)$$

$$= \frac{\sigma_y}{E} \left(1 - \frac{2}{m} \right)$$

$$= \frac{2.5 \times 10^6}{200 \times 10^9} \left(1 - (2 \times 0.3) \right) = 0.00005$$

$$\Rightarrow E_L = \frac{\delta L}{L}$$

$$\Rightarrow \delta L = E_L \times L = 0.00005 \times 3$$

$$= 0.15 \text{ mm (Increase)}$$

Change in Volume (δV)

$$E_v = E_1 + E_2 + E_3$$

$$= E_L + 2E_c$$

$$= 0.00005 + 2 \times 0.0002125$$

$$= 0.000475$$

$$E_v = \frac{\delta V}{V}$$

$$\delta V = E_v \times V$$

$$= 0.000475 \times \frac{\pi}{4} \times d^2 \times L$$

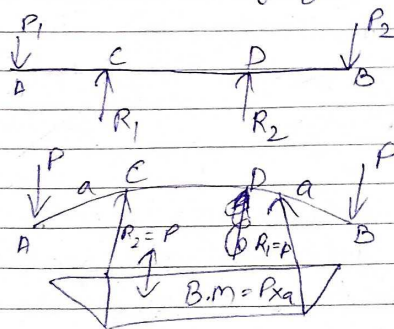
$$= 0.000475 \times \frac{\pi}{4} \times 3^2 \times 3 = 0.001119 \text{ m}^3$$

$$\delta V = 1119.3 \times 10^3 \text{ mm}^3$$

Unit-5

Theory of Simple bending

- ⇒ In general, along with bending, direct shearing due to transverse loads, twisting and buckling may occur in a beam.
- ⇒ Bending moment can come in two ways:-
Pure moment or transverse loads.
- ⇒ It is subjected to bending moment coming from transverse loads.
- ⇒ Only constant bending moment induced in the section (CD) is considered for pure bending and can be analyzed.
- ⇒ It must be noted that bending induces longitudinal stresses varying across the cross section.



Assumptions in theory of bending material

- ⇒ Perfectly homogeneous and isotropic
- ⇒ obeys Hooke's law.
- ⇒ modulus of Elasticity is same in tension and compression.
- ⇒ within the elastic limit

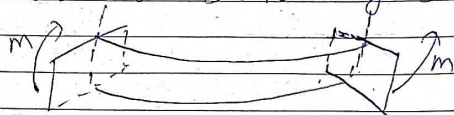
Assumptions in theory of bending - geometric

- ⇒ uniform cross-section
- ⇒ Beam is initially straight and long compared to the dimensions of the cross section.

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- ⇒ Transverse sections remain plane before and after bending.
 - ⇒ The loads are applied in the plane of bending.
 - ⇒ Resultant load lies in the plane of symmetry.
 - ⇒ Deflection of beam is small compared to its length.

Neutral axis

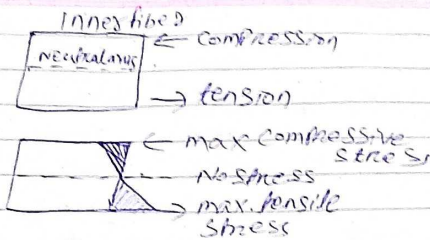
- ⇒ The beam is initially unstressed and subjected to a constant bending moment along its length.
- ⇒ Beam bends to a radius (R) and takes a circular curve shape along its length (L).



- ⇒ The normal stress due to bending are also called flexure stresses.
- ⇒ Outer fiber length of the beam increases due to tension.
- ⇒ Inner fiber length of the beam reduces due to compression.
- ⇒ There is a fiber between the two where the length does not change. It falls on the neutral axis.

Neutral axis

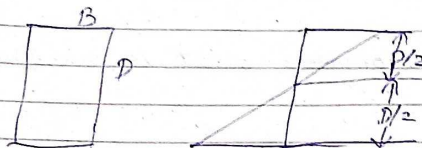
- ⇒ This axis of the beam where the stress is zero.
- ⇒ Length of the neutral axis remains same before and after bending.



Symmetrical Section

Neutral Axis lies exactly at the centre of the depth $Y_c = Y_t = \frac{D}{2}$

⇒ Extreme fibre stress are equal



Unsymmetrical Section

If the N.A. does not lie exactly at the centre of depth, the beam is known as unsymmetrical beam

⇒ Extreme fibre stress are not equal as Y_c and Y_t are not equal.

$$Y_c \neq Y_t$$

Section modulus :-

Rectangular section :-

width = b

depth = h

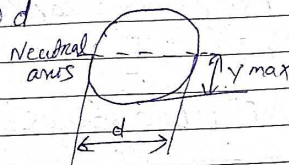
$$\text{Section modulus} = \frac{\text{Moment of inertia about neutral axis} = I}{\text{Distance from neutral axis } Y_{max}}$$

where $I = \frac{bh^3}{12}$ and $y_{max} = \frac{h}{2}$

$$Z_{rect} = \frac{\frac{bh^3}{12}}{\frac{h}{2}} = \frac{bh^3}{12} \times \frac{2}{h} = \frac{bh^2}{6}$$

Circular section:-

Diameter = d



(Moment of inertia about Neutral axis)

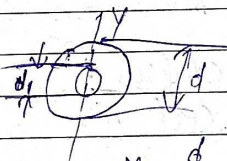
$$\Rightarrow I = \frac{\pi d^4}{64}$$

$$\Rightarrow y_{max} = \frac{d}{2}$$

$$Z = \frac{I}{y_{max}} = \frac{\frac{\pi d^4}{64}}{\frac{d}{2}} = \frac{\pi d^4}{64} \times \frac{2}{d} = \frac{\pi d^3}{32}$$

$$Z = \frac{\pi d^3}{32}$$

Hollow Circle



$$Area = \frac{\pi}{4}(d^2 - d_1^2)$$

$$y = \frac{d}{2}$$

$$I_{xx} = I_{yy} = \frac{\pi}{64}(d^4 - d_1^4)$$

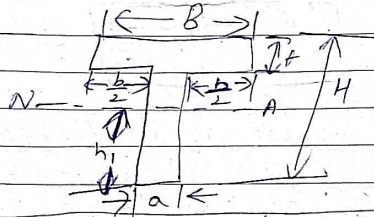
$$Z = \frac{I}{y} = \frac{\frac{\pi}{64}(d^4 - d_1^4)}{\frac{d}{2}} = \frac{\pi d^3}{32} \left(1 - \left(\frac{d_1}{d}\right)^4\right)$$

I Section



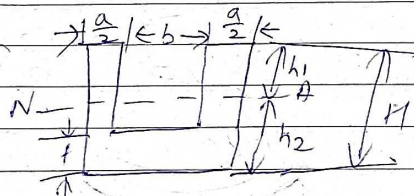
Area	Moment of Inertia (I)	y	Z = I/y
$bh - b_1h_1$	$I_{xx} = \frac{bh^3 - b_1h_1^3}{12}$	$\frac{h}{2}$	$Z_{xx} = \frac{bh^3 - b_1h_1^3}{2bh^2 \left[1 - \frac{b_1}{b}\left(\frac{h_1}{h}\right)^3\right]}$

T Section



Area	Moment of Inertia	y _{max}	Z = I/y _{max}
$BA + (H-t)a$	$I_{xx} = \frac{Bh^3 - b(h-t)^3 + ah_1^3}{3}$	$h_1 = aH^2/b^2$ $h_1 = H - h$	$Z_{xx} = \frac{2I_{xx}(aH + bt)}{aH^2 + bt^2}$

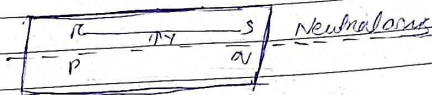
Channel Section



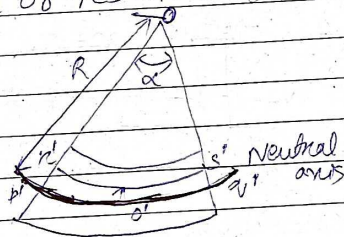
Area	Moment of Inertia	y _{max}	Z = I/y _{max}
$BA + (H-t)a$	$I_{xx} = \frac{Bh^3 - b(h-t)^3 + ah_1^3}{3}$	$h_1 = aH^2/b^2$ $h_1 = H - h$	$Z_{xx} = \frac{2I_{xx}(aH + bt)}{aH^2 + bt^2}$

Derivation of Simple bending formula

- Let us derive a relationship between
- ⇒ Radius of curvature (R) to which the beam N.A. bends
 - ⇒ Bending moment (M) applied
 - ⇒ Bending stresses (σ) and its cross-sectional dimension property (I).
 - ⇒ The equation which connects these variables is known as the bending equation



- ⇒ Length of Neutral layer of the beam before bending is $p'q'$.
- ⇒ length $RS = p'q'$ at a distance (y) from $p'q'$ radius of curvature ($R = OD$)
- ⇒ After bending neutral layer occupies $p'q'$. Since $p'q'$ lies on the N.A. its length does not change. $p'q' = p'q''$.
- ⇒ Length of RS after bending becomes ($r's'$)



$p'q'$ subtends an angle (α) at the centre of curvature, O .

$$p'q' = R\alpha \quad \dots (i)$$

$$r's' = (R - y)\alpha \quad \dots (ii)$$

Since beam is double, $2A$ is the area of the beam.

Since there is no change in length of the beam, $\Delta L = 0$.

$$\Delta L = \frac{FL}{EA} = 0$$

$$\text{Strain} = \frac{\Delta L}{L} = \frac{FL}{EA} = 0$$

Substituting $\Delta L = 0$ in (1)

$$\text{Strain} = \frac{\Delta L}{L} = \frac{FL}{EA} = 0$$

$$= \frac{FL}{EA} = 0$$

$$= \frac{FL}{EA} = 0$$

Let us take bending stress along Y -axis as σ .

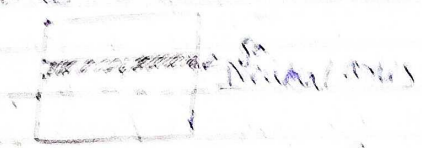
Young's modulus = E

$$\text{By Hooke's law, strain} = \frac{\sigma}{E}$$

$$\frac{\sigma}{E} = \frac{Y}{R} \Rightarrow \sigma = \frac{EY}{R}$$

Let a strip of area dA be at a distance Y from the neutral axis.

The neutral axis



$$F = \text{Stress} \times \text{Area} = \sigma \cdot dA = \frac{EY}{R} \cdot dA$$

$$\text{Moment of area about neutral axis} = \int Y \cdot dA \cdot Y$$

$$= \int Y^2 \cdot dA$$

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$$\begin{aligned}\text{Total resisting moment of the beam} &= \int \frac{E}{R} y^2 da \\ &= \frac{E}{R} \int y^2 da \\ \Rightarrow \int y da &= 0\end{aligned}$$

$y=0$ gives $\bar{y}$ the co-ordinate of the centroid of the cross-section

$\Rightarrow$ The term $\int y^2 da$ is the second moment of area about the neutral axis called moment of inertia (I_{NA}) = I

Resisting moment $\Rightarrow M = \frac{E}{R} \times I$

$$\Rightarrow \frac{M}{I} = \frac{E}{R} \quad \dots (iv)$$

from eqn (iv) = $\frac{E}{R} = \frac{\sigma}{y}$

Combine the eqn (iv) and (v)

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

Ex-1 A brass strip 80mm wide and 30mm thick is bent into an arc of radius 60m. What is the maximum stress developed in the strip if E for brass = $1 \times 10^5 \text{ MN/m}^2$

Given width of strip, $b = 80 \text{ mm}$

Thickness, $t = 30 \text{ mm}$

Radius of arc, $R = 60 \text{ m} = 60,000 \text{ mm}$

E for brass = $1 \times 10^5 \text{ MN/m}^2 = 1 \times 10^5 \text{ N/mm}^2$

maximum stress induced in both compression and tension = ?

$$y = t/2$$

$$= 30/2 = 15 \text{ mm}$$

From the relation, $\frac{\sigma}{y} = \frac{E}{R}$

$$\Rightarrow R = y \times \frac{E}{\sigma} = \frac{15 \times 10^5}{60,000}$$

$$\Rightarrow 25 \text{ N/mm}^2$$

Ex-2 To what radius an aluminium strip 100mm wide and 20mm thick can be bent if the maximum stress in the strip is not to exceed 50 N/mm^2 ? E for aluminium = $70 \times 10^3 \text{ N/mm}^2$

Given $b = 100 \text{ mm}$

$t = 20 \text{ mm}$

$$y = \frac{20}{2} = 10 \text{ mm}$$

$$\sigma = 50 \text{ N/mm}^2$$

$$E = 70 \times 10^3 \text{ N/mm}^2$$

Radius an aluminium = ?

$$\frac{\sigma}{y} = \frac{E}{R}$$

$$\Rightarrow R = y \times \frac{E}{\sigma}$$

$$= 10 \times \frac{70 \times 10^3}{50}$$

$$\Rightarrow 14000 \text{ mm} = 14 \text{ m}$$

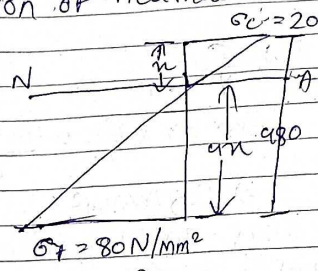
Ex-3 An unsymmetrical simply supported beam of depth 480mm is subjected to compressive stress of 20 N/mm^2 and tension stress of 80 N/mm^2 . Find the position of NA From the top?

Given: Unsymmetrical simply supported beam of depth 480mm

compressive stress = 20 N/mm^2

Tension stress = 80 N/mm^2

Position of neutral axis from the top = ?



$$\frac{n}{480-n} = \frac{20}{80}$$

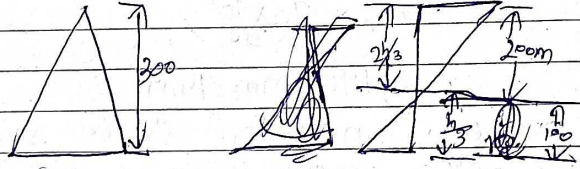
$$80n = 20(480-n)$$

$$80n + 20n = 9600$$

$$100n = 9600$$

$$n = \frac{9600}{100} = 96 \text{ mm}$$

EX-2 A triangular beam of depth 300mm is subjected to a max comp stress of 40 N/mm^2 at the top. Find the tensile stress at the bottom of the beam.
 Given: = Triangular beam depth = 300mm
 max. compression stress = 40 N/mm^2 at the top
 Tensile stress at the bottom of the beam = ?



$$\frac{\sigma_t}{40} = \frac{100}{200}$$

$$\sigma_t = \frac{40 \times 100}{2} = 20 \text{ N/mm}^2$$

Unit-4

Bending moment and Shear force

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* Beam:-

Structural member sufficiently long as compared to its lateral dimensions, supported at certain points along its length and subjected to loads transverse to its longitudinal axis is called a beam.

⇒ Generally the beam is horizontal and the applied loads are in a vertical plane acting perpendicular to the longitudinal axis of the beam.

Types of beams:-

⇒ Cantilever beam

⇒ Simply supported beam

⇒ fixed beam

⇒ continuous beam

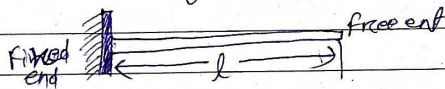
⇒ overhanging beam

•

* Cantilever beam

⇒ A beam which is fixed at one end and free at the other ends is called a cantilever.

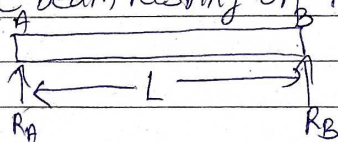
ex:- Beams projecting from the support to carry balcony, canopy etc.



* simply supported beam

⇒ A beam having its both ends freely resting on walls or columns is called simply supported beam.

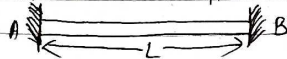
ex:- R.C.C beam resting on two walls.



* Fixed beam

A beam with both ends rigidly fixed or built in is called fixed beam or built in beam.

Ex: - R.C.C. beam in a framed structure.

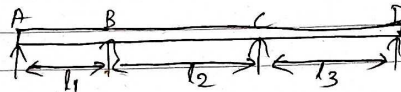


* Continuous beam

⇒ A beam which is supported over more than two supports is called continuous beam.

⇒ These span between the supports may be equal or unequal.

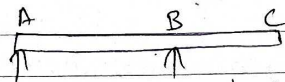
Ex: - R.C.C. beam continued over more than two supports.



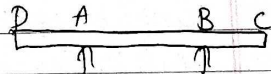
* Overhanging beam

⇒ A beam which is simply supported at two points and projected at one or both ends beyond the supports is called overhanging beam.

⇒ The additional length of the beam projecting beyond the support is called overhang.

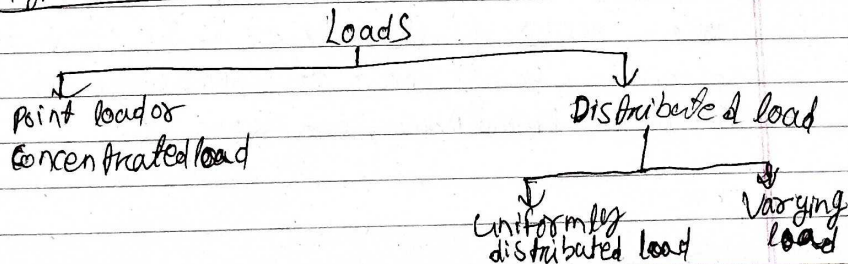


Single overhang beam



Double overhang beam.

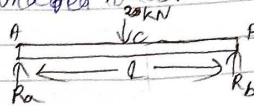
Types of loads



Point load

⇒ A load which acts at a point is called a point load or concentrated loads.

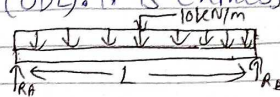
EX: 20 kN , R_A and R_B



Uniformly distributed load (UDL)

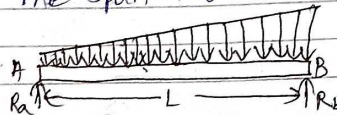
⇒ A load which is distributed over a length uniformly distributed load (UDL). It is expressed as load/m run.

EX: 10 kN/m



Varying load

⇒ A load which is varying uniformly from section to section over the span of a beam is called a varying load.



Shear force and bending moment

⇒ Shear force (S.F) at a section is defined as the algebraic sum of all vertical forces either to the left or to the right of the section including support reactions.

Unit: - Newton (N), kilo newton (kN)

Bending moment

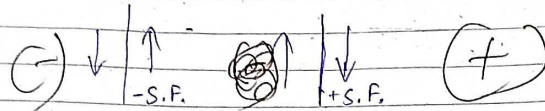
⇒ Bending moment at a section is defined as the algebraic sum of all movement caused by vertical forces either to the left or to the right of the section including those of supporting reactions.

Unit: - ~~Nmm~~ Nmm, Nm, kNm etc.

Sign Convention for shear force (S.F.)

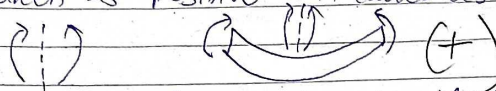
⇒ Shear force having an upward direction to the right hand side of a section or downwards to the left of the section will be negative

⇒ Shear force having upward direction to the left hand side of a section or downwards to the right of the section will be taken positive.

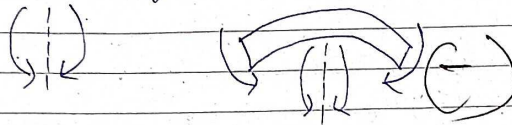


Sign convention for Bending moment (B.M.)

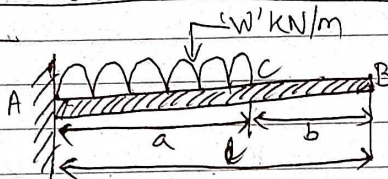
⇒ A bending moment causing concaving upwards will be taken as positive and called as sagging B.M.



⇒ A bending moment causing convexity upwards will be taken as negative and called as hogging B.M.



Shear force Diagram (SFD) and BMD for a cantilever subjected to UDL over a part span from the fixed end.



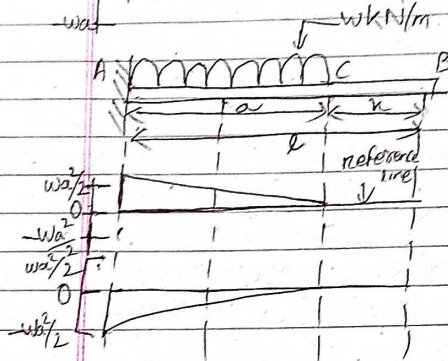
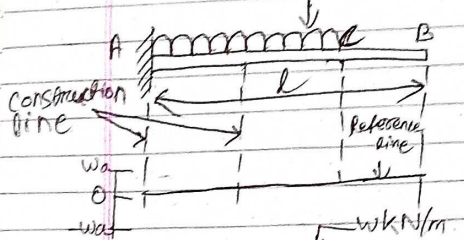
Free body diagram

The reaction at the fixed end

$$\text{Vertical upward reaction} = V_A = w \times a \\ = w a \text{ kN}$$

Moment at the fixed end = $M_A = -\frac{w l^2}{2}$

Bending moment diagram = $-\frac{w l^2}{2}$



B.M at B, $M_B = 0$

B.M at C, $M_C = 0$

B.M at the section X-X = $M_x = -\frac{wx \cdot x}{2}$
 $= -\frac{wx^2}{2}$ kN-m

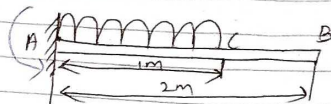
B.M at C (at $x = 0$) $M_C = -\frac{wx \cdot x}{2}$

$= 0$ kN-m

B.M at A (at $x = l$) $M_A = -\frac{wx \cdot x}{2}$

$= -\frac{wl^2}{2}$ kN-m

Shear force and bending moment diagrams for the cantilever



Free body diagram

⇒ The reactions at the fixed end

$$\text{Vertical upward reaction } V_A = wL = 25 \times 1 \\ = 25 \text{ kN}$$

$$\text{Moment at the fixed end, } M_A = -wL^2/2 = -\frac{(25 \times 1)^2}{2} \\ = -12.5 \text{ kN}\cdot\text{m}$$

The external moment at the fixed end will be in the anticlockwise direction. If the direction is the magnitude shall be taken +ve

Calculation of Shear force

$$\text{Shear force at B, } F_B = 0$$

$$\text{Shear force at C, } F_C = 0$$

In the portion AC

$$\text{SF at section } x-x, F_x = w \times x \text{ kN} = 25x \text{ kN (linear)}$$

$$\text{Shear force at C, (at } x=0) F_C = 25 \times 0 = 0 \text{ kN}$$

$$\text{Shear force at the fixed end, (at } x=1\text{m}) F_A = 25 \times 1 = 25 \text{ kN}$$

Calculation of Bending moment

$$\text{BM at B, } M_B = 0$$

$$\text{BM at C, } M_C = 0$$

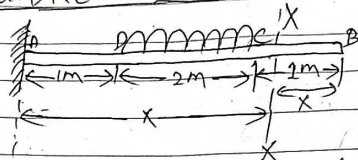
In the portion CA

$$\text{BM at section } x-x, M_x = -\frac{wx^2}{2} = -\frac{25x^2}{2} \text{ kN}\cdot\text{m}$$

$$\text{BM at C (at } x=0); M_C = -\frac{25 \times 0^2}{2} = 0 \text{ kN}\cdot\text{m}$$

$$\text{BM at the fixed end, A (at } x=1\text{m}), M_A = -\frac{25 \times 1^2}{2} \\ = -12.5 \text{ kN}\cdot\text{m}$$

* The cantilever beam of span



Since there is no load acting in the portion BC
The value of S.F is zero from B to C and SFD
coincides with reference line i.e zero S.F. line

$$\text{S.F. at any section } x-x, F_x = 2(x-1)$$

$$\text{S.F. at C (at } x=1\text{m)}, F_c = 2(0) = 0$$

$$\text{S.F. at D (at } x=3\text{m)}, F_D = 2(3-1)$$

$$\text{S.F. at any section } x-x, F_x = 2(x-1) \dots \text{linear}$$

$$\text{S.F. at D (at } x=3\text{m)}, F_D = 4\text{ kN}$$

$$\text{S.F. at A (at } x=4\text{m)}, F_D = 4\text{ kN}$$

$$\text{maximum S.F. } F_{\max} = F_A = 4\text{ kN}$$

Bending moment Diagram

The load is acting in the portion BC, the value
of B.M is zero from B to C and coincides with
reference line i.e zero B.M line

$$\text{B.M. at any section } x-x, M_x = -2(x-1) \times \frac{(x-1)}{2} = -(x-1)^2 \dots \text{parabolic}$$

$$\text{B.M. at D (at } x=1\text{m)}, M_c = -(1-1)^2 = 0$$

$$\text{B.M. at A (at } x=3\text{m)}, M_D = -(3-1)^2 = 4\text{ kN-m}$$

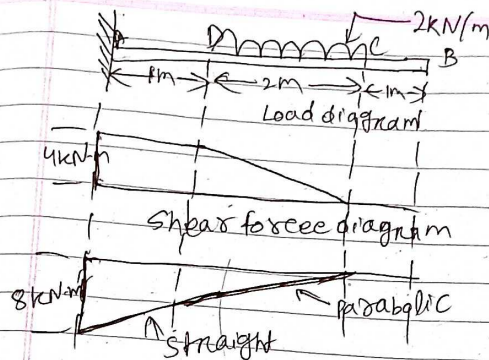
in the portion DA

$$M_x = -2 \times 2(x-2) = -4(x-2) \dots \text{linear}$$

$$M_D \text{ (at } x=3\text{m)} = -4(3-2) = -4\text{ kN-m}$$

$$M_A \text{ (at } x=4\text{m)} = -4(4-2) = -8\text{ kN-m}$$

Shear force and BMD are



B-M-D Bending moment Diagram

Bending moment

- ⇒ B.M at any section is defined as the algebraic sum of moments of all vertical forces acting either to the left or to the right of the section including those of support reactions.
- ⇒ B.M. causing concavity upwards will be taken as positive and called as sagging bending moment.
- ⇒ B.M. causing convexity upwards will be taken as negative and called as hogging bending moment.

Overhanging beams

- ⇒ If certain portion of a beam project beyond the support, the projected length of the beam is called overhang and such a beam is known as overhanging one side or both sides of the beam.
- ⇒ In simply supported beams (without overhangs), only sagging moments will occur.
- ⇒ In cantilever beams, only hogging moments will occur.
- ⇒ Sagging occurs in both supports.

- ⇒ In beams with overhangs, both sagging and hogging moments will occur.
⇒ hogging occurs in the overhang portion.

Point of Contraflexure

- ⇒ It is a point in the beam at which bending moment changes its sign from positive to negative and vice versa. Hence at the point of Contraflexure, the value of bending moment is zero.
⇒ Point of contraflexure is also known as point of inflexion.

Combined direct and bending stressesIntroduction

- ⇒ A machine subjected to axial compressive loads is called a strut or a column.
- ⇒ A strut is generally short and stout and may be horizontal or inclined.
- ⇒ A strut rarely buckles. A column is vulnerable to buckling.

Column

A column is a long slender member, normally vertical.

- ⇒ It is subjected to an axial compressive load and fixed rigidly or hinged at one or both ends or fixed at one end and either hinged or free at the other end.

Strut

- ⇒ A strut is a stouter member that may be fixed in any position.

difference betⁿ bending and buckling

- ⇒ A column is different from the beam even though they deflect in transverse direction, the forces are applying at perpendicular direction.

Direct and bending stresses

- ⇒ Whenever a body is subjected to an axial tension or compression, a direct stress comes into play at every section of the body.
- ⇒ we also know that whenever a body is subjected to a bending moment a bending stress comes into play.
- ⇒ It is thus obvious, that if a member is subjected to an axial loading along with a transverse bending a direct stress as well

- as a bending stress comes into play.

⇒ The magnitude and nature of these stresses may be easily found out from the magnitude and nature of the load and the moment. A little consideration will show that since both these stresses act normal to a cross-section, therefore the two stresses may be algebraically added into a single resultant stress.

Eccentric loading

⇒ A load, whose line of action does not coincide with the axis of a column or a strut is known as an ~~ex~~ eccentric load.

⇒ A bucket full of water, carried by a person in his hand, is an excellent example of an eccentric load.

⇒ A little consideration will show that the man will feel this load as more severe than the same load, if he had carried the same bucket over his head.

⇒ The simple reason for the same is that if he carries the bucket in his hand, then in addition to his carrying bucket, he has also to lean or bend on the other side of the bucket so as to counteract any possibility of his falling towards the bucket.

⇒ Thus we say that he is subjected to:-

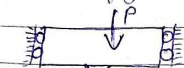
⇒ Direct load, due to the weight of the bucket (including water) and

⇒ moment due to eccentricity of the load

Slenderness ratio

⇒ The ratio of effective length of the column to the minimum radius of gyration of the cross sectional area of the column.

⇒ Radius of a gyration $k = \sqrt{\frac{I}{A}}$



Moment of Inertia ~~$I = Ak^2$~~
 $I = Ak^2$

where,

k = Radius of gyration

$$\frac{I}{A} = k^2$$

$$k = \sqrt{\frac{I}{A}}$$

Slenderness ratio

Slenderness ratio = $\frac{\text{Effective Length of column}}{\text{minimum radius of gyration}}$

⇒ It has no units

⇒ Slenderness ratio is an important factor in determining the instability of the column.

⇒ If the slenderness ratio is increased, the tendency to buckle increases and the capacity of the column to withstand the compressive load before buckling decreases.

Classification of column

⇒ Column can be classified into three types depending upon.

⇒ The Slenderness ratio

⇒ The length to least lateral dimension.

Short columns

⇒ Columns with slenderness ratio less than 12 are called short columns.

⇒ columns which have lengths less than 8 times their respective least lateral dimension.
⇒ considered to be subjected to direct compressive stresses only.

medium size columns

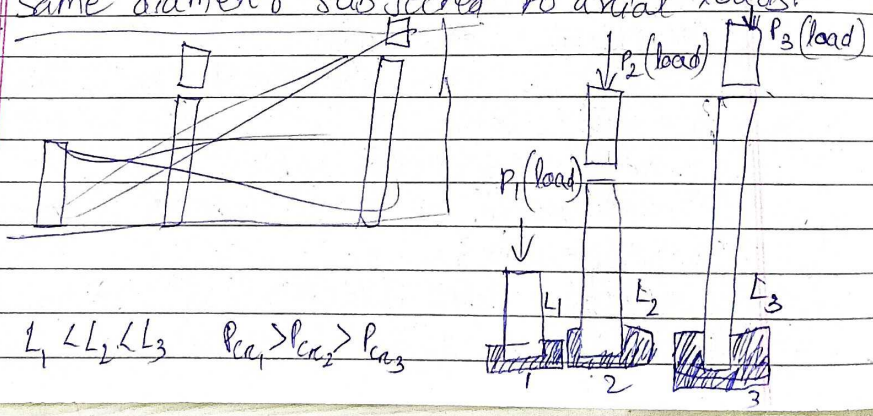
⇒ Columns with slenderness ratio lying between 32 and 120 are called medium size columns or intermediate columns.
⇒ Lengths varying from 8 times to 30 times their respective base dimension.
⇒ buckling as well as direct stresses are of significant factors.

long columns

⇒ Columns with slenderness ratio more than 120 are called long columns.
⇒ Length more than 30 times their respective least lateral dimension.
⇒ Long columns are usually considered to be subjected to buckling only.

comparison of critical load and stress for different column lengths

Let us take three flag pole columns of same diameter subjected to axial loads.



⇒ failure in the shortest column (1) is due to direct compression.

⇒ failure in medium column is due to combined buckling and direct compressive stress.

⇒ failure in long column is due to buckling.

⇒ failure stress values at failure by buckling are lowest for long column.

Comparison

Long Column	Short Column
Slenderness ratio is greater than 120	Slenderness ratio is less than $\frac{32}{3}$
⇒ long column fails primarily due to buckling	⇒ short column fails primarily due to compression.
⇒ Length is more than 30 times the least lateral dimensions	⇒ length is less than 8 times the least lateral dimension (or the smaller side).

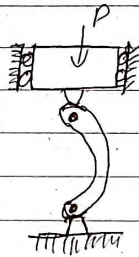
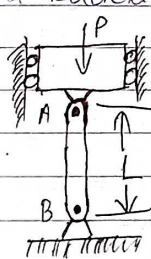
Failure of Columns

⇒ Columns fail due to excessive compressive loads.

⇒ by Pure Compressive Stress

⇒ by buckling

⇒ by combination of Pure Compressive Stress and buckling.



Failure of Columns due to Buckling.

Uniqueness of Buckling failure.

For cases of axial, torsion and bending, excessive stress or deformation is the significant factor for failure (yield or fracture)

⇒ Buckling of the member is uniquely different as the failure load bears no unique relationship to the stress and deformation.

⇒ Buckling refers to the unstable state of force and the force which causes this instability is called the critical force, the state of Buckling is also called indifferent equilibrium.

⇒ The column is presumed to have failed once instability occurs and little concern is given then to the degree of buckling.

* Definition of Buckling load.

⇒ the limiting load at which the column tends to have lateral displacement or tends to buckle.

⇒ This load is also called as crippling load.

⇒ The buckling takes place about the axis having minimum radius of gyration or least moment of inertia of a cross section.

* Definition of Safe load.

⇒ safe load is well below the buckling load.

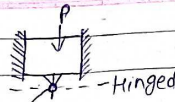
⇒ It is obtained by dividing the buckling load by a suitable factor of safety (F.S).

Different end Conditions of column

End conditions

The end conditions of a loaded column are of four types.

⇒ Both ends are pin jointed or hinged.



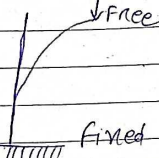
Hinged



Hinged

Both ends hinged

⇒ one end fixed and other end free

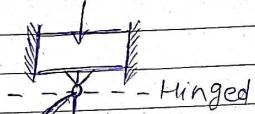


Free

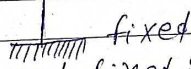
Fixed

one end fixed and other end free

⇒ one end fixed and other end pin jointed



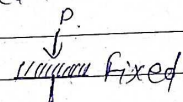
Hinged



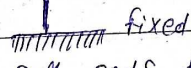
fixed

one end fixed and other end hinged.

Both ends fixed



Fixed



fixed

Both ends fixed.

Equivalent length of column

→ The distance between adjacent points of inflexion is called equivalent length or effective length or simple column length.

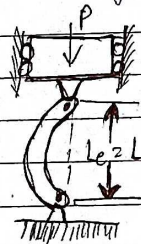
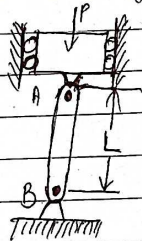
⇒ A point of inflexion is found at ends that are free to rotate (hinged or free) and at every point where there is a change of the sign of curvature. In both cases, the curvature is zero and the radius of curvature is ∞ .

⇒ An equivalent length of a column (or strut) of a given length, given section and given end conditions is defined as the length of a column (or strut) of the same material, same section and having the same buckling load but having both of its hinged.

⇒ Equivalent length for different end conditions will be given later. However this is given now as an information.

* Both ends hinged

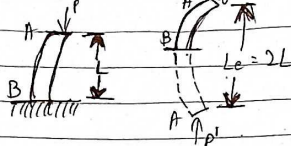
⇒ Equivalent length $L_e = \text{actual length } L$



equivalent length for Both ends hinged

* one end is fixed, and other end is free

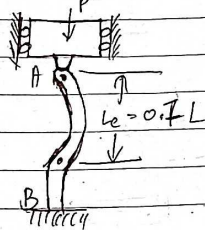
⇒ equivalent length $L_e = 2L$



* one end is fixed and other end is Pin Jointed.

⇒ between the top of the column and inflexion point

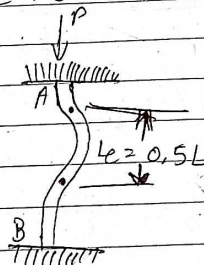
⇒ Equivalent length $L_e = \frac{L}{\sqrt{2}} = 0.707L$



* Both ends fixed

⇒ The distance between the two inflexion points

$L_e = \frac{L}{2}$



Euler Theory / Buckling load

* Euler formula

⇒ Euler's formula is used for calculating the critical load for a column or strut,

$$P_{\text{Euler}} = \frac{\pi^2 EI_{\text{min}}}{L_e^2}$$

where

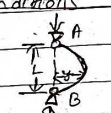
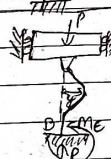
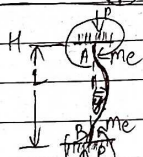
P_{Euler} = Critical load

E = Modulus of elasticity

I = Least moment of inertia of section of the column

L_e = Equivalent length of the column

Various end conditions of Euler's formula.

Case	End Condition	Figure with end conditions	Equivalent length
1	Both ends hinged or pin jointed or rounded or free		L
2	One end fixed other end free		$2L$
3	One end fixed other end pin jointed		$\frac{L}{2}$
4	Both ends fixed or encastered		$\frac{L}{2}$

Limitations for uses of Euler formula

- ⇒ While deriving the expression of the buckling load for a column, we have considered that
- ⇒ column has already buckled under the load P and then P_{os} determined.
- ⇒ The column is very long and it fails only by buckling.

⇒ In other words, formula is not valid for short or medium sized columns.

$$\text{Euler's buckling load, } P = \frac{\pi^2 EI}{l^2}$$

where l = equivalent length

I = least moment of inertia

$= Ak^2$ where k being the least radius of gyration.

$$= P = \frac{\pi^2 E Ak^2}{l^2} \quad \text{or} \quad \frac{P}{A} = \frac{\pi^2 E}{\left(\frac{l}{k}\right)^2}$$

where

l/k is called the Slenderness ratio.

⇒ This shows that Euler's buckling load is inversely proportional to the square of the slenderness ratio, or

⇒ As the length of the column decreases, buckling load goes on increasing.

⇒ Hence this is not valid for short columns which fails by crushing.

⇒ Therefore the ~~the~~ Euler's buckling theory is valid for a column beyond a certain value of Slenderness ratio.

⇒ Let f_c is the ultimate crushing strength or compressive strength of the materials of the column.

$$\begin{aligned} \frac{P}{A} &< f_c \\ &= \frac{\pi^2 E}{\left(\frac{l}{k}\right)^2} < f_c \end{aligned}$$

$$= \left(\frac{l}{k}\right)^2 > \frac{\pi^2 E}{f_c}$$

$$\text{Slenderness ratio } \frac{l}{k} > \pi \sqrt{\frac{E}{f_c}}$$

⇒ For a mild steel column with $E = 210 \text{ kN/mm}^2$

⇒ and ultimate stress in compression $f_c = 0.33 \text{ kN/mm}^2$

$$\frac{L}{k} > \pi \sqrt{\frac{E}{f_c}}$$

or

$$\frac{L}{k} > 79.25 \approx 80$$

⇒ From the above it is clear that for a mild steel column, Euler's theory is applicable only when the slenderness ratio is greater than 80.

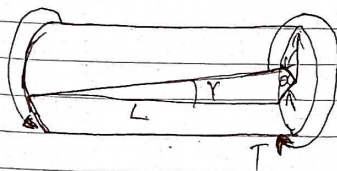
⇒ When the ends of the column are hinged the slenderness ratio should be more than 80.

Torsion

- ⇒ Shafts are used for transmitting power.
- ⇒ Power transmitting shafts are subjected to twist.
- ⇒ Twisting moment is called as torque or turning moment.

Torsion equation:-

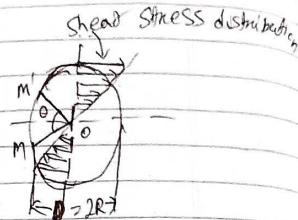
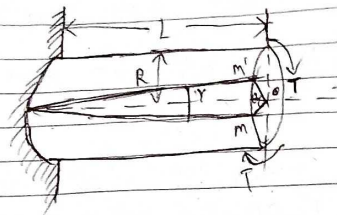
- ⇒ The equation which gives the magnitude of shear stress at any point on the shaft subjected to torsion is known as torsion equation.
- ⇒ Torsion equation is developed to establish a relation between applied torque, angle of twist and shear stress induced.



Assumptions

- ⇒ The material is assumed to be homogeneous.
- ⇒ The twist along the length of the shaft is uniform.
- ⇒ Cross section is uniform circular throughout.
- ⇒ The cross-section of the shaft, which remains plane and circular, before and after twist.
- ⇒ All radii remain straight and unchanged after twist.
- ⇒ The maximum shear stress induced in the shaft is within the elastic limit.

Torsion equation for shaft



Torsional Shear Stress in shaft

L = Length of the shaft

D = diameter of the shaft

T = maximum twisting moment

Let us take a line (OM) along the shaft from fixed end.

Line OM occupies a new position, OM' due to application of torque. Cross section of the shaft is twisted by an angle θ and surface by an angle γ .

θ = The angle of twist (radians)

Surface angle = Shear strain = γ (radians)

J = Polar moment of inertia

C = modulus of rigidity

Shear stress (τ) is induced in the cross section due to the application of torque.

Choose a ring with radius = r

Shear strain in the shaft due to twisting

$$\gamma_r = \frac{mm}{L} = r \theta$$

where R = Radius of shaft

$$mm' = R \times \theta$$

Therefore, $\gamma_r = \frac{R\theta}{L}$

Also shear strain (γ_r) by Hooke's = $\frac{\text{shear stress}}{\text{modulus of rigidity}}$

$$\Rightarrow \gamma_r = \frac{\tau_r}{G} \quad \text{--- eq (2)}$$

Combining eq (1) and (2)

Substituting for γ_r , $\frac{R\theta}{L} = \frac{\tau_r}{G}$

$$\frac{\tau_r}{R} = \frac{G\theta}{L} \quad \text{--- eq (3)}$$

$$\tau_r = \frac{G\theta}{L} R \quad \text{--- eq (3A)}$$

$\Rightarrow$ Shear stress is proportional to the radius. Therefore at center of the shaft cross-section shear stress is zero and at the radius R the shear stress is maximum.

$\Rightarrow$ In a shaft of given material's maximum permissible shear stress is fixed.

$\Rightarrow$ Thus the greatest twisting moment that the shaft can withstand is proportional to the selection Polar modulus of the shaft.

$\Rightarrow$ Shaft is an important machine element and is used on power transmission device.

$\Rightarrow$ Usually of circular cross section and are supported on bearings.

$\Rightarrow$ may be made either in solid or hollow sections.

functions of shaft

$\Rightarrow$ Shafts are used to transmit power.

$\Rightarrow$ Shafts are used to transmit the motion

$\Rightarrow$ Shafts are used to transmit the torque

Properties of Shafts

- ⇒ High strength
- ⇒ Good wear resistance
- ⇒ Low sensitivity to stress concentration.
- ⇒ Good machinability.
- ⇒ Ability to be heat treated.

Shafts materials

- ⇒ Usually made of mild steel and carbon steel.
- ⇒ Alloy steels are used where ⇒ high strength
⇒ wear resistance required
- ⇒ Commercial shafts are made of low carbon steel.
- ⇒ Cold drawn shafts are stronger than hot rolled shafts but with high residual stresses
- ⇒ For special purposes the shafts are forged.

Uses	Material
Cam shafts, spindles	C10, C14
Crank shafts, spindles, axles	C10, C14
above of high ductility	C50, C60

Standard Shaft Sizes

Diameter in mm	Increment
25-60	5 mm
60-110	10 mm
110-140	15 mm

- ⇒ Standard lengths are 5m, 6m and 7m.
- ⇒ Length is limited on account of manufacture and transport difficulties.
- ⇒ Machine shafts are
 - * 10-75 mm by increment of 5 mm
 - * 40-100 mm by increment of 10 mm

Polar section modulus

⇒ The term J/R is referred to as polar section modulus and is denoted by Z_p .

⇒ Let T be the moment of torsional resistance of the section of a shaft of radius R .

* J be polar moment of inertia of the shaft section.

⇒ Shear stress intensity τ at any point on the section, at a distance r from the axis of the shaft is given by, $\tau = \left(\frac{T}{J}\right)r$

⇒ The maximum shear stress τ_s occurs at the outer most radius R ,

$$\tau_s = \left(\frac{T}{J}\right)R$$

$$T = \tau_s \times J/R$$

$$T = \tau_s \times Z_p$$

where,

$$Z_p = \frac{J}{R}$$

Z_p = polar moment of inertia of the shaft

maximum radius

⇒ The above ratio is called Polar modulus of shaft section.

⇒ For a shaft magnitude of the Polar modulus of section is a measure of its strength in torsional rigidity.

⇒ shafts of same material and length

- having the same Polar modulus have the same strength.

Solid shaft

Let the solid circular shaft be of diameter 'd'
Polar moment of inertia,

$$J = \frac{\pi d^4}{32}$$

Polar modulus, $Z_p = \frac{J}{R}$

$$= \frac{2\pi d^4}{32d}$$

$$\Rightarrow Z_p = \frac{\pi d^3}{16}$$

Hollow shaft

Let hollow circular shaft be of external diameter 'D' and internal diameter 'd'

Polar moment of inertia,

$$J = \frac{\pi (D^4 - d^4)}{32}$$

Polar section modulus,

$$Z_p = \frac{J}{R}$$

$$= \frac{\left(\frac{\pi (D^4 - d^4)}{32} \right)}{\frac{D}{2}}$$

Torsional rigidity

$\Rightarrow$ The product of 'G' and 'J' is called Torsional

rigidity for the shaft.

$$\text{Torsional rigidity} = G \times J$$

From the torsion equation, the angle of twist is given as, $\theta = \frac{Tl}{GJ}$

⇒ For a given shaft length and torque, the angle of twist is inversely proportional to torsional rigidity.

Torsional stiffness

⇒ The torque required to produce a unit angular displacement is called torsional stiffness.

$$\text{Torsional stiffness, } k = \frac{T}{\theta} = \frac{GJ}{l}$$

Power

⇒ The ability or the capacity to do work is called power,

$$\text{Power} = \frac{W}{t}$$

Where, W = work done N-m

t = Time in sec

⇒ In general the power output of a machine is expressed in horse power.

⇒ Hp is an FPS unit of power.

⇒ 1 hp = 746 watts.

Power transmitted by shaft

$$\begin{aligned} \text{Power} &= \text{work done by torque per second} \\ &= \text{Torque} \times \text{Angular displacement per sec} \\ &= \frac{2\pi NT}{60} \text{ watts} \end{aligned}$$

Where T = Min torque, N-m

N = Speed of the shaft, rpm

$$\omega (\text{omega}) = \text{Angular velocity} = \frac{2\pi N}{60} \text{ Radians/Sec}$$

$$P = T \times \omega \text{ watts } T \text{ in N.m}$$

$$= \text{N-m/sec}$$

Comparison between solid and hollow shaft

- ⇒ The material of the solid shaft is not effectively used.
- ⇒ The central part of the shaft does not contribute to transmit the torque.
- ⇒ Hollow shafts are used for large power transmission.
- ⇒ Hollow shaft has greater strength and stiffness than solid shaft of equal weight.
- ⇒ Assuming a hollow shaft to be equal in torsional strength of a solid shaft.
- ⇒ Their resisting moments must be equal as they are made of same material and subjected to same stress intensity.
- ⇒ If $k = 0.5$ then $D = 1.022d$.
- ⇒ From the above results we can see that a hollow shaft having inside diameter one half of outside diameter would be only 2.2% lighter in diameter.
- ⇒ 2.2% lighter in weight than a solid shaft of equal strength.
- ⇒ If both the shafts are of the same diameter, the hollow shaft would be:
 - * 25% lighter in weight
 - * 93.7% as strong as the solid shaft.

Class work
Practice

Beam

which is subjected to transverse shear load bearing its functional load. Due to transverse shear load beam are subjected to variable shear force and variable bending moment.

hence to know this type of variation a maximum values of shear force and bending moment, we need to shear force diagram and bending moment diagram need to be drawn.

beam: - (i) statically determinate beam
(ii) statically indeterminate beam

- (a) simply supported beam
- (b) cantilever beam
- (c) overhanging beam
- (a) fixed beam
- (b) continuous beam

Assignment

Types of supports based on force resistance :-

- (1) Axial support :- Resists forces along the axis
- (2) Shear support :- Resists forces perpendicular to the axis
- (3) moment support :- Resists rotational forces

Types

Fixed support :- It prevents movement or rotation in any direction. A pole or column in concrete is an example. It is most rigid type.

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Roller Support :-

Allows rotation about an axis perpendicular to the plane of the structure, as well as lateral displacement along the support.

Pinned or hinged support

Allows for rotation but does not allow movement either vertically or horizontally.
Example :- A human knee.

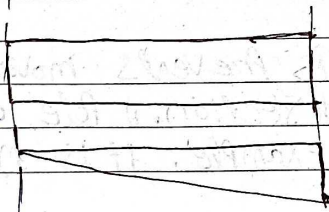
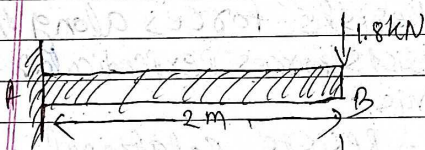
Simple Support

It provides vertical support at the end of the beam but allows for both rotation and some horizontal movement. Simple supports are often used in floor joists or beams with minimal loads.

Rockers support

It is similar to roller support, it also resists vertical force and allows horizontal translation and rotation.

Q17 A cantilever beam 2m long carries a point load of 1.8 kN at its free end. Draw shear force and bending moment diagram for the cantilever.



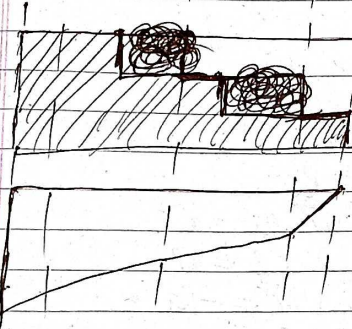
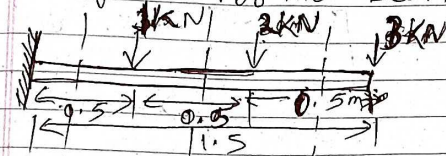
$$S.F. = -1.8 \text{ kN}$$

$$\text{Bending moment} = 1.8 \times 2 = 3.6 \text{ kNm}$$

Point B = $+w_1 = 1.8 \text{ kN}$

$F_c = 1.8 \times 2 = 3.6 \text{ kN-m}$

- (2) A cantilever beam 1.5 m long carries point loads of 1 kN, 2 kN and 3 kN at 0.5 m, 1.0 m and 1.5 m from the fixed end respectively. Draw the shear force and Bending moment diagram for the beam. $+6 \text{ kN}, -7 \text{ kN-m}$



~~$3 \times 1.5 = 4.5$~~
 $F_{\max} = 3 + 2 + 1 \text{ kN} = 6 \text{ kN}$
 26 kN

~~Reactions~~

~~Bending moment =~~

~~$3 \times 0.5 = 1.5 \text{ kN}$~~

~~$5 \times 0.5 = 2.5$~~

~~$6 \times 1.5 = 9$~~

~~$B.M. = -3 \times 0.5 = -1.5 \text{ m}$~~

~~$= -5 \times 1 = -5 \text{ m}$~~

~~$= -6 \times 1.5 = -9.0$~~

~~$= -9.0 + 5 - 0.5$~~

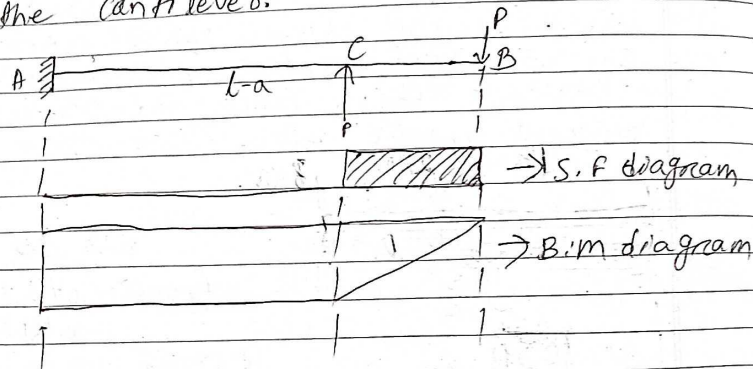
$B.M._B = 0$

$B.M._D = -3 \times 0.5 = -1.5$

$B.M._C = -(3 \times 1) + (2 \times 0.5) = -(3 + 1) = -4$

$B.M._A = -((3 \times 1.5) + (2 \times 1) + (1 \times 0.5)) = -(4.5 + 2 + 0.5)$
 $= -7 \text{ kN}$

- ① A cantilever AB of length l is subjected to a downward load P at its free end and an upward load P at distance a from the free end. Draw S.F and B.M diagram for the cantilever.



Reaction force at $A = 0$,

Reaction moment at A

$$M_A = Pl - P(l-a) = Pa$$

S.F at any section in AC = 0

S.F at any section in CB = $+P$

B.M at any section in AC = $-Pa$

B.M at B = 0

- (2) a cantilever subjected to a system of loads. Draw S.F. and B.M. diagrams

At any section between D and E, Distance x

From E, S.F. $S_x = +500N$; B.M. $= m_x = -500x$

At $x = 0$

$m_x = 0$

At $x = 0.5m$

$$m_x = -500 \times 0.5N = -250$$

At any section between C and D, Distance x from E,

$$S.F = S_x = +500 + 800 = 1300N$$

$$B.M = m_x = -500x - 800(x - 0.5) \\ = -1300x + 400$$

At $x = 0.5$

$$M_x = -1300 \times 0.5 + 400 \text{ Nm} = -250 \text{ Nm}$$

At $x = 1 \text{ m}$

$$M_x = -1300 + 400 \text{ Nm} = -900 \text{ Nm}$$

At any section betⁿ B and C distant x from B.

$$S.F. = S_x = 500 + 800 + 300 \text{ N} = 1600 \text{ N}$$

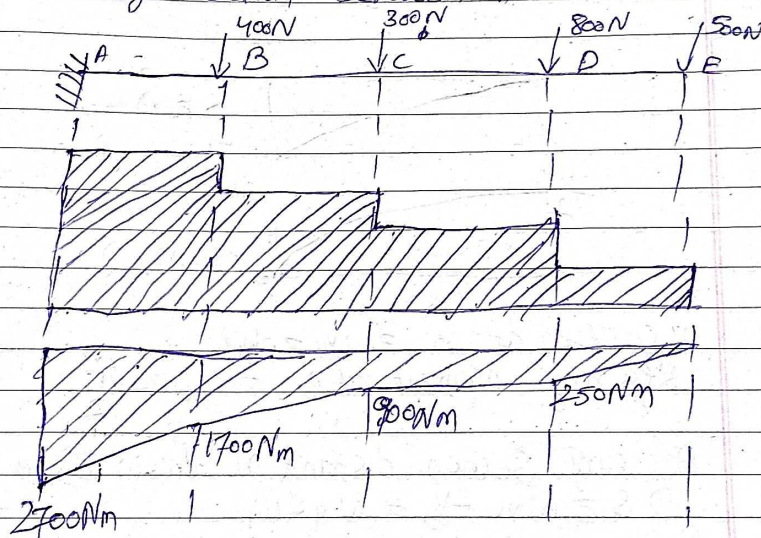
$$B.M. = M_x = -500x - 800(x - 0.5) - 300(x - 1) \text{ Nm}$$

$$= -1600x + 700 \text{ Nm}$$

At $x = 1 \text{ m}$, $M_x = -1600 + 700 = -900 \text{ Nm}$

At $x = 1.5 \text{ m}$, $M_x = -1600 \times 1.5 + 700 = -1700 \text{ Nm}$

At any section between A and B distant x from E



$$S.F. = S_x = 500 + 800 + 300 + 400 = 2000 \text{ N}$$

$$B.M. = M_x = -500x - 800(x - 0.5) - 300(x - 1) - 400(x - 1.5)$$

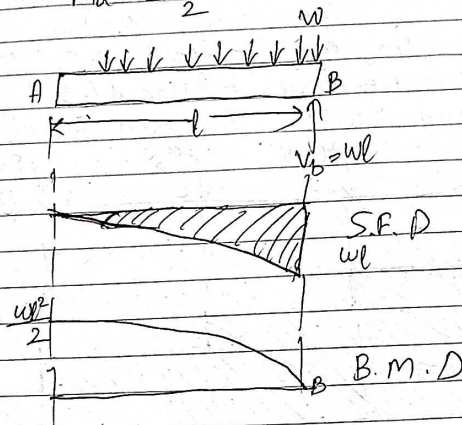
$$= -2000x + 1300 \text{ Nm}$$

At $x = 1.5 \text{ m}$, $M_x = -2000 \times 1.5 + 1300 \text{ Nm} = -1700 \text{ Nm}$

At $x = 2 \text{ m}$, $M_x = -2000 \times 2 + 1300 = -2700 \text{ Nm}$

- (3) Draw shear force and bending moment diagram for the beam AB. It is supported by a smooth plan A which slides within the groove and so it cannot exert a vertical reaction although it can offer reacting moment and an axial reaction.

$$M_A = \frac{wl^2}{2}$$



$$\text{Reaction at B} = V_B = wl$$

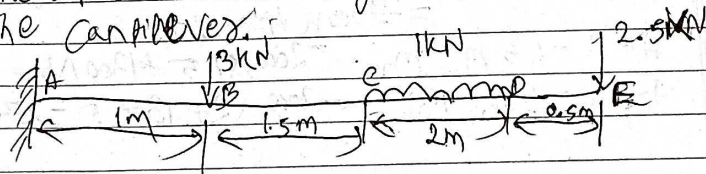
$$\text{Reacting moment at A} = M_A = \frac{wl^2}{2}$$

At any section distant x from B, shear force

$$\Rightarrow S = wx - V_B = wx - wl$$

$$\text{Bending moment} = m = \frac{wx^2}{2}$$

- (4) Shear Force and Bending moment Diagrams for the cantilever.

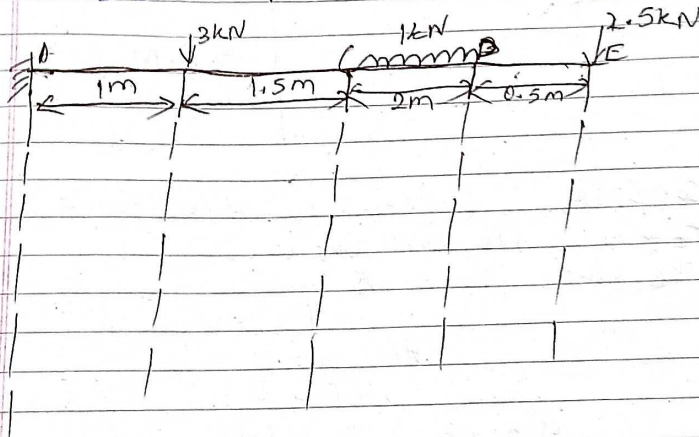


The Vertical and Horizontal forces at the
 Top E of the arm BE will be transmitted
 to the beam at B along with a $\odot$ Clockwise
 Couple of $43.2 \times 1.5 = 64.8 \text{ kNm}$
 The forces and couple transmitted to the
 beam ABCD are:-

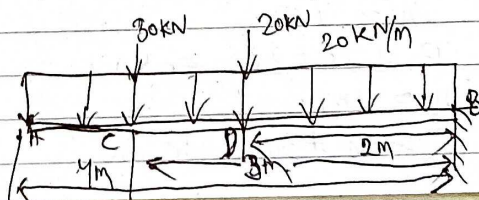
Normal thrust at Section just right of B
 $= 43.2 \text{ kN}$

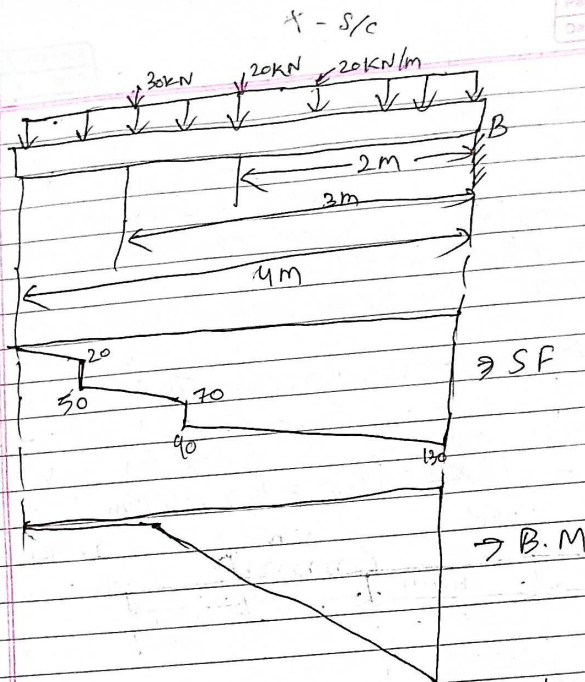
Shear force at a section just right B =
 $18 - 32.4 = -14.4 \text{ kN}$

Bending moment at a section just right
 of B = $18 \times 2 + 64.8 = 100.8 \text{ kNm}$



(5) A cantilever beam is loaded. Draw the
 BM and S.F. Diagrams





Consider a section at a distance x from A.
 For x to lie betn A and C.
 The shear force is-

$$F = 20x$$

At $x = 0$

$$F = 0$$

And at $x = 1m$

$$F = 20kN$$

For x to lie betn C and D

$$F = 20x + 30$$

At $x = 1m$

$$F = 20 + 30 = 50kN$$

At $x = 2m$

$$F = 40 + 30 = 70kN$$

for x to lie betⁿ D and B,

$$F = 20x + 30 + 20$$

$$\Rightarrow 20x + 50$$

At $x = 2m$

$$F = 40 + 50 = 90kN$$

At $x = 4m$

$$F = 80 + 50 = 130kN$$

B.M. Diagram Between A and C and bending moment is,

$$M = 20x \times \frac{x}{2} = 10x^2$$

$$\text{At } \Rightarrow x = 0, m = 0$$

$$x = 1m, m = 10kN \cdot m$$

at for x to lie betⁿ C and D,

$$M = 20x + \frac{x^2}{2} + 30(x-1)$$

$$= 10x^2 + 30x - 30$$

At $x = 1m$

$$m = 10 + 30 - 30 = 10kN \cdot m$$

At $x = 2m$

$$m = 40 + 60 - 30 = 70kN \cdot m$$

for x to lie between D and B,

$$M = 20x \times \frac{x}{2} + 30(x-1) + 20(x-2)$$

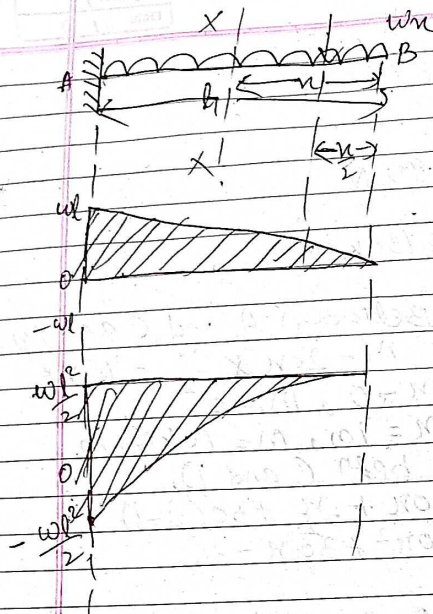
$$= 10x^2 + 50x - 70$$

At $x = 2m$

$$m = 40 + 100 - 70 = 70kN \cdot m$$

At $x = 4m$

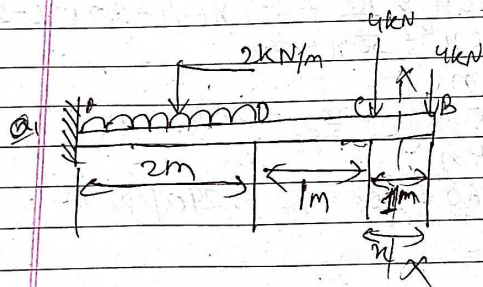
$$m = 160 + 200 - 70 = 290kN \cdot m$$



$$\begin{aligned} SF &= w(x-l) \\ \text{at } x=0 & \\ \Rightarrow w(x-l) &= 0 \\ SF &= 0 \\ SF &= w(l-x) \\ BM &= w \left[l \cdot x - \frac{x^2}{2} \right] \\ \text{For } x &= l \\ BM &= w \left[2l^2 - 2l^2 + l^2 \right] \\ &= \frac{wl^2}{2} \end{aligned}$$

$$\begin{aligned} BM &= B = 0 \\ \therefore x &= l \\ BM_A &= w \left[0 + \frac{l^2}{2} \right] \\ &= \frac{wl^2}{2} \end{aligned}$$

$$\Rightarrow w \cdot l = \frac{w \cdot l^2}{2}$$

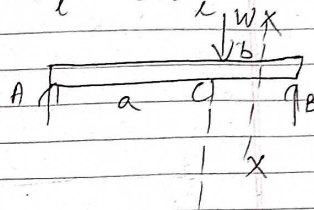


$$R_A + R_B = W \quad \text{--- (i)}$$

$$R_A = \frac{W a}{l} \quad R_B = \frac{W b}{l}$$

Taking moment about B

$$(R_A \times l) - (W \times a) = 0$$



$$R_A \times l = W a$$

$$R_A = \frac{W a}{l} \quad \text{put eqn (i)}$$

$$\frac{W a}{l} + R_B = W$$

$$R_B = W - \frac{W a}{l}$$

$$= \frac{W l - W a}{l}$$

$$= \frac{W(l-a)}{l}$$

$$= \frac{W a}{l} + \frac{W(l-a)}{l}$$

$$= \frac{W a + W(l-a)}{l}$$

$$W - \frac{W a}{l}$$

$$\frac{W l - W a}{l}$$

$$\frac{W(l-a)}{l}$$

$$\frac{W b}{l}$$

$$R_A = \frac{W a}{l}$$

$$R_B = \frac{W b}{l}$$

$$a + b = l$$

$$b = (l-a)$$

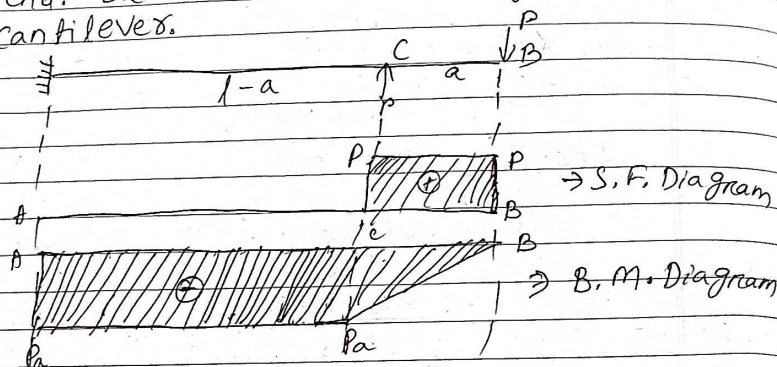
$$R_A \times l - W(l-a)$$

$$R_A \times a = \frac{W b}{l} \times a = \frac{W a b}{l}$$

$$R_A \times l - W(l-a)$$

$$= \frac{W a b}{l} - W a = 0$$

- ① A cantilever AB of length l is subjected to a downward load P at its free end and an upward load P at distance a from the free end. Draw S.F. and B.M. diagrams for the cantilever.



Sol Reacting force at A = 0

Reacting moment at A

$$M_A = Pl - P(l-a) = Pa$$

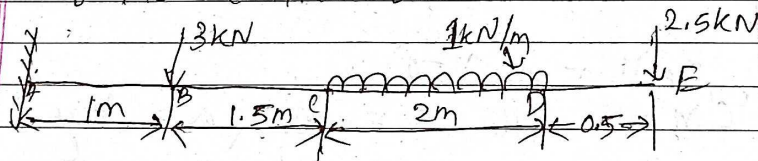
S.F. at any section in AC = 0;

S.F. at any section in CB = +P

B.M. at any section in AC = -Pa

B.M. at B = 0.

- ② Shear force and Bending moment diagrams for the cantilever.



Sol Let V_A be the vertical reaction at A.

Since there is only one support

$$V_A = 3 + 1 \times 2 + 2.5 = 7.5 \text{ kN (upward)}$$

There will also be a reacting moment or fixing moment at A in an anticlockwise order which will be equal and opposite to the moment of the forces on the cantilever about A.

$$\text{Reacting moment} = 3 \times 1 + 1 \times 2 \times 3.5 + 2.5 \times 5 \\ = 22.5 \text{ kNm}$$

S.F. calculations:-

$$\text{S.F. between A and B} = +7.5 \text{ kN}$$

$$\text{S.F. between B and C} = +7.5 - 3 = 4.5 \text{ kN}$$

$$\text{S.F. at D} = +2.5 \text{ kN}$$

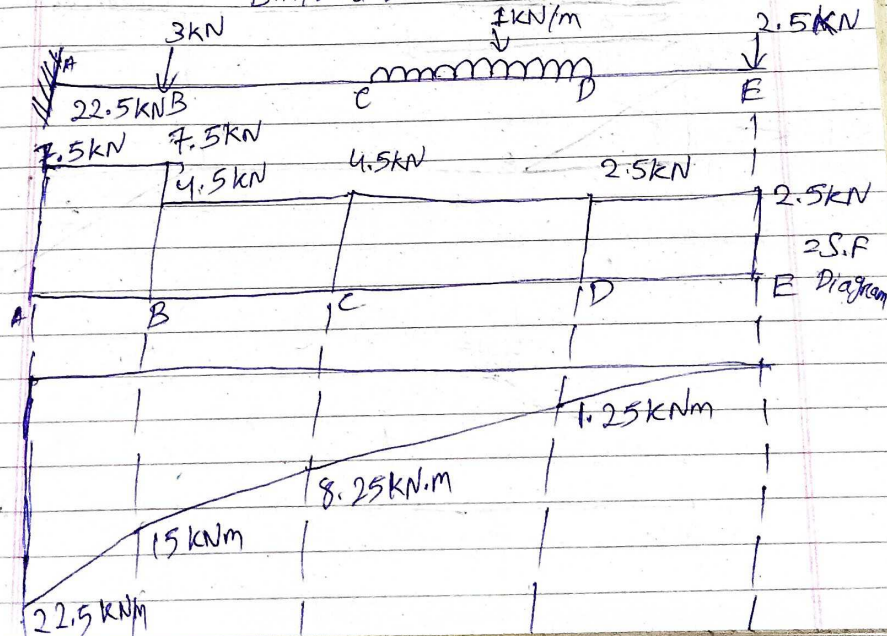
From C to D the

S.F. will change uniformly from +4.5 kN to +2.5 kN.

$$\text{S.F. between D and E} = +2.5 \text{ kN}$$

$$\text{B.M. at E} = 0$$

$$\text{B.M. at D} = -2.5 \times 0.5 = -1.25 \text{ kNm}$$



$$\text{B.m. at C} = -2.5 \times 2.5 - 1 \times 2 \times 1 = -8.25 \text{ kNm}$$

alternatively, $\therefore -22.5 + 7.5 \times 2.5 - 3 \times 1.5$
 $= -8.25 \text{ kNm}$

$$\text{B.m. at B} = -25 \times 4 - 1 \times 2 \times 2.5$$

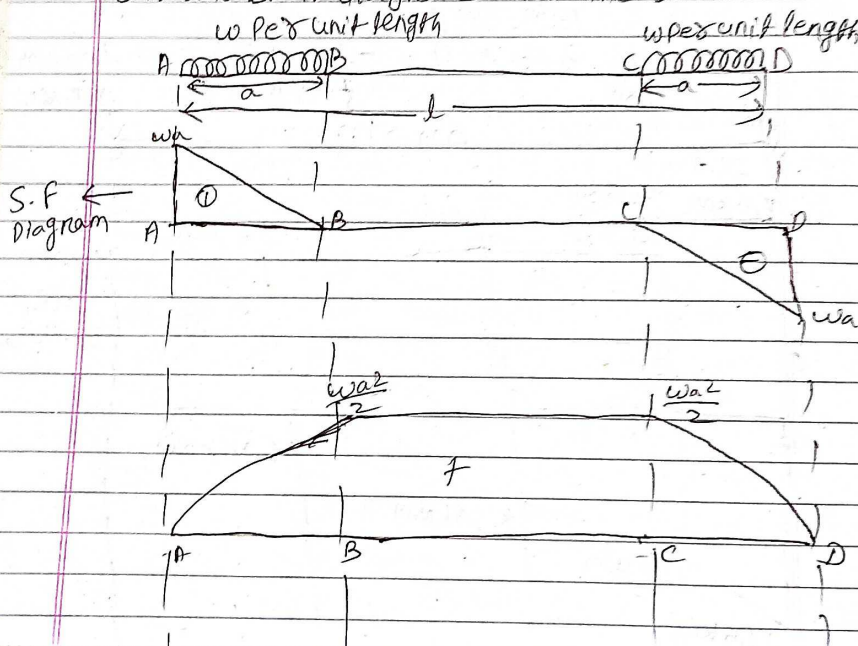
$$= -15 \text{ kNm}$$

alternatively

$$-22.5 + 7.5 \times 1 = -15 \text{ kNm}$$

$$\text{B.M. at A} = -22.5 \text{ kNm}$$

- (3) A beam ABCD of length l is simply supported at its ends A and D. It carries a uniformly distributed load of w per unit length on the parts AB and CD each of length a . Draw S.F. and B.M. diagrams for the beam.



V_m

Sol N_a and V_d at the supports are equal.

$$V_a = V_d = w a$$

S.F. Analysis

At any section in AB distant x

From A.

$$S.F. = S = w a - w x$$

At $x = 0$, i.e., at A

$$S_a = w a$$

At $x = a$, i.e., at B.

$$S_b = w a - w a = 0$$

at any section in BC, $S.F. = V_b - w x = 0$

The Shear force uniformly decreases from $+w a$ at A to zero at B.

B.M. Analysis

At any section in AB distant x from A

$$B.M. = m = w a x - \frac{w x^2}{2}$$

At $x = 0$, i.e., at A, $m_a = 0$

At $x = a$, i.e., at B, $m_b = w a a - \frac{w a^2}{2} = \frac{w a^2}{2}$

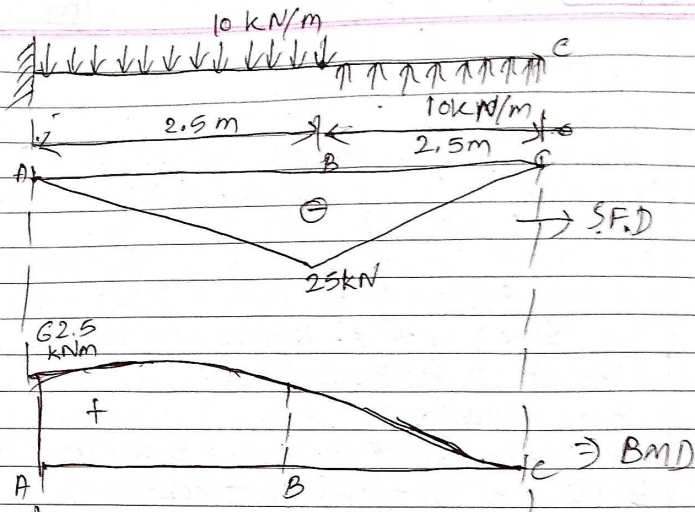
At any section in BC distant x from A

$$B.M. = m = w a x - w a \left(x - \frac{a}{2} \right) = \frac{w a^2}{2}$$

The bending moment is constant at the value $\frac{w a^2}{2}$ between B and C

From C to D the B.M. varies parabolically from the value $\frac{w a^2}{2}$ to zero.

(Q) Draw Shear force and bending moment diagram for the beam.



Taking moment about A

(i) $10 \times 2.5 \times 3.75 = 93.75 \text{ kNm} \curvearrowright$

(ii) $10 \times 2.5 \times 1.25 = 31.25 \text{ kNm} \curvearrowright$

Reacting force at A

$\Rightarrow M_A = 62.5 \text{ kNm}$

$\Rightarrow V_A = 0$

S.F. analysis

At any section in CB distant x from C.
The Shear force $= S = -10x$

At $x = 0, S = 0$

At $x = 2.5\text{m}, S = -25 \text{ kN}$

Similarly for the part AB

The Shear force varies linearly from 0 at A to -25 kN at B.

B.M. Analysis

Range AB At a section in AB distant x from A

The bending moment $= m = 62.5 - \frac{10x^2}{2}$
 $= 62.5 - 5x^2$

At $x=0$, i.e. at A, $m_a = +62.5 \text{ kNm}$
 At $x=2.5 \text{ m}$, i.e. at B, $m_b = 62.5 - 5 \times 2.5^2$
 $= +31.25 \text{ kNm}$

Range CB

At a section distant x from C

The bending moment $= M = \frac{10x^2}{2} = 5x^2$

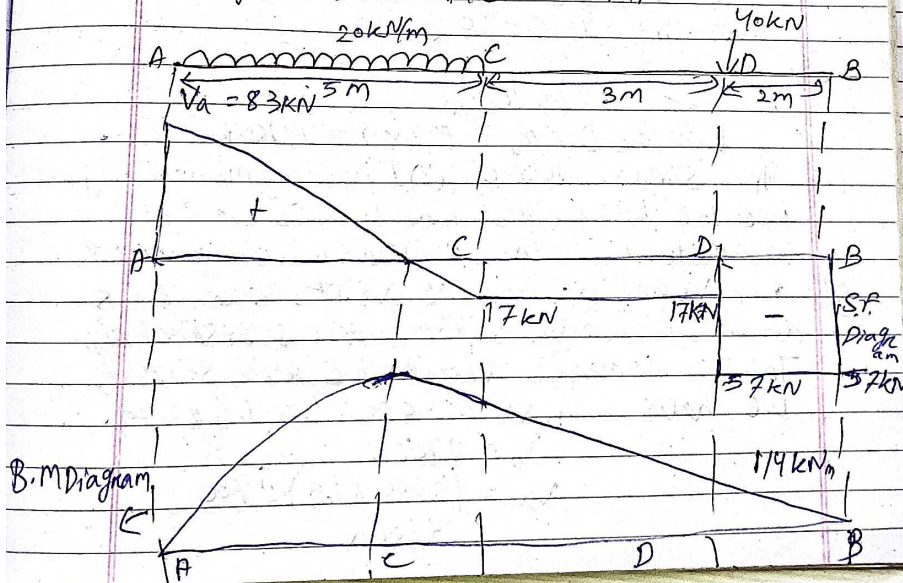
at $x=0$, i.e. at C

$m_c = 0$

At $x=2.5 \text{ m}$ i.e. at B

$m_b = 5 \times 2.5^2$
 $= 31.25 \text{ kNm}$

- (5) A beam AB 10m long is simply supported at its ends A and B. It carries a UDL of 20 kN/m for a distance of 5m from the left end A and a concentrated load of 40 kN at a distance of 2m from the right end B. Draw S.F. and B.M. diagrams for the beam.



Taking moments about the left end A,
 $V_b \times 10 = (20 \times 5 \times 2.5) + (40 \times 8)$
 $\therefore V_b = 57 \text{ kN}$
 $\therefore V_a = (20 \times 5) + 40 - 57 = 83 \text{ kN}$

SF Analysis

$$\text{S.F at A} = 83 \text{ kN}$$

$$\text{S.F at C} = 83 - 20 \times 5 = -17 \text{ kN}$$

$$\text{S.F bet}^n \text{ C and D} = -17 \text{ kN}$$

$$\text{S.F bet}^n \text{ D and B} = -57 \text{ kN}$$

Let the S.F. be zero at a distance of x metres

From A. Equating the S.F. to zero.

$$83 - 20x = 0$$

$$\therefore x = 4.15 \text{ m}$$

B.M Analysis B.M at A = $M_a = 0$

$$\text{B.M at C} = M_c = 83 \times 5 - 20 \times 5 \times 2.5$$

$$= 165 \text{ kNm}$$

The Section of zero shear, i.e. at $x = 4.15 \text{ m}$

$$\therefore M = 83 \times 4.15 - 20 \times \frac{4.15^2}{2}$$

$$= 172.225 \text{ kNm}$$

$$\text{B.M at D} = M_d = 57 \times 2 = 114 \text{ kNm}$$

- (6) The shear force and bending moment diagrams for the beam. Also find the positive and ~~top~~ negative of the maximum bending moment.

Sol Let V_a and V_b be the vertical reactions at the left and right supports respectively.

Taking moments about the left support, we have $V_b \times 5 = 4000 \times 1.5 (1.5 + 0.75)$

$$V_b = 2700 \text{ N}$$

$$V_a = (4000 \times 1.5) - 2700$$

$$= 3300 \text{ N}$$

S.F betⁿ A and C = 3000N

SF betⁿ D and B = -2700N

Equating the S.F at E to zero, we have
 $3300 - 4000(x - 1.5) = 0 \quad \therefore x = 2.325m$

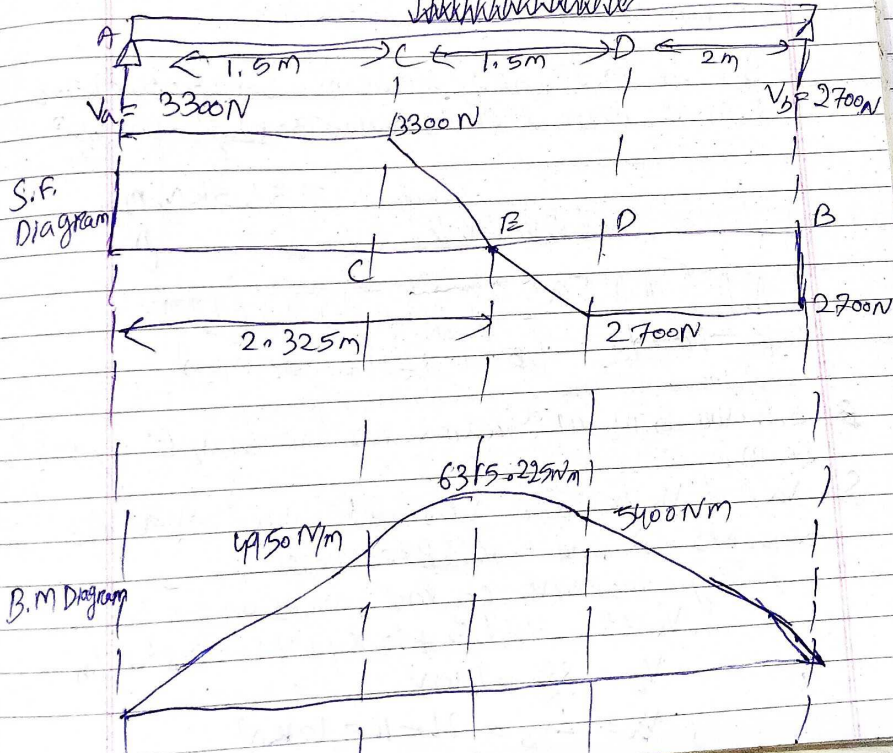
B.M = B.M. at A = $M_A = 0$

B.M at C = $M_C = 3300 \times 1.5 = 4950$ ^{Nm}

B.M at D = $M_D = 2700 \times 2 = 5400$ ^{Nm}

$A = M_E = 3300 \times 2.325 - 4000 \left(\frac{2.325 - 1.5}{2} \right)^2$

$= 6311.25 \text{ Nm}$
 4000 N/m



- (7) A 5m long reinforced concrete foundation beam ABCD supports a uniformly distributed load of 50 kN/m on its middle 3m length. Assuming a uniformly distributed upward soil reaction, determine
 (i) S.F. and bending moment at B
 (ii) Shear force and bending moment at the midpoint of the beam.

Sol Total load applied on the beam

$$= 50 \times 3 = 150 \text{ kN}$$

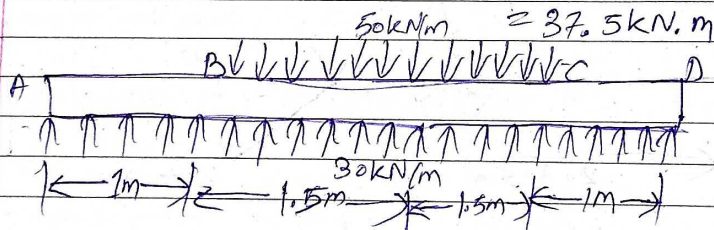
$$\text{Uniform upward soil reaction} = \frac{150}{3} = 30 \text{ kN/m}$$

$$\text{S.F. at B} = 30 \times 1 = 30 \text{ kN}$$

$$\text{B.M. at B} = \frac{30 \times 1^2}{2} = 15 \text{ kN/m}$$

$$\text{S.F. at midpoint of the beam} = 30 \times 2.5 - 50 \times 1.5 = 0$$

$$\text{B.M. at midpoint of the beam} = 30 \times \frac{2.5^2}{2} - 50 \times \frac{1.5^2}{2}$$



- (8) S.F. and B.M. diagrams for the simply supported beam.

Sol V_a and V_b be the vertical reactions at the supports A and B respectively.

taking moment of the forces on the beam.

$$V_b \times 8 = 4 \times 1.5 + 10 \times 4 + 7 \times 6 = 88 \text{ kN/m}$$

$$V_b = \frac{88}{8} = 11 \text{ kN}$$

$$V_a = -V_b = 21 - 11 = 10 \text{ kN}$$

S.F. between A and C = 10 kN

S.F. between C and D = $10 - 4 = 6$ kN

S.F. between D and E = $10 - 4 - 10 = -4$ kN

alternatively = $-11 + 7 = -4$ kN

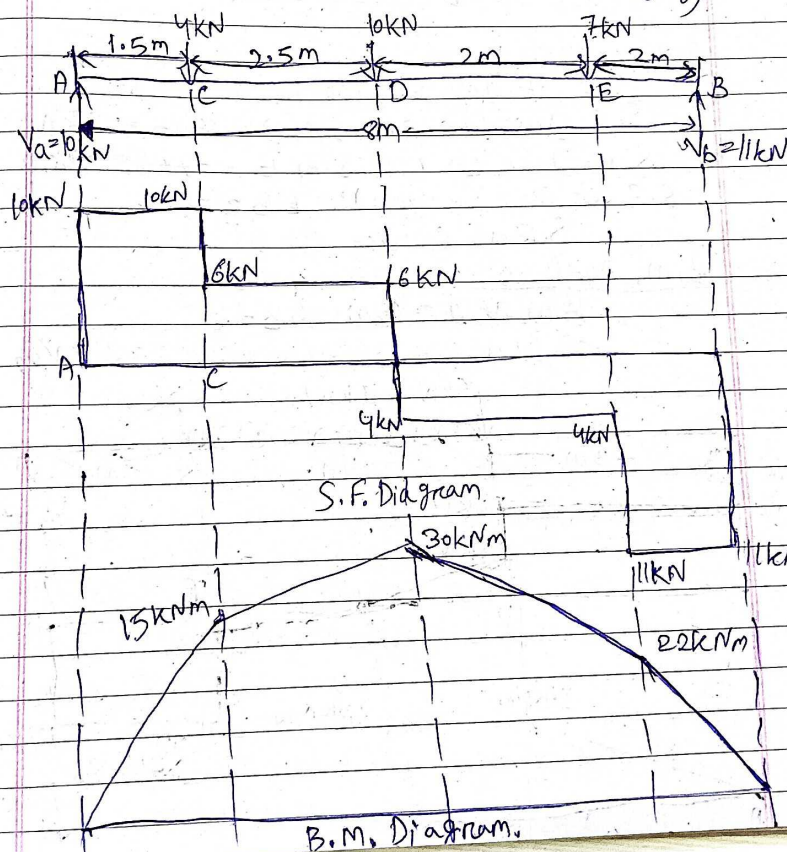
S.F. between E and B = $10 - 4 - 10 - 7 = -11$ kN
 $= -V_B = -11$ kN

B.M. at A = 0

B.M. at C = $10 \times 1.5 = 15$ kNm (Sagging)

B.M. at D = $10 \times 4 - 4 \times 2.5 = +30$ kNm (Sagging)

B.M. at E = $11 \times 2 = 22$ kNm (Sagging)



- (9) A beam AB 10 metres long has supports at its ends A and B. It carries a point load of 5 kN at 3 metres from A and a point load of 5 kN at 7 metres from A and a U.D.L of 1 kN per metre betⁿ the point loads. Draw S.F and B.M. diagrams for the beam.

Sol Reaction :-

Since the loading is symmetrical reaction at each support equals half the total load.

$$V_a = V_b = \frac{5 + 5 + 1 \times 4}{2} = 7 \text{ kN}$$

S.F. Analysis :-

S.F at any section in AD = +7 kN

S.F Just on R.H.S of D = 7 - 5 = 2 kN

S.F Just on L.H.S of E = -7 + 5 = -2 kN

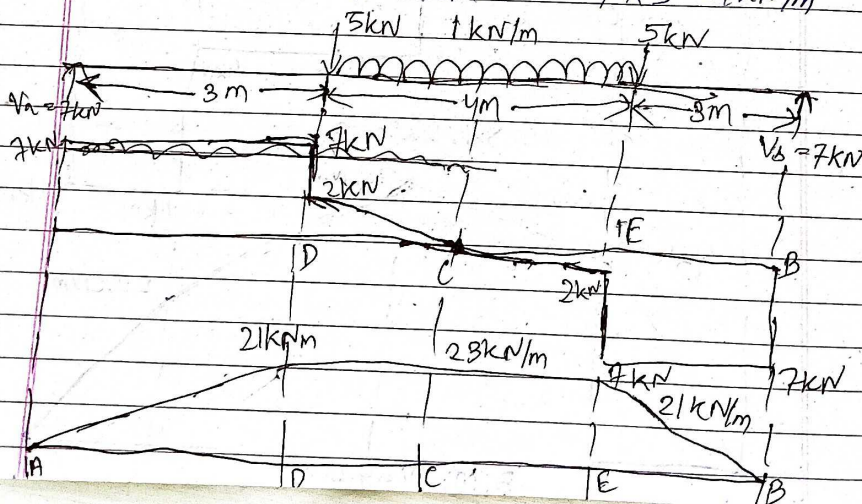
S.F at any section in EB = -7 kN

S.F at centre C = 0

B.M. Analysis

B.M. at A = B.M. at B = 0

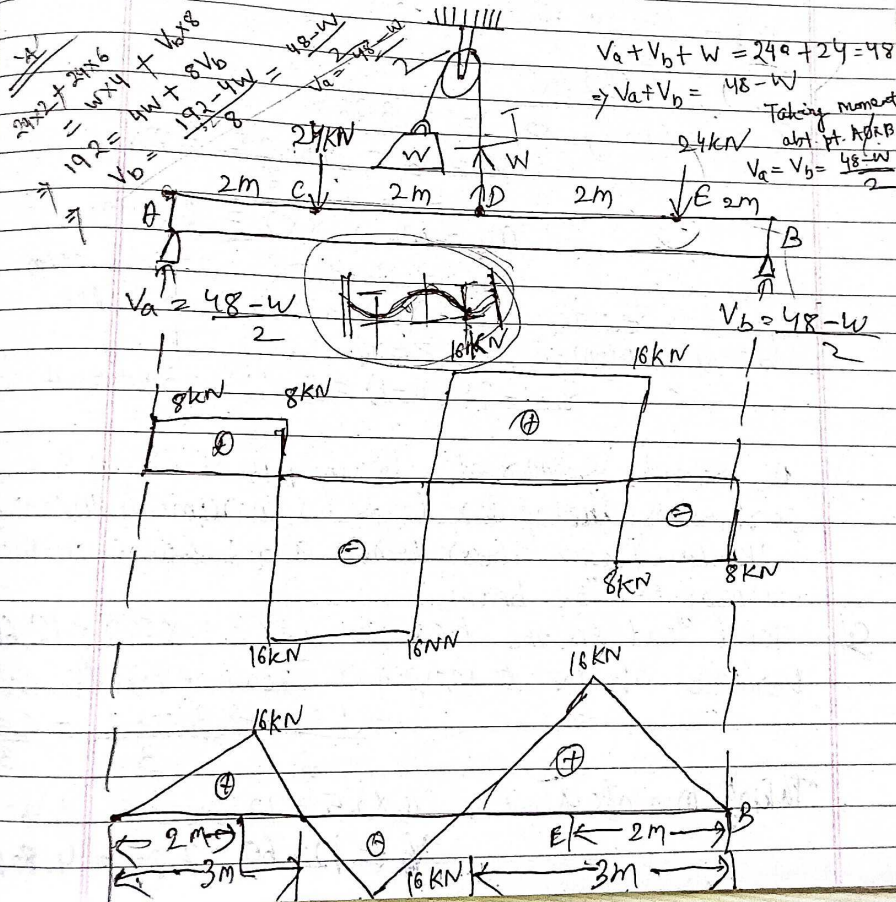
B.M. at D = B.M. at E = 7 × 3 = 21 kN/m



$$\text{B.M. at C} = 7 \times 5 - 5 \times 2 - 1 \times \frac{2^2}{2}$$

$$= 35 - 10 - 2 = 23 \text{ kNm}$$

(10) For the beam, determine the magnitude of the counter weight W for which the absolute maximum bending moment for the beam is as small as possible. Corresponding to this condⁿ draw B.M. and S.F. diagrams



Sol Reaction at each support $\Rightarrow V_a = V_b = \frac{48-W}{2}$ kN

$$\text{B.M at C} = \text{B.M at E} = M_c = M_e = \left(\frac{48-W}{2} \right) 2 \\ = (48-W) \text{ kNm}$$

$$\text{B.M at D} = m_d = \left(\frac{48-W}{2} \right) 4 - (24 \times 2) \\ = 96 - 2W - 48 = 48 - 2W = -(2W - 48) \text{ kNm}$$

Sagging moment of C or E and the hogging moment at D

$$48 - W = 2W - 48$$

$$\therefore 3W = 96$$

$$W = 32 \text{ kN}$$

Corresponding to $W = 32 \text{ kN}$

$$V_a = V_b = \frac{48-32}{2} = 8 \text{ kN}$$

$$M_c = M_e = 8 \times 2 = 16 \text{ kNm}$$

$$m_d = 8 \times 4 - 24 \times 2 = -16 \text{ kNm}$$

Let at a section in CD distant x from A, the bending moment be zero

$$8x - 24(x-2) = 0 \therefore x = 3 \text{ m from A}$$

(11) A simple supported beam AB of span 3.5m carries a triangular load on maximum intensity 7.2 kN/m. Draw shear force and bending moment diagram for the beam.

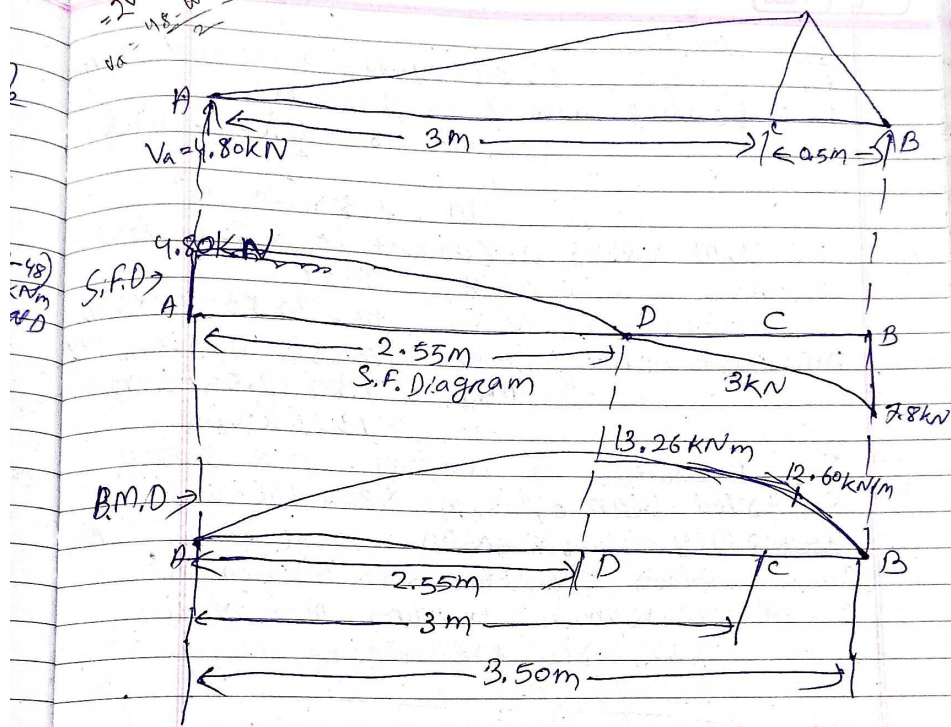
Sol Total load on the beam $= \frac{1}{2} \times 3.5 \times 7.2 = 12.6 \text{ kN}$

$$\text{Distance of the triangular load from the end A} \\ = \frac{1}{3} \times 3.5 = \frac{3.5}{3} = 1.167 \text{ m}$$

$$\text{Taking moments about A. } V_b \times 3.5 = 12.6 \times \frac{3.5}{3} \therefore V_b = 7.8 \text{ kN}$$

$$V_a = 12.60 - 7.80 = 4.80 \text{ kN}$$

$$\begin{aligned} & - (48 - 2W) \\ & = 2W - 48 = 1 \\ & = 48 - W \\ & \frac{110 - 32}{2} = \frac{16}{2} = 8 \\ & V_A = 4.80 \text{ kN} \end{aligned}$$



S.F. Analysis

S.F. at a section X distant x from A

$$S = 7.80 - \frac{1}{2} \times x \times (2.4x)$$

$$S = 7.80 - 1.2x^2$$

Shear force varies according to a parabolic law

At $x = 0$ i.e. at A, $S = 7.8 \text{ kN}$

At $x = 3\text{m}$ i.e. at B, $S = 7.80 - 1.2 \times 3^2 = -3.6 \text{ kN}$

Section of zero shear

$$7.80 - 1.2x^2 = 0$$

$$x^2 = \frac{7.8}{1.2}$$

$$\therefore x = 2.55 \text{ m from A}$$

B.M. Analysis

At any section IS AC distant x from the end A.
The bending moment is given by

$$m = 7.80 - \frac{1}{2}x \cdot (2.4x) \cdot \frac{x}{3}$$

$$m = 7.8x - 0.4x^3$$

The B.M. varies according to a cubic law

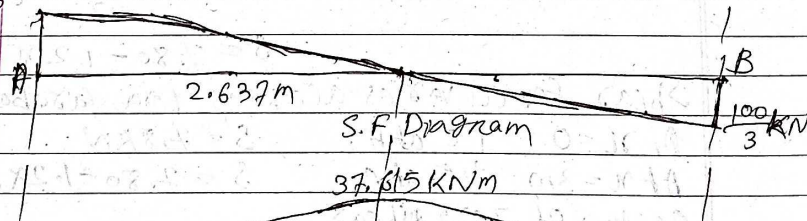
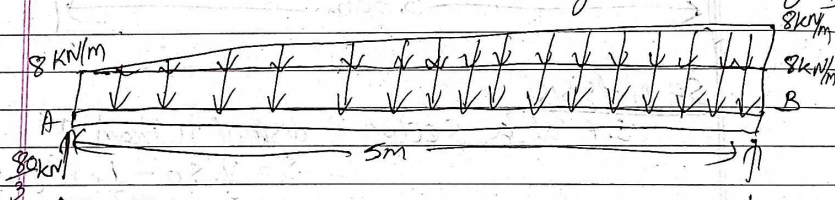
At $x=0$ i.e. at A, $M=0$

$x=3m$ i.e. at C, $m = 7.8 \times 3 - 0.4 \times 3^3 = 13.26 \text{ kNm}$

maximum bending moment occurs at $x=2.55m$

$$M_{\max} = 7.80 \times 2.55 - 0.4 \times 2.55^3 = 13.26 \text{ kNm}$$

- (12) The intensity of loading on a simply supported beam of 5m. span increases uniformly from 8 kN/m at one end to 16 kN/m at the other end. Find the position and magnitude of the maximum bending moment. Also draw shear force and bending moment diagrams.



Taking moment about the end A, we have

$$V_b \times 5 = [8 \times 5 \times \frac{5}{2}] + [\frac{1}{2} \times 5 \times 8 \times \frac{10}{3}]$$

$$V_b = \frac{100}{3} \text{ kN}$$

$$V_A = \text{total load} - V_b$$

$$= (8 \times 5 + \frac{1}{2} \times 8 \times 5) - \frac{100}{3} \text{ kN}$$

$$= 60 - \frac{100}{3} = \frac{80}{3} \text{ kN}$$

$$\text{Load intensity at the section } xx = 8 + \frac{x}{5} \times 8$$

$$= 8 + 1.6x$$

$$\text{S.F at the section } xx \quad S = \frac{80}{3} - 8x - \frac{1}{2} \times 1.6x$$

S.F. diagram follows a parabolic law

$$x = 0, \text{ i.e at A,}$$

$$S = \frac{80}{3} \text{ kN}$$

$$x = 5\text{m, i.e at B}$$

$$S = \frac{80}{3} - 8 \times 5 - 0.8 \times 5^2$$

$$= -\frac{100}{3} \text{ kN}$$

Section of zero shear

$$\frac{80}{3} - 8x - 0.8x^2 = 0$$

$$3x^2 + 30x - 100 = 0$$

$$\text{Solving, we get } x = 2.637 \text{ m}$$

B. M analysis

B. M at any section $xx = m = \frac{80}{3}x - \frac{8x^2}{2} - \frac{1}{2} \times 1.6x \times \frac{x}{3}$

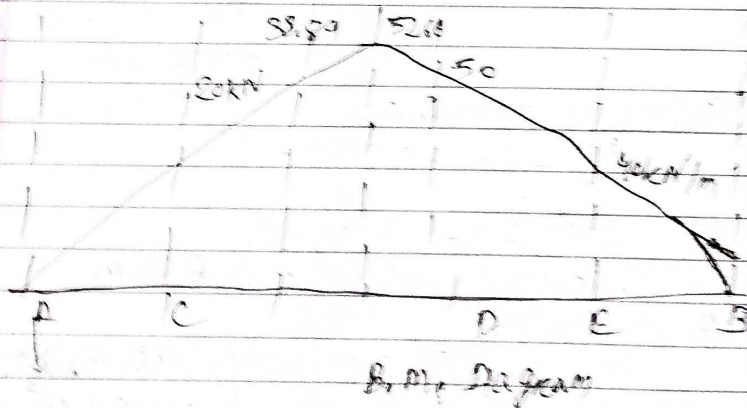
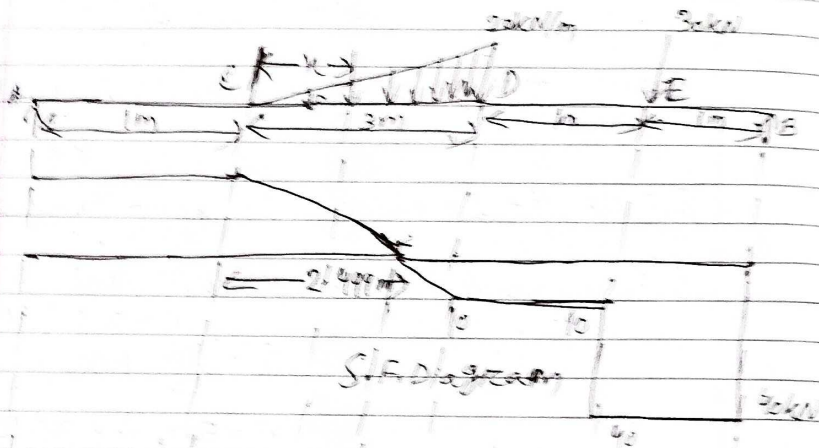
$$xx = m = \frac{80}{3}x - \frac{8x^2}{2} - \frac{1}{2} \times 1.6x \times \frac{x}{3}$$

$$M = \frac{50}{3}x - 4x^2 - \frac{0.8}{3}x^3$$

$$\therefore \text{maximum bending moment} = \frac{50}{3} \times 2.637 - 4(2.637)^2 - \frac{0.8}{3}(2.637)^3$$

$$= 37.615 \text{ kNm}$$

(3) Draw S.F and B.M diagram for the simply supported beam.



Taking moment about A.

$$V_b \times 6 = 30 \times 5 + \frac{1}{2} \times 3 \times 20 \times 3 \therefore V_b = 40 \text{ kN}$$

$$\text{Total load applied on the beam} = 30 + \frac{1}{2} \times 3 \times 20 = 60 \text{ kN}$$

$$V_a = 60 - 40 = 20 \text{ kN}$$

S.F. Analysis :- S.F. at any section in AC = +20 kN
consider any section X in CD distant x from C.
Load intensity at the section $x = \frac{20x}{3}$

$$\text{S.F. at the section } X = S_x = 20 - \frac{1}{2} x \cdot \frac{20x}{3}$$

$$S_x = 20 - \frac{10}{3} x^2$$

$$X = 0, S.F. = +20 \text{ kN}$$

$$X = 1 \text{ m}, S.F. = 20 - \frac{10}{3} (1)^2 = 16.67 \text{ kN}$$

$$X = 2 \text{ m}, S.F. = 20 - \frac{10}{3} (2)^2 = 6.67 \text{ kN}$$

$$X = 3 \text{ m}, S.F. = 20 - \frac{10}{3} (3)^2 = -10 \text{ kN}$$

Section of Zero

Equating the general expression for Shear force to zero.

$$20 - \frac{10}{3} x^2 = 0, x^2 = 6 \therefore x = \sqrt{6} = 2.449$$

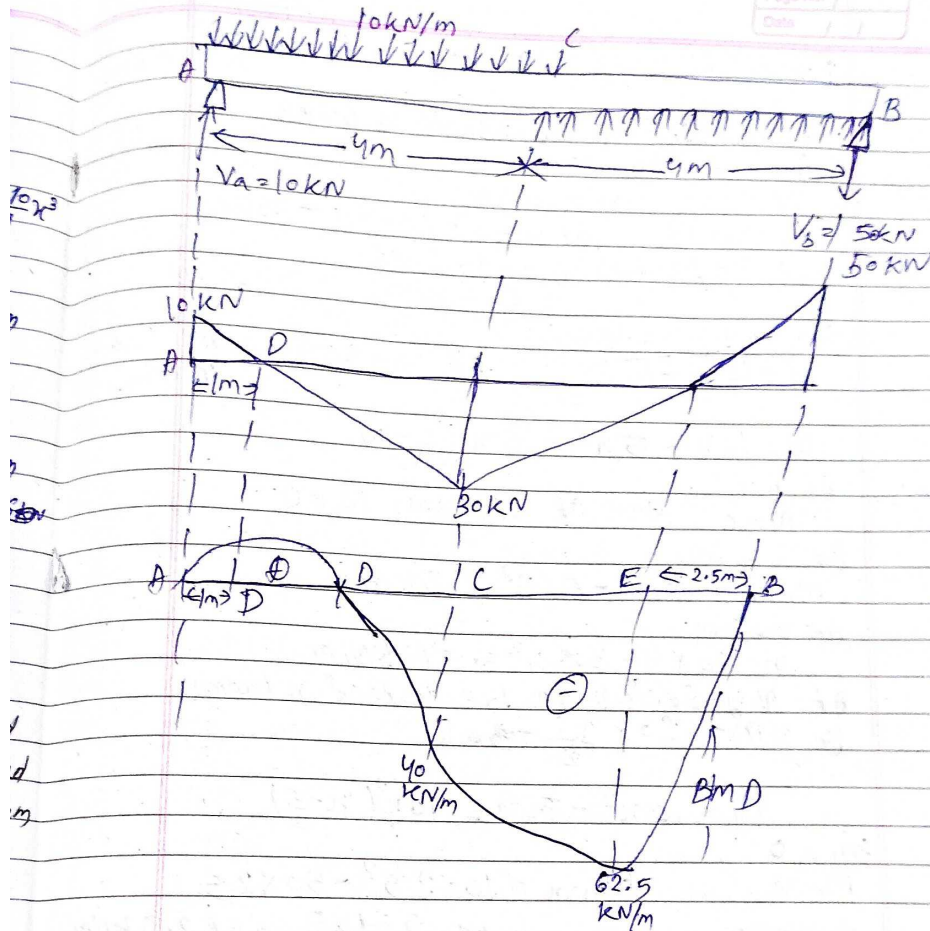
$$S.F. \text{ at any section in DE} = 30 - 40 = -10 \text{ kN}$$

$$S.F. \text{ at any section in EB} = -40 \text{ kN}$$

B.M. Analysis

$$\text{B.M. at A} = 0$$

$$\text{B.M. at C} = 20 \times 1 = 20 \text{ kNm}$$



S.F Analysis

At any section in AC distant x from A

$$\text{Shear Force} = S = 10 - 10x$$

$$\text{At } x = 0$$

$$\text{Shear Force} = 10 \text{ kN}$$

$$\text{At } x = 4 \text{ m}$$

$$S = 10 - 10 \times 4 = -30 \text{ kN}$$

For the section of zero shear $10 - 10x = 0$

$\therefore x = 1\text{m from A}$

At any section in BC distant x from B

$$S.F. = 50 - 20x$$

$$\text{At } x=0 \quad SF = 50\text{ kN}$$

$$\text{At } x=4\text{m} \quad SF = 50 - 20 \times 4 = -30\text{ kN}$$

For the section of zero shear

$$50 - 20x = 0 \therefore x = 2.5\text{m}$$

Bending moment:-

$$m = 10x - 10 \times \frac{x^2}{2}$$

At any section in AC distant x from A

$$m = 10x - 5x^2 = 5x(2-x)$$

$$\text{At } x=0$$

$$m=0 \quad \text{and at } x=2\text{m}, m=0$$

$$\text{At } x=1\text{m}$$

$$M_{\text{max}} = 10(1) - 5(1)^2 = 5\text{ kNm}$$

$$\text{At } x=4\text{m}$$

$$m = 10 \times 4 - 5 \times 4^2 = -40\text{ kNm}$$

At any section in BC distant x from B

$$m = 20 \times \frac{x^2}{2} - 50x$$

$$m = 10x^2 - 50x = 10x(x-5)$$

$$\text{At } x=0$$

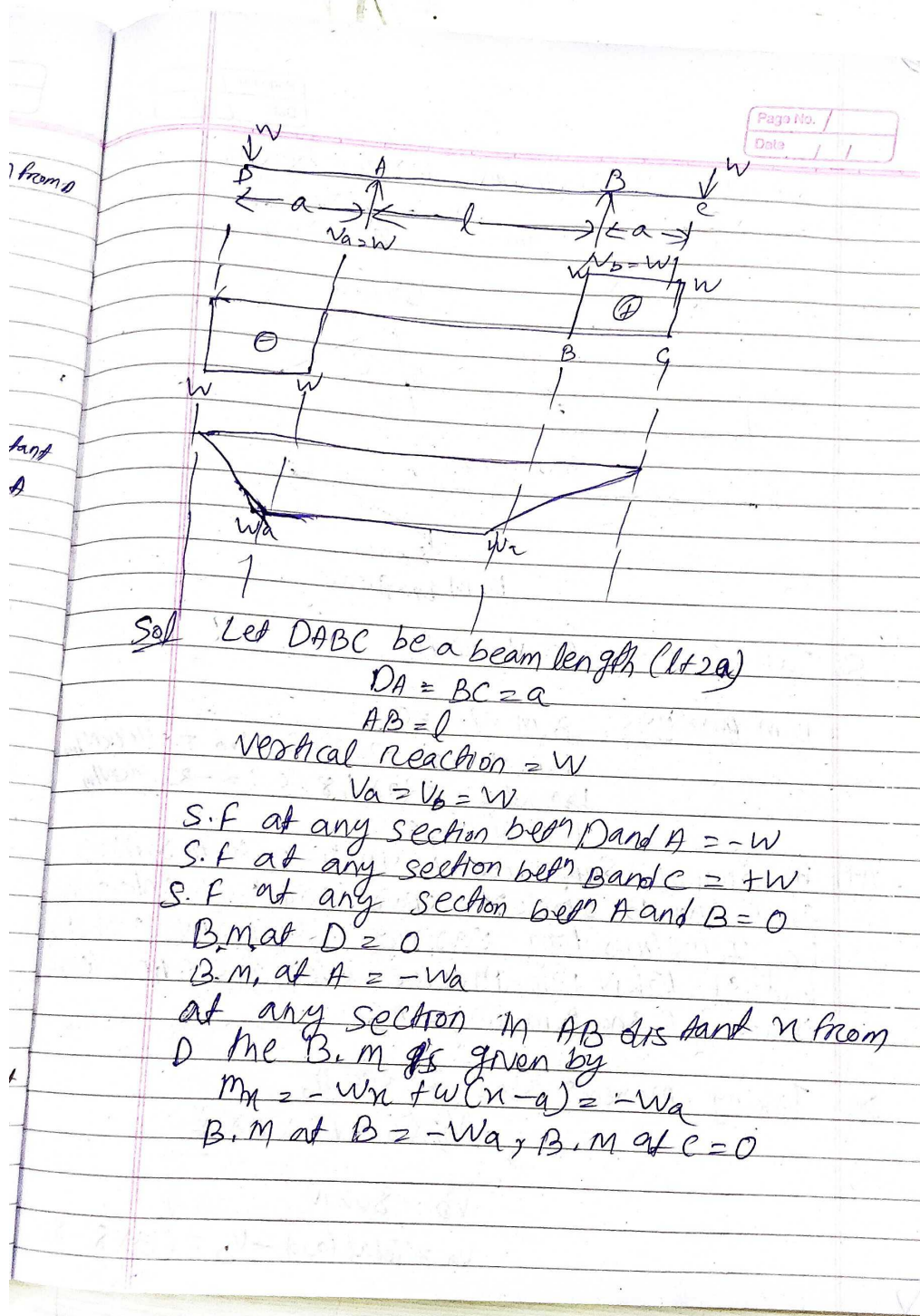
$$m=0$$

$$\text{At } x=4\text{m}$$

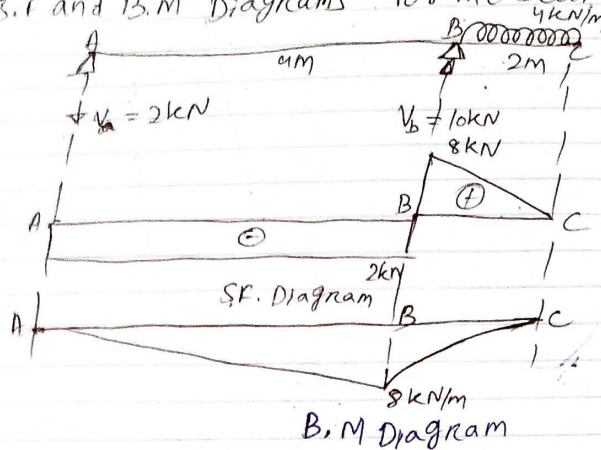
$$M_{\text{min}} = 10 \times 2.5^2 - 50 \times 2.5$$

$$= 62.5 - 125 = -62.5\text{ kNm}$$

(12) A beam of length $(l+2a)$ has supports l apart with an overhang a on each side. The beam carries a concentrated load w at each end. Construct shear force and B.M. diagrams.



(18) S.F and B.M Diagrams for the beam.



Sol Taking moments about A,

B.M Analysis B.M at C = 0

$$B.M \text{ at } A = -20 \times 1.2 \times 0.6 = -14.4 \text{ kNm}$$

$$B.M \text{ at } B = -20 \times 1.8 \times 0.9 = -32.4 \text{ kNm}$$

B.M at D = 0

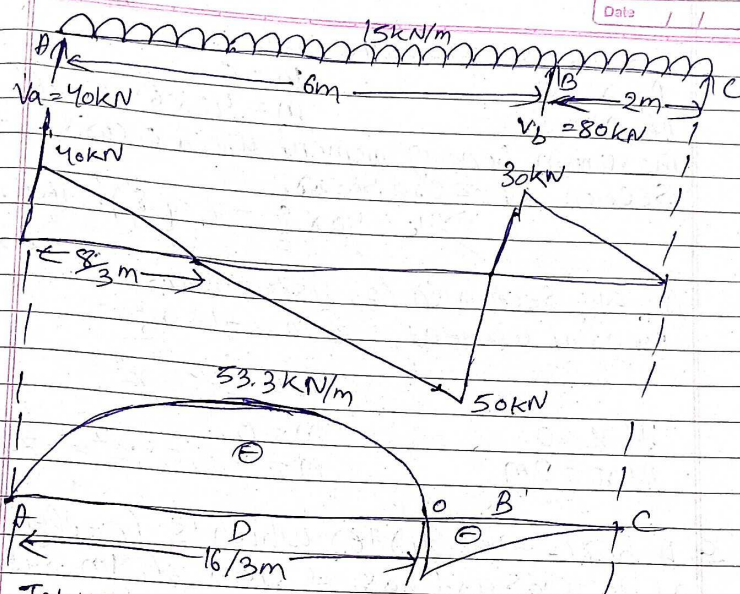
(19) A Simply Supported beam ABC with supports at A and B, 6 meters apart and with an overhang BC 2 metres long carries a uniformly distributed load of 15 kN per metre over the whole length. Draw S.F and B.M diagrams.

Sol Taking moments about the end A,

$$V_b \times 6 = 15 \times 8 \times \frac{8}{2}$$

$$V_b = 80 \text{ kN}$$

$$V_a = \text{Total load} - V_b = (15 \times 8) - 80 = 40 \text{ kN}$$



~~Taking moments about the end A,~~
S.F analysis, At any section AB distant x from A

$$\text{Shear force} = S = 40 - 15x$$

$$\text{At } x = 0$$

$$S = 40 \text{ kN}$$

$$\text{At } x = 6 \text{ m}$$

$$S = 40 - 15 \times 6 = -50 \text{ kN}$$

Section of zero shear:-

$$40 - 15x = 0$$

$$x = \frac{40}{15} = \frac{8}{3} \text{ m} = 2.67 \text{ m}$$

At any section in CB, distant x from C,

$$\text{At } x = 0$$

$$\text{Shear force} = S = 15x$$

$$\text{At } x = 2 \text{ m}$$

$$S = 0$$

$$S = 15 \times 2 = 30 \text{ kN}$$

B.M analysis At any section in AB, distant x from A

$$M = 40x - 15 \times \frac{x^2}{2} \quad M = 40x - 7.5x^2$$

This is a parabolic law.

At $x=0$

$$m=0$$

At $x=6m$

$$m = 40 \times 6 - 7.5 \times 6^2 = -30 \text{ kN/m}$$

maximum bending moment which occurs at the section of zero shear.

$$\Rightarrow M_c = 40 \times \frac{8}{3} - 7.5 \left(\frac{8}{3} \right)^2 = \frac{160}{3} \text{ kNm}$$

$$\approx 53.33 \text{ kNm}$$

At any section on CB, distant x and C
Bending moment $\Rightarrow M = -15 \frac{x^2}{2}$

$$M = -7.5 x^2$$

At $x=0$

$$m=0$$

At $x=2m$

$$m = -7.5 \times 2^2 = -30 \text{ kN/m}$$

(20) A horizontal girder which is freely supported at its ends and has a span of 9m supports a UDL of 20 kN/m run over the whole span and also two concentrated loads of 30 kN and 40 kN at points 6m and 7.5m respectively from the left support. Draw the bending moment and shearing force diagrams and state the values of the maximum bending moment and maximum shear force.

Ans Taking moments about B, we get

$$R_A \times 9 = 30 \times 3 + 40 \times 1.5 + 20 \times 9 \times 4.5$$

$$= 90 + 60 + 810 = 960$$

$$R_A = \frac{960}{9} = 106.67 \text{ kN}$$

$$R_B = 250 - 106.67 = 143.33 \text{ kN}$$

S.F diagram. Consider a section at a distance x from A. When x lies between A and C,

$$F = R_A - 20x = 106.67 - 20x$$

$$x=0, F = 106.67 \text{ kN}$$

$$x=6 \text{ m}$$

$$F = 106.67 - 120 = -13.33 \text{ kN}$$

$$F=0 \text{ for } x = 5\frac{1}{2} \text{ m}$$

for x to lie between C and D,

$$F = R_A - 20x - 30 = 106.67 - 20x - 30$$

$$x=6 \text{ m}$$

$$F = 106.67 - 120 - 30 = -43.33 \text{ kN}$$

$$x=7.5 \text{ m}$$

$$F = 106.67 - 150 - 30$$

$$= -73.33$$

for x to lie betⁿ D and B

$$F = R_A - 20x - 30 - 40$$

$$= 106.67 - 20x - 70$$

$$x=7.5 \text{ m}$$

$$F = 106.67 - 150 - 70 = -113.33 \text{ kN}$$

$$x=9 \text{ m}$$

$$F = 106.67 - 180 - 70 = -143.33 \text{ kN}$$

B.M Diagram betⁿ A and C.

$$M = R_A \times x - 20x \times \frac{x}{2} = 106.67x - 10x^2$$

$$x=0, M=0$$

$$x=6$$

$$M = 640 - 360 = 280 \text{ kN.m}$$

$$x = 5\frac{1}{2} \text{ m}$$

$$M = \frac{320}{3} \times \frac{16}{3} - \frac{2560}{9} = \frac{2560}{9} = 284.4 \text{ kN.m}$$

$$\text{Betⁿ C and D } M = R_A \times x - 10x^2 - 30(x-6) \\ = \frac{320}{3}x - 10x^2 - 30x + 180$$

$$= \frac{230}{3} x - 10x^2 + 180$$

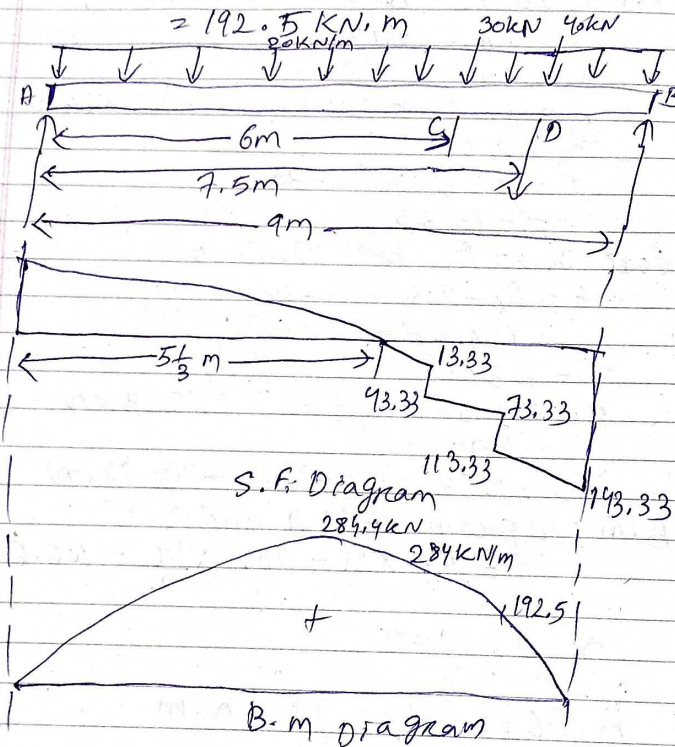
$$x = 6m$$

$$M = 460 - 360 + 180 = 280 \text{ kN.m}$$

$$x = 7.5m$$

$$\Rightarrow M = \frac{230}{3} \times 7.5 - 10 \times 56.25 + 180$$

$$= 192.5 \text{ kN.m}$$



(21) Draw the shear force and B.M.D of the beam
Find the position and magnitude of maximum bending moment.

Sol Taking moment about B, we get

$$R_A \times 5 = 4 \times 1.5 \times 2.75$$

$$R_A = 3.3 \text{ kN}$$

$$R_2 = 6 - 3.3 = 2.7 \text{ kN}$$

S.F Diagram :- Shear force betn A and C is equal to R_A . Consider a section at a distance x from A, for x to lie betn C and D

$$F = R_A - 4(x - 1.5) = 3.3 - 4x + 6$$

$$= 9.3 - 4x$$

$$x = 1.5 \text{ m}$$

$$F = 3.3 \text{ kN}$$

$$x = 3 \text{ m}$$

$$F = 9.3 - 12 = -2.7 \text{ kN}$$

$$F = 0$$

$$x = \frac{9.3}{4} = 2.325 \text{ m}$$

B.M Diagram, betn A and C

$$M = R_A \times x = 3.3x$$

$$\text{At } x = 0, M = 0$$

$$x = 1.5 \text{ m}$$

$$M = 4.95 \text{ kN.m}$$

for x to lie betn C and D,

$$M = R_A \times x - \frac{4(x - 1.5)^2}{2} = 3.3x - 2(x - 1.5)^2$$

$$x = 1.5 \text{ m}$$

$$M = 4.95 \text{ kN.m}$$

$$x = 3 \text{ m}$$

$$M = 3.3 \times 3 - 2(3 - 1.5)^2 = 9.9 - 2 \times 2.25$$

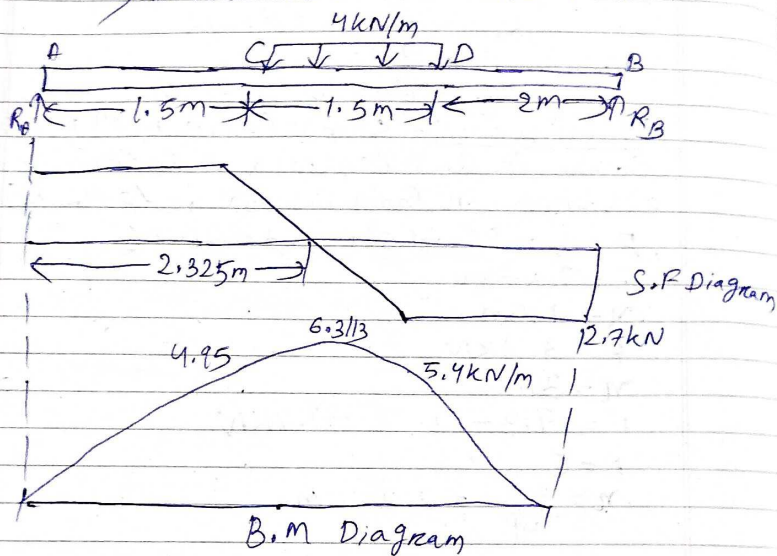
$$= 9.9 - 4.5 = 5.4 \text{ kN.m}$$

$$x = 2.325 \text{ m}$$

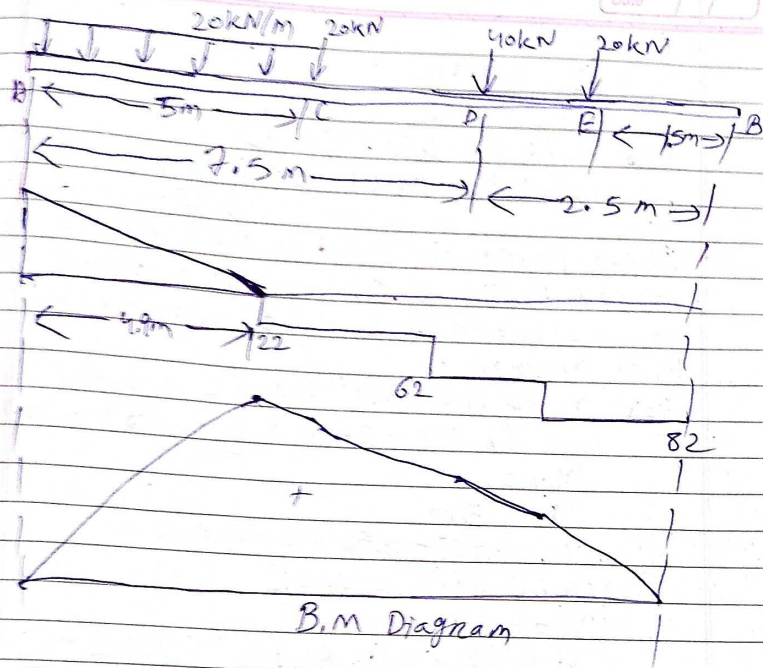
$$M = 3.3 \times 2.325 - 2(2.325 - 1.5)^2$$

$$= 7.6725 - 2 \times 0.68$$

$$\Rightarrow 7.6725 - 1.3612 = 6.3113 \text{ kN.m}$$



- (22) A beam of 10m length is simply supported at its ends. It carries a UDL of 20 kN/m run over the length of left half of its span, together with concentrated load of 20, 40 and 20 kN situated at 1.5, 2.5 and 5 m respectively from right hand support. Draw the B.M. and S.F. Diagrams for this beam and find out the magnitude and position of the maximum B.M. taking place in this beam.



Taking moments about B

$$R_A \times 7.5 = 20 \times 1.5 + 40 \times 2.5 + 20 \times 5 + 20 \times 5 \times 2.5$$

$$= 30 + 100 + 100 + 250 = 480$$

$$R_A = 98 \text{ kN}$$

$$R_B = (20 + 40 + 20 + 100) - 98$$

$$= 180 - 98 = 82 \text{ kN}$$

S.F Diagram

in the beam A and C,

$$F = R_A - 20x = 98 - 20x$$

$$x = 0$$

$$F = 98 \text{ kN}$$

$$x = 5$$

$$F = 98 - 20 \times 5 = 98 - 100 = -2 \text{ kN}$$

$$F = 0$$

$$x = \frac{98}{20} = 4.9 \text{ m}$$

for x to lie betⁿ C and D

$$F = R_A - 20 \times 5 - 20 = 98 - 100 - 20$$

$$\Rightarrow 98 - 120 = -22 \text{ kN}$$

Shear force at Just to the Right = -22 kN

Shear force at D = -22 - 40 = -62 kN

Shear force at E = -62 - 20 = -82 kN

B. M. Diagram

for x to lie betⁿ A and C, the b. m. is

$$M = R_A \times x - 20x \times \frac{x}{2} = 98x - 10x^2$$

$$x = 0, M = 0$$

$$x = 5 \text{ m}$$

$$M = 490 - 250 = 240 \text{ kN.m}$$

Parabolic betⁿ A and C

$$x = 4.9 \text{ m}$$

$$M = 98 \times 4.9 - 10(4.9)^2 = 480.2 - 240.1$$

$$= 240.1 \text{ kN.m}$$

for x to lie betⁿ C and D

$$M = R_A \times x - 20 \times 5(x - 2.5) - 20(x - 5)$$

$$= 98x - 100x + 250 - 20x + 100$$

$$\Rightarrow 350 - 22x$$

$$x = 5 \text{ m}$$

$$M = 350 - 110 = 240 \text{ kN.m}$$

$$x = 7.5 \text{ m}$$

$$M = 350 - 165 = 185 \text{ kN.m}$$

x lie betⁿ D and E

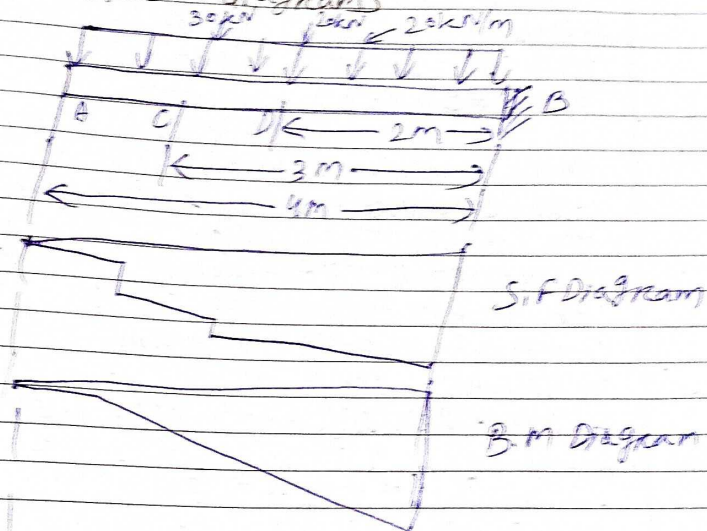
$$M = R_A \times x - 20 \times 5(x - 2.5) - 20(x - 5) - 40(x - 7.5)$$

$$= 98x - 100x + 250 - 20x + 100 - 40x + 300$$

$$= 650 - 62x$$

$$x = 8.5 \text{ m} \quad M = 650 - 527 = 123 \text{ kN.m}$$

(23) A cantilever beam is loaded as shown. Draw the B.M. and S.F. diagrams.



$$F = 20 \text{ kN}$$

$$x = 0$$

$$F = 0$$

$$x = 1 \text{ m}$$

$$F = 20 \text{ kN}$$

x to lie betⁿ C and D

$$F = 20x + 30$$

$$x = 1 \text{ m}$$

$$F = 20 + 30 = 50 \text{ kN}$$

$$x = 2 \text{ m}$$

$$F = 40 + 30 = 70 \text{ kN}$$

x to lie betⁿ D and B

$$F = 20x + 30 + 20 = 20x + 50$$

$$x = 2 \text{ m}$$

$$F = 40 + 50 = 90 \text{ kN}$$

$$x = 4m$$

$$F = 80 + 50 = 130kN$$

B.M Diagram, betⁿ A and C

$$M = 20x \times \frac{x}{2} = 10x^2$$

$$\text{At } x=0, M=0$$

$$x=1m, M = 10kN.m$$

x lie betⁿ C and D

$$M = 20x \times \frac{x}{2} + 30(x-1) = 10x^2 + 30x - 30$$

$$\text{At } x=1m$$

$$M = 10 + 30 - 30 = 10kN.m$$

$$\text{At } x=2m$$

$$M = 40 + 60 - 30 = 70kN.m$$

x lie betⁿ D and B

$$M = 20x \times \frac{x}{2} + 30(x-1) + 20(x-2)$$

$$= 10x^2 + 50x - 70$$

$$x = 2m$$

$$M = 40 + 100 - 70 = 70kN.m$$

$$x = 4m$$

$$M = 160 + 200 - 70 = 290kN.m$$

(24) Draw the B.M and S.F diagrams for the overhanging beam carrying loads the values of the principal ordinates and locate the point of contraflexure, if any

Sol Taking moment about B, we get

$$R_A \times 6 + 50 \times 3 + 20 \times 3 \times 1.5 = 20 \times 6 \times 3$$

$$R_A \times 6 + 150 + 90 = 360$$

$$R_A = 20kN$$

$$R_B = (20 \times 9 + 50) - 20$$

$$= 230 - 20 = 210kN$$

S.F. Diagram

in the beam A and B $F = R_A - 20x$
 $= 20 - 20x$

$x = 0$ $x = 6m$
 $F = 20$ $F = -10 kN$
 $F = 0$ for $x = 1m$

for x to lie bet B and C

$F = R_A - 20x + R_B$
 $= 230 - 20x$

$x = 6m$

$F = 230 - 120 = 110 kN$

$x = 9m$

$F = 230 - 180 = 50 kN$

B.M Diagram bet A and B

$m = R_A \times x - 20x \times \frac{x}{2}$
 $= 20x - 10x^2$

$x = 0, m = 0$

$x = 6m$ $m = 120 - 360 = -240 kNm$

$m = 0$ at $x = 2m$

$x = 1m$; $m = 20 - 10 = 10 kNm$

x to lie bet B and C

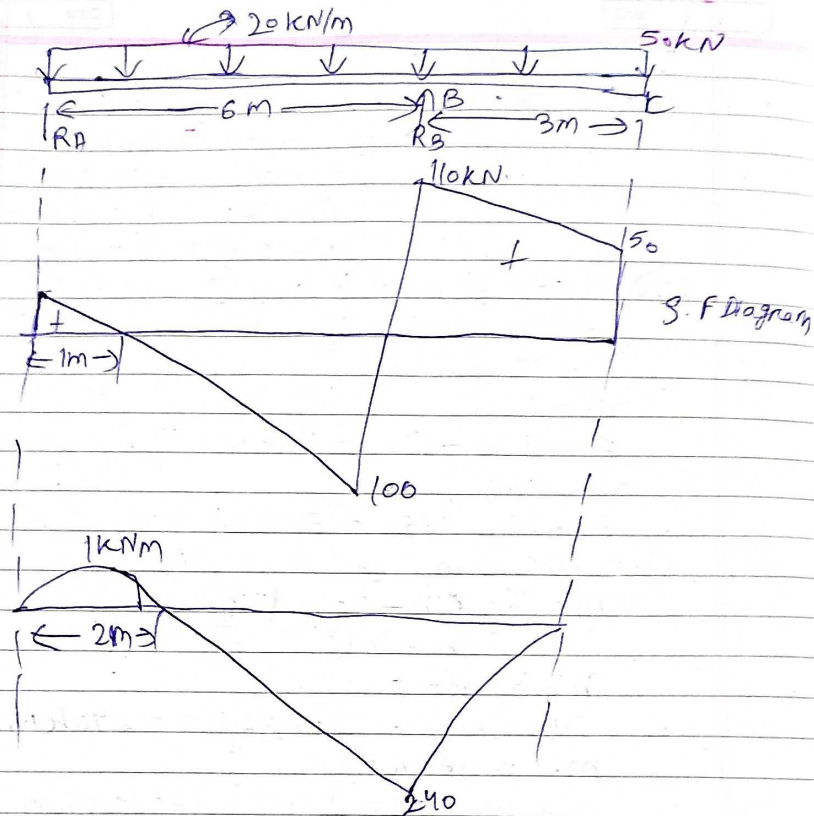
$M = R_A \times x - 20x \times \frac{x}{2} + R_B(x-6)$
 $= 20x - 10x^2 + 210x - 1260$
 $= 230x - 10x^2 - 1260$

$x = 6m$

$m = 1380 - 360 - 1260 = -240 kNm$

$x = 9m, m = 2070 - 810 - 1260 = 0$





(23) Calculate the reactions in the case of the beam. Construct the bending moment and Shearing force diagrams. Determine the location of the max^m bending moment and mark it on each of the diagrams.

Sol Taking moments about B, we get.

$$R_A \times 6 + 15 \times 2 = 80 \times 4.5$$

$$R_A \times 6 + 30 = 360$$

$$R_A = \frac{330}{6} = 55 \text{ kN}$$

$$R_B = (80 + 15) - 55 = 40 \text{ kN}$$

S.F Diagram :- at a distance x from A
for x to lie betⁿ A and C

$$F = R_A - \frac{80}{3}x = 55 - \frac{80}{3}x$$

$$x=0, F=55\text{ kN}$$

$$x=3\text{ m}, F=-25\text{ kN}$$

$$F=0$$

$$\text{when } x = \frac{165}{80} = 2.06\text{ m}$$

S: F betⁿ C and B is constant

for x to lie betⁿ B and D

$$F = R_A + R_B - 80 = 95 - 80 = 15\text{ kN}$$

B.M Diagram

$$\text{betⁿ A and C } M = R_A x - \frac{80x \times x}{2}$$

$$= 55x - \frac{40}{3}x^2$$

$$\text{At } x=0, m=0$$

$$x=3\text{ m}$$

$$m = 165 - 120 = 45\text{ kN.m}$$

$$x = 2.06\text{ m}$$

$$m = 113.3 - 56.6 = 56.7\text{ kN.m}$$

② x to lie betⁿ C and B

$$m = R_A x - 80(x - 1.5)$$

$$= 55x - 80x + 120$$

$$= 120 - 25x$$

$$x = 3\text{ m}$$

$$m = 120 - 75 = 45\text{ kN.m}$$

$$x = 6\text{ m}$$

$$m = 120 - 150 = -30\text{ kN.m}$$

$$m=0 \text{ at } x = \frac{120}{25} = 4.8\text{ m}$$

Let x lie between B and D

$$M = R_A x - 80(x - 1.5) + R_B(x - 6)$$

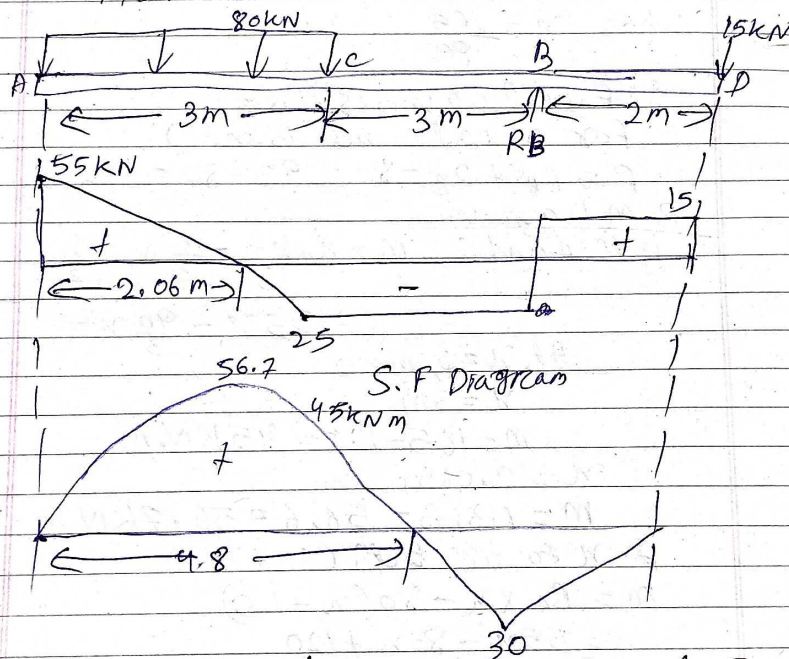
$$= 55x - 80x + 120 + 40x - 240 = 15x - 120$$

$$x = 6m$$

$$M = 90 - 120 = -30 \text{ kN.m}$$

$$x = 8m$$

$$M = 120 - 120 = 0$$



- (20) A beam 8.5m long rests on supports 5m apart. The right hand end overhangs its support by 2m and left hand end by 1.5m. The beam carries a uniformly distributed load 50kN per metre run between the supports only. The beam also carries a point load of 60kN at the extreme right hand end, and a point load of 40kN.

at the left hand end. Construct the shear force and B.M.D. Stating there on all the important values of shear force and bending moment. State the position of the points of inflexion on the beam.

2-120

kN

Sol Taking moments about B we get

$$R_B \times 5 + 60 \times 2 = 40 \times 6.5 + 250 \times 2.5$$

$$R_B \times 5 + 120 = 260 + 625$$

$$R_B \times 5 = 765$$

$$R_B = 153 \text{ kN}$$

$$R_C = (40 + 250 + 60) - 153$$

$$= 350 - 153 = 197 \text{ kN}$$

S.F Diagram

into lie betⁿ B and C

$$F = R_B - 50x - 40$$

$$= 153 - 50x - 40 = 113 - 50x$$

$$\text{At } x=0, F = 113 \text{ kN and at } x=5\text{m}$$

$$F = -137 \text{ kN. } F=0 \text{ at } x=2.26\text{m}$$

x lie betⁿ C and D

$$R = R_A + R_B - 250 - 40$$

$$= 350 - 290 = 60 \text{ kN}$$

B.M Diagram

betⁿ A and B,

$$M = -40(1.5 - x) = 90x - 60$$

$$x=1.5\text{m, } M=0 \text{ for } x=0$$

$$M = -60 \text{ kN.m}$$

x lie betⁿ B and C

$$M = R_B \times x - 50x \times \frac{x}{2} - 40(x - 1.5)$$

$$= 153x - 25x^2 - 40x + 60$$

$$= 113x - 25x^2 - 60$$

$$\text{At } x=0, M = -60 \text{ kN.m}$$

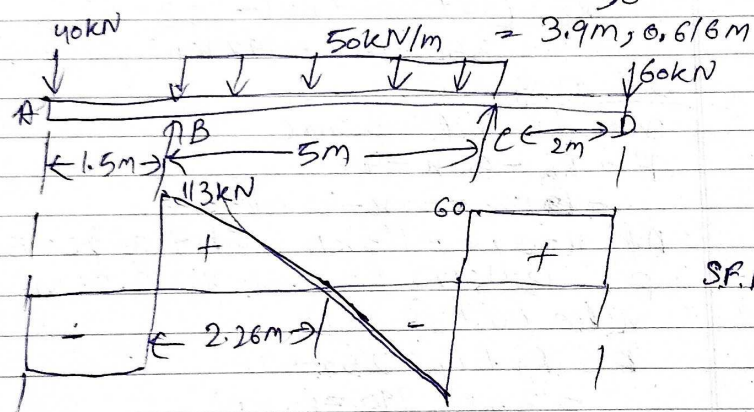
$$\text{At } x=5\text{m}, M = 565 - 625 - 60 \\ = -120 \text{ kN.m}$$

$$\text{At } x = 2.26 \text{ m}$$

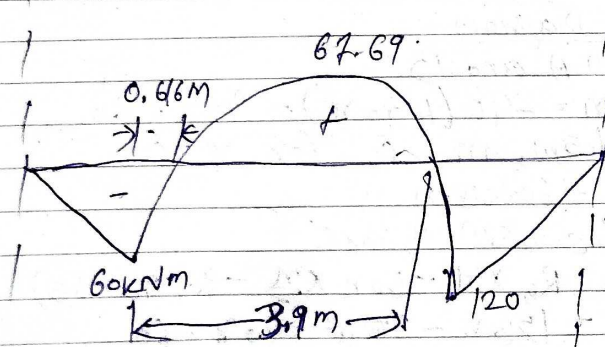
$$M = 113 \times 2.26 - 25 \times (2.26)^2 - 60 \\ = 225.38 - 127.69 - 60 \\ = 67.69 \text{ kN.m}$$

$$M=0, 25x^2 - 113x + 60 = 0$$

$$x = \frac{113 \pm \sqrt{12769 - 600}}{50} = \frac{113 \pm \sqrt{6769}}{50} \\ = \frac{113 \pm 82.2}{50}$$



SF Diagram



B.M Diagram

(27) Compute the values of max^m and min^m B.M and S.F for the simple supported beam loaded. ~~as~~ Draw the B.M and S.F diagrams to ~~the~~ Scale indicating the significant values along the beam.

Sol- Taking moments about B,

$$R_D \times 6 = 1.5 \times 1 + 3 \times 3 = 10.5$$

$$R_D = 1.75 \text{ kN}$$

$$R_B = (1.5 + 3) - 1.75 = 2.75 \text{ kN}$$

S.F Diagram

S.F betⁿ A and C = 1.75 kN

for x to lie betⁿ C and D

$$3 = \frac{W \times x^2}{2}$$

$$W = 2 \text{ kN}$$

$$\therefore \frac{n-1}{3} = \frac{Wn}{W}$$

$$\Rightarrow \frac{n-1}{3} = \frac{Wn}{2}$$

$$\Rightarrow Wn = \frac{2}{3}(n-1)$$

$$F = R_D - \frac{Wn(n-1)}{2}$$

$$= 1.75 - \frac{2}{6}(n-1)^2$$

$$n = 1 \text{ m}, F = 1.75 \text{ kN}$$

$$n = 2 \text{ m},$$

$$F = 1.75 - \frac{2}{6} = 1.75 - 0.333 = 1.417 \text{ kN}$$

$$n = 3 \text{ m}$$

$$F = 1.75 - \frac{4}{3} = 1.75 - 1.333 = 0.417 \text{ kN}$$

$$x = 4m, F = 1.75 - 3 = -1.25kN$$

$$S.F. \text{ between D and E} = -1.25kN$$

$$S.F. \text{ at E} = -1.25 - 1.50 = -2.75kN$$

$$S.F. \text{ at B} = -2.75kN$$

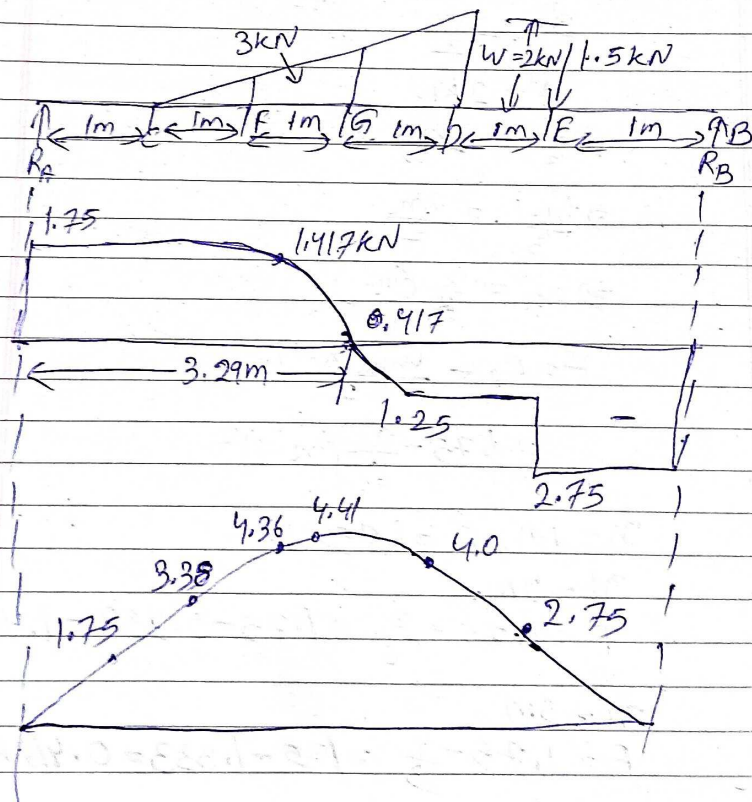
$$S.F. \text{ between C and D is zero}$$

$$1.75 - \frac{2}{6}(x-1)^2 = 0$$

$$(x-1)^2 = 1.75 \times \frac{6}{2} = 5.25$$

$$x-1 = \sqrt{5.25} = 2.29m$$

$$x = 3.29m$$



B M Diagrams

$$M = R_A \times x = 1.75x$$

$x=0, M=0$ and at $x=1m$

$$M = 1.75 \text{ kNm}$$

for x lie betⁿ C and D

$$M = R_A \times x - \frac{wx(x-1)}{2} \times \frac{(x-1)}{3}$$

$$= 1.75x - \frac{2}{3} \times \frac{1}{6} (x-1)^3$$

$$= 1.75x - \frac{1}{9} (x-1)^3$$

$$x = 1m$$

$$M = 1.75 \text{ kNm}$$

$$x = 2m$$

$$M = 3.50 - \frac{1}{9}$$

$$= 3.50 - 0.111 = 3.389 \text{ kNm}$$

$$x = 3m$$

$$M = 5.25 - \frac{1}{9} \times 8 = 5.25 - 0.889$$

$$= 4.361 \text{ kNm}$$

$$x = 3.29 m$$

$$M = 1.75 \times 3.29 - \frac{1}{9} (3.29-1)^3$$

$$= 4.423 \text{ kNm}$$

$$x = 4m$$

$$M = 7 - \frac{1}{9} \times 27 = 7 - 3 = 4 \text{ kNm}$$

x to lie betⁿ D and E

$$M = R_A \times x - 3(x-3)$$

$$= 1.75x - 3x + 9 = 9 - 1.25x$$

$$x = 5m$$

$$M = 9 - 6.25 = 2.75 \text{ kNm}$$

(28) Draw the Shear force and B.M.D for the beam. Indicate the numerical values at all important sections.

Sol Taking moments about D

$$R_A \times 12.5 + 16 \times 2.5 \times 12.5 = 10 \times 5 \times 10 + 80 \times 7.5 + 80 \times 5$$

$$R_A \times 12.5 + 50 = 500 + 600 + 400$$

$$R_A = \frac{1450}{12.5} = 116 \text{ kN}$$

$$R_D = 250 - 116 = 134 \text{ kN}$$

S.F diagram, Consider a section at a distance x from A. betⁿ A and B

$$F = R_A - 10x = 116 - 10x$$

$$x = 0, F = 116 \text{ kN}$$

$$x = 5 \text{ m}, F = 116 - 50 = 66 \text{ kN}$$

betⁿ B and C

$$F = R_A - 10 \times 5 - 80$$

$$= 116 - 50 - 80 = -14 \text{ kN}$$

betⁿ C and D

$$F = R_A - 10 \times 5 - 80 - 80$$

$$= 116 - 50 - 160 = -94 \text{ kN}$$

betⁿ D and E

$$F = R_A - 10 \times 5 - 80 - 80 + R_D - 16(x - 12.5)$$

$$= 116 - 50 - 160 + 134 - 16x + 200$$

$$= 240 - 16x$$

At $x = 12.5 \text{ m}$

$$F = 240 - 200 = 40 \text{ kN}$$

B.M diagram betⁿ A and B.

$$M = R_A \times x - 10 \times \frac{x^2}{2} = 1160 - 5x^2$$

$$\text{At } x = 0, M = 0$$

$$\text{At } x = 5 \text{ m}$$

$$M = 580 - 125 = 455 \text{ kN-m}$$

Parabolic betⁿ Band C

$$M = R_A \times x - 10 \times 5(x-2.5) - 80(x-5)$$

$$= 116x - 50x + 125x - 80x + 400$$

$$= -14x + 525$$

$$x = 7.5m$$

$$M = -105 + 525 = 420kN-m$$

B.M linear betⁿ Band C.

betⁿ Cand D $M = R_A \times x - 10 \times 5(x-2.5) - 80(x-5)$

$$= 116x - 50x + 125x - 80x + 400 - 80(x-7.5)$$

$$= 116x - 50x + 125x - 80x + 400 - 80x + 600$$

$$= -94x$$

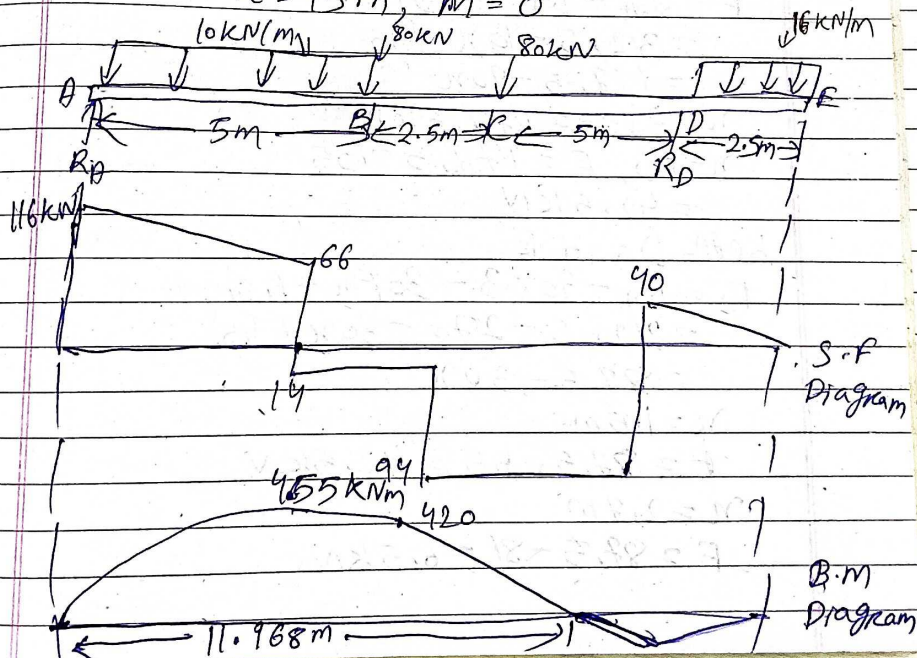
$$x = 12.5m$$

$$M = -50kN-m$$

B.M linear betⁿ Cand D $M = 0, x = 11.968m$

Betⁿ DE $M = \frac{-16(15-x)^2}{2} = -8(15-x)^2$

$$x = 15m, M = 0$$



(29) Draw the complete shear force diagram for the overhanging beam. Hence determine the position in the central bay at which maximum positive bending moment occurs. Find also magnitude of the maximum positive and negative moments.

Sol Taking moments about B

$$R_A \times 4.5 = 90 \times 3 \times 4.5 + 30 \times 3 \times 1.5 - 30 \times 1.5 \times 0.75 + 50 \times 1.5$$

$$R_A \times 4.5 = 1215 + 135 - 33.75 + 90 = 1406.25$$

$$R_A = 312.5 \text{ kN}$$

$$R_B = (90 \times 3 + 50) - 30 \times 4.5 - 312.5$$

$$= 455 - 312.5 = 142.5 \text{ kN}$$

$$F = -90(-x + 1.5) = 90x - 135$$

$$x = 1.5, F = 0$$

$$x = 0, F = -135 \text{ kN}$$

bet A and D

$$F = R_A - 90(x + 1.5)$$

$$= 312.5 - 90x - 135$$

$$= 177.5 - 90x$$

$$x = 0, F = 177.5 \text{ kN}$$

$$x = 1.5, F = 177.5 - 135$$

$$= 42.5 \text{ kN}$$

bet D and E

$$F = R_A - 90 \times 3 - 30(x - 1.5)$$

$$= 312.5 - 270 - 30x + 45$$

$$= 87.5 - 30x$$

$$x = 1.5 \text{ m}$$

$$F = 87.5 - 45 = 42.5 \text{ kN}$$

$$x = 2.7 \text{ m}$$

$$F = 87.5 - 81 = 6.5 \text{ kN}$$

betⁿ E and B: $F = R_A - 90 \times 3 - 50 - 30(x - 1.5)$
 $= 312.5 - 270 - 50 - 30x + 45$
 $= 37.5 - 30x$

$x = 2.7 \text{ m}$

$F = 37.5 - 81 = -43.5 \text{ kN}$

$x = 4.5 \text{ m}$

$F = 37.5 - 135 = -97.5 \text{ kN}$

bet Band F: $F = R_A - 90 \times 3 - 50 + R_B - 30(x - 1.5)$
 $= 312.5 - 270 - 50 + 142.5 - 30x + 45$
 $= 180 - 30x$

$x = 4.5 \text{ m}$

$F = 180 - 135 = 45 \text{ kN}$

$x = 6 \text{ m}, F = 0$

B.M. diagram

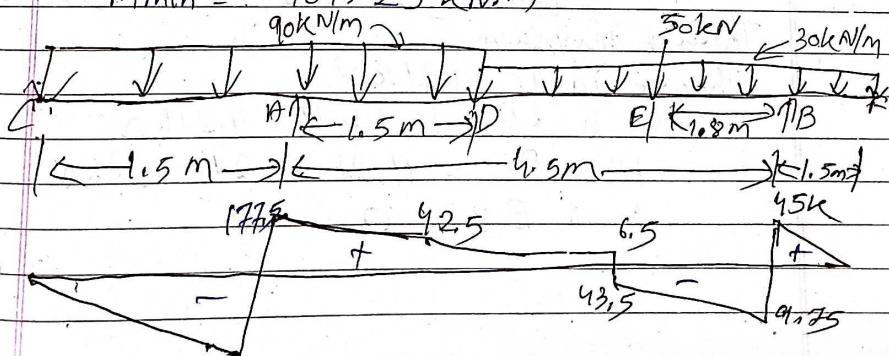
$M_A = -90 \times 1.5 \times 0.75 = -101.25 \text{ kN.m}$

$M_E = R_A \times 2.7 - 90 \times 3 \times 2.7 - 30 \times 1.2 \times 0.6$
 $= 312.5 \times 2.7 - 270 \times 2.7 - 30 \times 0.72$
 $= 843.75 - 72.9 - 21.6 = 93.15 \text{ kN.m}$

$M_B = R_A \times 4.5 - 90 \times 3 \times 4.5 - 90 - 135$
 $= 1406.25 - 1215 - 90 - 135 = +33.75 \text{ kN}$

$M_{\max} = 93.15 \text{ kN.m}$

$M_{\min} = -101.25 \text{ kN.m}$



30. A beam AB 10m long is loaded. Draw the B.M. S.F. and axial thrust diagrams

Sol. The total horizontal force on the beam is
 $= -10 + 21.21 - 10$
 $= 1.21 \text{ kN to the right}$

Taking moment about B, we get

$$V_A \times 10 = 17.32 \times 7 + 21.21 \times 5 + 17.32 \times 3$$

$$= 279.25$$

$$V_A = 27.925 \text{ kN}$$

$$V_B = (17.32 + 21.21 + 17.32) - 27.925$$

$$= 55.85 - 27.925$$

$$= 27.925 \text{ kN}$$

S.F. Diagram

$$F_A = 27.925 \text{ kN}$$

$$F_C = 27.925 - 17.32 = 10.605 \text{ kN}$$

$$F_D = 10.605 - 21.21$$

$$= -10.605 \text{ kN}$$

$$F_E = -10.605 - 17.32$$

$$= -27.925 \text{ kN}$$

The S.F. B.M. diagram

$$M_C = M_A = 27.925 \times 3 = 83.775 \text{ kNm}$$

$$M_D = 27.925 \times 5 - 17.32 \times 2$$

$$= 139.625 - 34.64 = 104.985 \text{ kNm}$$

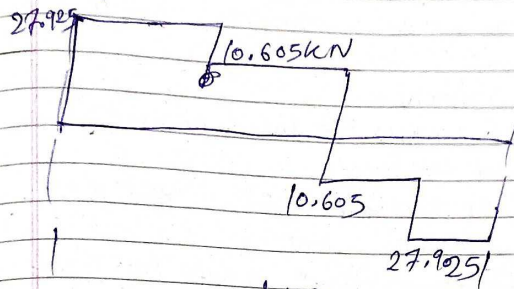
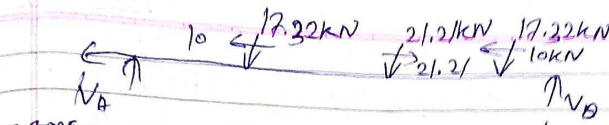
Thrust diagram

$$\text{Thrust in AC} = 1.21 \text{ kN}$$

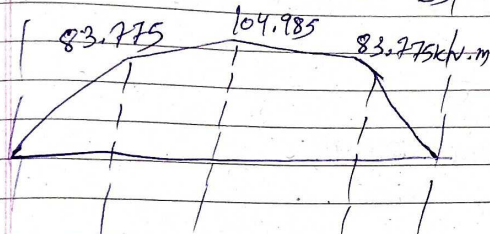
$$\text{" " CD} = 10 + 1.21 = 11.21 \text{ kN}$$

$$\text{" " DE} = 11.21 - 21.21 = -10 \text{ kN}$$

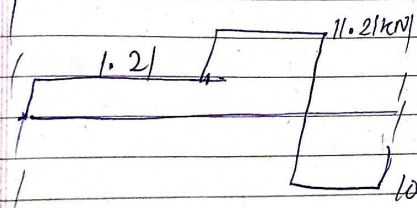
$$\text{" " EB} = 0$$



S.F. Diagram



B.M. Diagram



Thrust Diagram

(*) Thrust diagram (Note: not for MP)

Graphical representation of variation of axial load that acts on the beam,

where

M = resisting moment

I = moment of inertia of the section of (N.A.)

E = modulus of elasticity

R = Radius of curvature of Neutral axis

σ = Bending stress at a point

Y = Distance of a point from the Neutral axis

$$\frac{M}{I} = \frac{\sigma}{Y} = \frac{E}{R}$$

What is effective length

$$d = 33.85 \quad \approx \frac{\pi d^2}{\frac{64}{\pi d^2} \frac{R d^2}{4}}$$

length = 3m
dia = 5cm

Assignment = Example-1 (B.1)

3×10^3

$5 \text{ cm} \times 10^{-6} = 5 \times 10^{-5}$

(i) Fixed and free = effective length = $l_e = 2l = \frac{\pi^2 EI}{4l^2} \times 2 \times 10^5$

(ii) Both fixed = $l_e = \frac{l}{2} = \frac{\pi^2 EI}{4l^2} \times \frac{l}{2}$

(iii) Fixed and hinged = $l_e = \frac{l}{\sqrt{2}} = \frac{\pi^2 EI}{4l^2} \times \frac{l}{\sqrt{2}}$

(iv) Both end hinged = $l_e = l$

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Problem-1

* A column 1 metre long has an area of cross section of 900 mm^2 . Find the slenderness ratio if the section is (a) circular (b) square (c) hollow circular with inner radius half the outer radius (d) hollow circular with outer radius of 60 mm and (e) rectangular with depth twice the width which of the column is strongest?

Sol Circular section

$$A = \frac{\pi d^2}{4} = 900 \text{ mm}^2$$

$$d = 33.85 \text{ mm}$$

$$K = \sqrt{\frac{I}{A}} = \sqrt{\frac{\frac{\pi d^4}{64}}{\frac{\pi d^2}{4}}} = \sqrt{\frac{\pi d^4 \times 4}{64 \times \pi d^2}} = \sqrt{\frac{d^2}{16}}$$

$$K = \frac{d}{4} = \frac{33.85}{4}$$

$$K = 8.46 \text{ mm}$$

$$\text{Slenderness ratio} = \frac{L}{K} = \frac{1000}{8.46} = 118.2$$

Square section :-

$$A = d^2 = 900$$

$$d = \sqrt{900} = 30 \text{ mm}$$

$$K = \sqrt{\frac{I}{A}} = \sqrt{\frac{d^4}{12 \times d^2}} = \frac{d}{\sqrt{12}}$$

$$K = \frac{30}{\sqrt{12}} \text{ mm}$$

$$\text{Slenderness ratio} = \frac{L}{K} = \frac{1000 \times \sqrt{12}}{30} = 115.47$$

Hollow circular column with inner radius (r) equal to half of outer radius (R)

$$r = \frac{B}{2} \text{ and } d = \frac{D}{2} \text{ or } D = 2d$$

$$\text{Area} = \frac{\pi}{4} (D^2 - d^2) = 900 = \frac{\pi}{4} (4d^2 - d^2)$$

$$3d^2 = \frac{900 \times 4}{\pi}$$

$$d = \sqrt{\frac{900 \times 4}{3 \times \pi}} = 19.54 \text{ mm}$$

$$D = 2 \times 19.54$$

$$D = 39.08 \text{ mm}$$

$$\text{Radius of gyration } k = \sqrt{\frac{I}{A}} = \sqrt{\frac{\frac{\pi}{64} (D^4 - d^4)}{\frac{\pi}{4} (D^2 - d^2)}}$$

$$\Rightarrow \sqrt{\frac{D^2 + d^2}{4}} = \frac{\sqrt{5}d^2}{4}$$

$$\Rightarrow \sqrt{5 \times 19.54^2} = 10.92 \text{ mm}$$

$$\text{Slenderness ratio} = \frac{L}{k} = \frac{1000}{10.92} = 91.57$$

Hollow circular section with outer radius 60mm

$D = 120 \text{ mm}$ and $d = \text{internal diameter}$.

$$\frac{\pi}{4} (D^2 - d^2) = 900$$

$$\Rightarrow D^2 - d^2 = \frac{900 \times 4}{\pi} = 1146$$

$$\Rightarrow d^2 = (120)^2 - 1146$$

$$\Rightarrow d^2 = 13254$$

$$\Rightarrow d = 115.12 \text{ mm}$$

$$\text{From previous ratio case } k = \sqrt{\frac{D^2 + d^2}{4}}$$

$$k = \sqrt{\frac{(120)^2 + (115.12)^2}{4}}$$

$$k = 41.57 \text{ mm}$$

Slenderness ratio $= \frac{L}{k} = \frac{1000}{41.57} = 24.05$
Rectangular Section

$d = 2b$

$A = d \times b = 2b \times b = 2b^2 = 900$

$b = 21.21 \text{ mm}$

$d = 42.42 \text{ mm}$

$k = \sqrt{\frac{I}{A}} = \sqrt{\frac{b^2}{12}} = \frac{b}{\sqrt{12}} = \frac{21.21}{\sqrt{12}}$

$\Rightarrow k = 6.12 \text{ mm}$

Slenderness ratio $= \frac{L}{k} = \frac{1000}{6.12} = 163.39$

Previous Year Questions with Answers

(a) Define Poisson's Ratio?

Ans Poisson's Ratio :- The ratio of the lateral strain to the longitudinal strain. It is denoted by $\frac{1}{m}$. Poisson's ratio $= \frac{1}{m} = \frac{\text{Lateral strain}}{\text{Longitudinal strain}}$

(b) What is meant by Shear force diagram and Bending moment diagram?

Ans The Shear force diagram is represent the variation of shear force along a beam, the bending diagram is a graphical ~~represent~~ representation of the variation of bending moment along the length of the beam. ~~The bending diagram is a graphical representation~~

(c) ~~or~~ Define the point of contra-flexure

$\Rightarrow$ The point where the bending moment is zero is known as point of contra-flexure. Point of Contraflexure is always in overhanging beam.

(d) Define Column?

Ans Column:- A long vertical slender member or bar subjected to an axial compressive force is known as column.

(e) What is meant by Temperature Stress?

Ans Temp. stress or thermal stress is the stress developed in a body due to change in temp.

(f) What do you mean by ~~tem~~ section modulus?

Ans:- Section modulus is a geometric property used to calculate the bending stresses in a structural member. Mathematically, it is defined as the ratio of the moment of inertia of a section about its centroidal axis and to the distance of the extreme layer from the neutral axis.

(g) Define Principal Stress and Principal Plane

Ans Principal Stress:- The magnitude of direct stress across a Principal Plane is known as principal stress.

Principal Plane:- The perpendicular planes which have no shear stresses are known as principal planes.

(h) What is the function of Shaft? Write down the formula for Power Transmitted by a Shaft.

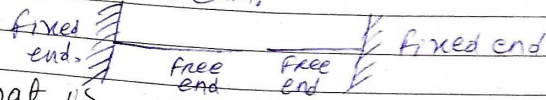
Ans- A Shaft is a rotating machine element usually in cross section which is used to transmit power from one part to another.

Formula for Power Transmitted by Shaft.

$$P = \frac{2\pi NT}{60}$$

(i) Define cantilever beam with example.

Ans The beam whose one end is fixed and the other end is free is known as cantilever beam.



(j) What is meant by thin cylinder shell? State its application.

Ans Closed vessels are used for storing fluids under pressure. If the ratio of thickness of shell to internal radius is less than $\frac{1}{10}$, then the cylindrical vessel is known as thin cylinder.

Applications :-

- ⇒ Boiler shells,
- ⇒ Pressure tanks
- ⇒ Pipes.

2(a) A hollow cylinder 2m long has an outside diameter of 50mm and inside diameter of 30mm. If the cylinder is carrying a load of 25 kN, find the stress in the cylinder. Also find the deformation of the cylinder, if the modulus of elasticity for the cylinder material is 100 GPa.

Ans $L = 2\text{m} = 2000\text{mm}$

$$d_o = 50\text{mm}$$

$$d_i = 30\text{mm}$$

$$\text{Load (P)} = 25\text{kN} = 25 \times 10^3 \text{N}$$

Modulus of elasticity

$$E = 100\text{GPa} = 100 \times 10^3 \text{N/mm}^2$$

$$\text{Area} = \frac{\pi}{4} (d_o^2 - d_i^2)$$

$$= \frac{\pi}{4} (50^2 - 30^2)$$

$$= \frac{\pi}{4} (2500 - 900)$$

$$= \frac{\pi}{4} \times 1600 = 1256.4 \text{ mm}^2$$

$$\text{Stress} = \frac{\text{Load}}{\text{Area}} = \frac{25 \times 10^3}{1256.4} = 19.89 \text{ N/mm}^2$$

$$\text{Modulus of elasticity} = \frac{\text{Stress}}{\text{Strain}}$$

$$\Rightarrow 100 \times 10^3 = \frac{19.89}{\text{Strain}}$$

$$\Rightarrow \text{Strain} = \frac{19.89}{100 \times 10^3} = 0.0001989$$

$$\Rightarrow \text{Strain} = \frac{\delta L}{L}$$

$$\Rightarrow 0.0001989 = \frac{\delta L}{2000}$$

$$\Rightarrow \delta L = 2000 \times 0.0001989 = 0.3978 \text{ mm}$$

(b) The stresses at point of a machine component are 150 MPa and 50 MPa both tensile. Find the intensities of normal, shear and resultant stresses on a plane inclined at an angle of 55° with the axis of major tensile stress.

Ans. Given that,

$$P_1 = 150 \text{ MPa}, P_2 = 50 \text{ MPa}$$

$$\theta = 55^\circ$$

~~Q.1~~ Normal Stress:-

$$\sigma_n = \left(\frac{P_1 + P_2}{2} \right) + \left(\frac{P_1 - P_2}{2} \right) \cos 2\theta$$

$$= \left(\frac{150 + 50}{2} \right) + \left(\frac{150 - 50}{2} \right) \cos (2 \times 55^\circ)$$

$$= \left(\frac{200}{2} \right) + \left(\frac{100}{2} \right) \cos 110^\circ$$

$$= 100 + 50 \cos 110^\circ$$

$$= 100 - 17.10 = 82.9 \text{ MPa}$$

Shear Stress

$$\tau_s = \frac{P_1 - P_2}{2} \sin 2\theta$$

$$= \frac{(150 - 50)}{2} \sin (2 \times 55^\circ)$$

$$= \left(\frac{100}{2} \right) \sin 110^\circ$$

$$= 50 \times 110^\circ = 46.98 \text{ MPa}$$

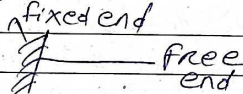
$$\text{Resultant Stress} = \sqrt{(\sigma_x)^2 + (\tau_s)^2}$$

$$= \sqrt{(82.9)^2 + (46.98)^2}$$

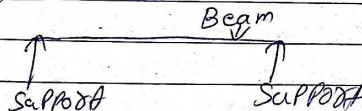
$$= \sqrt{6872.41 + 2207.12}$$

$$= \sqrt{9079.53} = 95.28 \text{ MPa}$$

(C) Show diagrammatically different types of beam and loads

(Ans) Cantilever Beam:- 

(2) Simply Supported Beam



(3) Fixed Beam:-

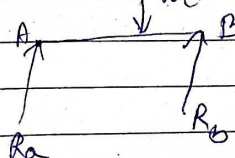


(4) Over-hanging beam:-

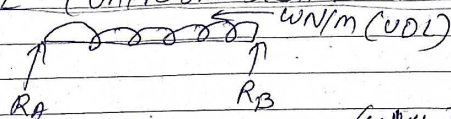


Different types of load:- w (unit load)

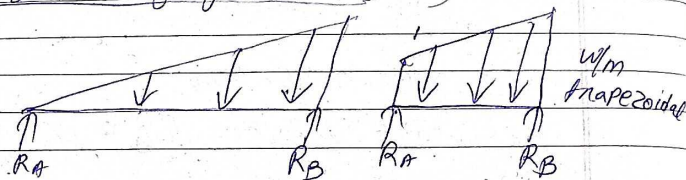
(1) Point Load:-



(2) UDL = (Uniform Distributed Load):-



(3) Uniform Varying Load:- (UVL)



(d) What are the various Assumptions taken while deriving bending equations under theory of simple bending?

Ans:- (i) The material of the beam is homogeneous and isotropic.

(ii) The value of modulus of elasticity is the same in tension and compression.

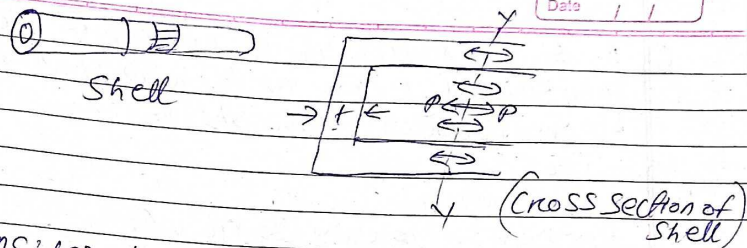
(iii) The material of the beam obeys Hooke's law and is stressed within elastic limit.

(iv) The transverse section of the beam which were plane before bending remain plane after bending.

(v) The loads are applied in the plane of bending.

(vi) The resultant push or pull on the transverse section of the beam is zero.

(e) Derive a formula for the longitudinal stress in a thin cylindrical shell subjected to an internal pressure.



Consider the cylindrical shell subjected to the internal pressure. As a result of this pressure, the shell has also a tendency to torn away into two pieces as

Let $L \rightarrow$ Length of the shell.

$d \rightarrow$ Internal diameter of shell

$t \rightarrow$ Thickness of the shell

$P \rightarrow$ Intensity of internal pressure

This pressure let the cylindrical shell be torn away longitudinally along some section (say Y-Y).

Total Pressure along Y-Y.

$=$ Intensity of Stress $\times$ Area

$$= P \times \frac{\pi}{4} d^2$$

Total resisting section to the failure of shell

$$= \pi d t$$

Longitudinal Stress in the shell

$$\sigma_L = \frac{\text{Total Pressure}}{\text{Resisting section}}$$

$$= \frac{P \times \frac{\pi}{4} d^2}{\pi d t} = \frac{P d}{4 t}$$

$$\Rightarrow \sigma_L = \frac{P d}{4 t}$$

- (F) A Solid circular shaft of 100mm diameter is transmitting 120kW at 150 rpm. Find the intensity of shear stress on the shaft.

Ans Given data

$$d = 100 \text{ mm}$$

$$\text{Power (P)} = 120 \text{ kW} = 120 \times 10^3 \text{ watt}$$

$$N = 150 \text{ rpm.}$$

$$P = \frac{2\pi NT}{60}$$

$$\Rightarrow 120 \times 10^3 = \frac{2 \times \pi \times 150 \times T}{60}$$

$$\Rightarrow T = \frac{120 \times 10^3 \times 60}{2\pi \times 150}$$

$$\Rightarrow T = 7640.878 \text{ N-m}$$

$$\Rightarrow T = 7640.878 \times 10^3 \text{ N-mm}$$

Torque transmitted by shaft

$$T = \frac{\pi}{16} \sigma_s \times d^3$$

$$\Rightarrow 7640.878 \times 10^3 = \frac{\pi}{16} \sigma_s \times (100)^3$$

$$\Rightarrow \sigma_s = \frac{7640.878 \times 16 \times 10^3}{\pi (100)^3}$$

$$\Rightarrow \sigma_s = 122.254 \text{ N/mm}^2$$

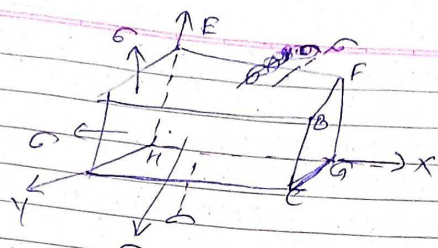
- (3) Prove $E = 3k(1-2/m)$

Where E = Young's modulus

k = Bulk modulus

$1/m$ = Poisson's Ratio.

Sol Consider a cube ABCDEFGH subjected to tensile stress ' σ ' on all the faces.



Let us consider the strain of one side AB of the cube under the action of 3 mutually perpendicular stresses.

(i) Strain of AB due to stresses on the faces ADHE and BCFG (tensile strain) = $\frac{\sigma}{E}$

(ii) Strain of AB due to stresses on the faces ABFE and DCGH = $\frac{\sigma}{E}$

(iii) Strain of AB due to stresses on the faces ABCD and EFGH. $\left[\frac{1}{m} = \frac{\text{Lateral strain}}{\text{Linear strain}} \right]$

$\therefore$ Net strain on

$$AB = \frac{\sigma}{E} - \frac{\sigma}{mE} - \frac{\sigma}{mE}$$

$$= \frac{\sigma}{E} \left(1 - \frac{2}{m} \right)$$

Volumetric strain in the unit cube,

$$e_v = 3 \times \text{Strain in AB}$$

$$= 3 \times \frac{\sigma}{E} \left(1 - \frac{2}{m} \right)$$

$$\text{Bulk modulus } k = \frac{\text{Stress}}{\text{Volumetric strain}} = \frac{\sigma}{e_v}$$

$$= \frac{\sigma \times E}{3\sigma \left(1 - \frac{2}{m} \right)}$$

$$\therefore 3k = \frac{E}{\left(1 - \frac{2}{m} \right)} \quad E = 3k \left(1 - \frac{2}{m} \right) \text{ Proved.}$$

(4) Prove the relation $\frac{\sigma}{y} = \frac{E}{R} = \frac{M}{I}$

where, m = Bending moment

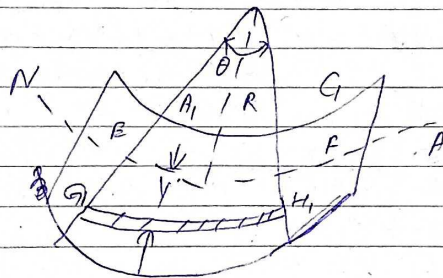
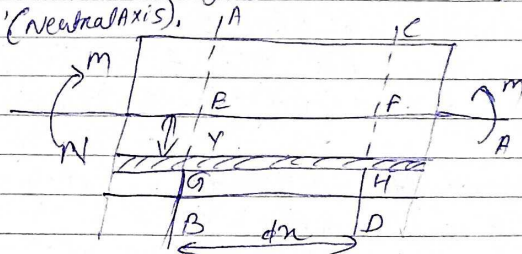
I = moment of Inertia

σ = Bending Stress from neutral axis

E = Young's modulus

R = Radius of curvature

Sol Consider a layer GH at a distance y from 'NA' (Neutral Axis).



original length of this fibre $GH = dx$.

Length of the fibre

$$GH = EF = dx = R\theta$$

$$\text{Length of fibre } GH_1 = (R+y)\theta$$

Change of length of fibre

$$GH = GH_1 - GH$$

$$= (R+y)\theta - R\theta = y\theta$$

Strain in fibre GH ,

$$e = \frac{\text{Change in length}}{\text{original length}}$$

$$e = \frac{Y\theta}{R\theta} = \frac{Y}{R} \quad \dots (i)$$

But strain, $e = \frac{\text{Stress}}{E}$

$$e = \frac{\sigma}{E} \quad \dots (ii)$$

From (i) and (ii),

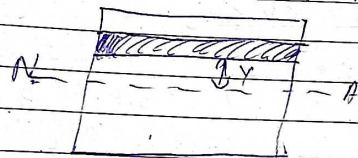
$$\text{we get, } \frac{\sigma}{E} = \frac{Y}{R}$$

$$\Rightarrow \frac{\sigma}{Y} = \frac{E}{R}$$

$$\Rightarrow \sigma = \frac{E}{R} Y$$

$$\Rightarrow \sigma @ Y = \frac{E}{R} \quad \dots (iii)$$

Consider an elementary area δa of the beam section at a distance 'Y' from the NA



$$\begin{aligned} \text{force on the elementary area} &= \sigma \times \delta a \\ &= \left(\frac{E}{R} Y \right) \delta a \end{aligned}$$

$$\begin{aligned} \text{moment of resistance offered by this elementary area about NA} &= \left(\frac{E}{R} \right) Y \times \delta a \times Y \\ &= \frac{E}{R} Y^2 \delta a \end{aligned}$$

$$M = \int \frac{E}{R} y^2 \delta a$$

$$= \frac{E}{R} \int y^2 \delta a$$

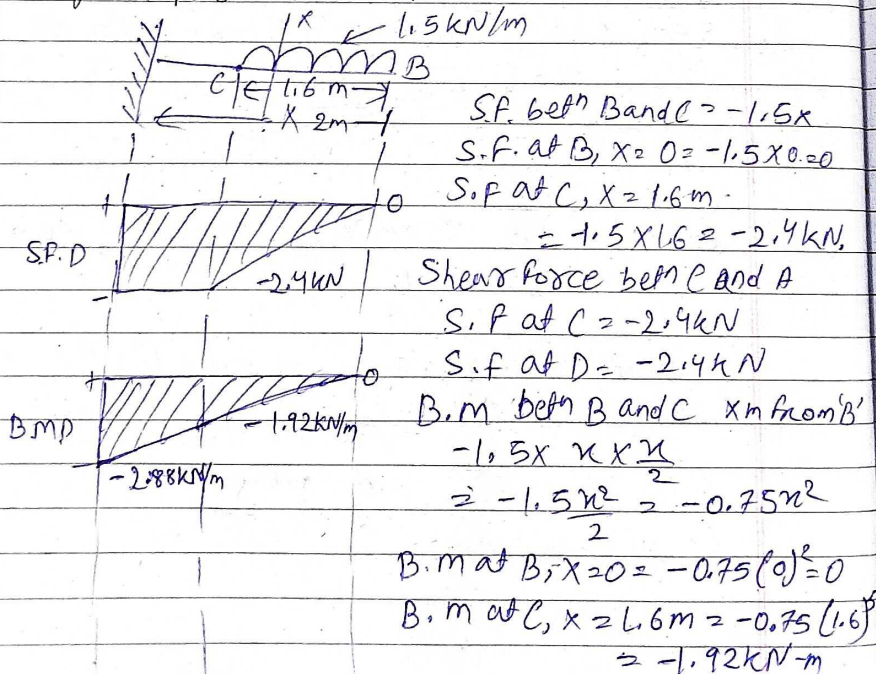
$$\Rightarrow \frac{E}{R} I \quad \left[\because \int y^2 \delta a = I \right] \text{ The beam section about NA}$$

$$\Rightarrow \frac{M}{I} = \frac{E}{R} \quad \therefore (4)$$

Comparing eqn (3) and eqn 4

$$\left[\frac{M}{I} = \frac{\sigma_y}{R} = \frac{E}{R} \right] \text{ Proved}$$

- (5) A cantilever beam AB, 2m long carries a uniformly distributed load of 1.5 kN/m over a length of 1.6 m from the free end. Draw the shear force diagram and bending moment diagram for the beam.



B.m. betn C and A:-
 $= -1.5 \times 1.6 \times \left(\frac{x - 1.6}{2} \right)$

$$= -2.4 \times \left(\frac{2x - 1.6}{2} \right)$$

$$= -1.2 (2x - 1.6) = -2.4x + 1.92$$

$\Rightarrow$ B.m. at C, $x = 1.6 = -2.4 \times 1.6 + 1.92$

$$= -3.84 + 1.92 = -1.92 \text{ kN-m}$$

B.m. at D $= -2.4 \times 2 + 1.92$

$$x = 2 \text{ m} = -4.8 + 1.92 = -2.88 \text{ kN-m}$$

- (6) A cylindrical shell of 1.3m diameter is made up of 18mm thick plates. Find the circumferential and longitudinal stress in the plates, if the boiler is subjected to an internal pressure of 2.4 MPa. Take efficiency of the joints as 70%.

Ans $d = 1.3 \text{ m} = 1300 \text{ mm}$

$t = 18 \text{ mm}$

Internal pressure (p) = 2.4 MPa

$$= 2.4 \text{ N/mm}^2$$

$\eta = 70\%$

$$(C_c) = \frac{pd}{2t\eta} = \frac{2.4 \times 1300}{2 \times 18 \times 0.7} = 123.8 \text{ N/mm}^2$$

Longitudinal stress

$$(C_L) = \frac{pd}{4t\eta} = \frac{2.4 \times 1300}{4 \times 18 \times 0.7} = 61.9 \text{ N/mm}^2$$

- (7) A rectangular beam 60mm wide and 150mm deep is simply supported over a span of 6m. If the beam is subjected to central point load of 12kN. Find the maximum bending stress induced in the beam section.

Ans wide (b) = 60 mm
Deep (d) = 150 mm
length (l) = 6 m = 6000 mm
Load = 12 kN = 12×10^3 N
I $\rightarrow$ moment of inertia
 $I = \frac{1}{12} bd^3$

$$= \frac{1}{12} \times 60 \times (150)^3$$
$$= \frac{1}{12} \times 60 \times (150)^3$$
$$= 16875000 \text{ mm}^4$$

$$Y_{\max} = \frac{d}{2} = \frac{150}{2} = 75 \text{ mm}$$

$$Z = \text{Section modulus}$$
$$= \frac{I}{Y_{\max}} = \frac{16875000}{75}$$

$$\Rightarrow Z = 225,000 \text{ mm}^3$$

maximum bending moment $= \frac{WL}{4}$

$$\text{maximum B.M.} = \frac{12 \times 10^3 \times 6000}{4} = 18000 \times 10^3 \text{ N-m}$$

Applying bending equation,

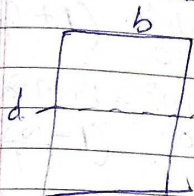
$$\frac{M}{I} = \frac{\sigma_b}{Y_{\max}}$$

$$\Rightarrow \frac{18000 \times 10^3}{16875000} = \frac{\sigma_b}{75}$$

$$\Rightarrow \sigma_b = \frac{18000 \times 10^3 \times 75}{16875000} = 80 \text{ N/mm}^2$$

Assignment
 * Section: - Circular * Bending moment eqn I in neutral axis
 hollow circular * Bending moment eqn I in neutral axis
 Square * Bending of column eqn I in
 Rectangular
 Triangular
 I section
 T section
 channel (U) section

* all types Problem are - 6.5 and 6.6 in ed/syll
 * Sample Paper 2019 all question.



$$I = \frac{bd^3}{12}$$

$$y = \frac{d}{2}$$

$$Z = \frac{bd^3}{12} \times \frac{2}{d} = \frac{bd^2}{6}$$

$$S_{max, min} = \frac{P}{A} \pm \frac{M}{Z}$$

$$= \frac{P}{bd} \pm \frac{6Pe}{bd^2}$$

$$= \frac{P}{bd} \left(1 \pm \frac{6e}{d} \right)$$

$$= \frac{P}{A} \left(1 \pm \frac{6e}{d} \right)$$

Exam: Imp long question

S.F.D and BMD

E, C, k derivation

Torsion eqn or Bending equation Derivation of Problem
 Bending Problem

① Example - 1

A rectangular strut is 150 mm and 120 mm thick. It carries a load of 180 kN at an eccentricity of 10 mm in a plane bisecting the thickness. Find the max^m and min^m intensities of stress in the section.

Given width (b) = 150 mm Load = 180 kN = 1800×10^3 N
Thickness (d) = 120 mm eccentricity (e) = 10 mm

Area of the strut,

$$A = b \times d = 150 \times 120 = 18000 \text{ mm}^2$$

$$\sigma_{\max} = \frac{P}{A} \left(1 + \frac{6e}{b} \right) = \frac{1800 \times 10^3}{18000} \left(1 + \frac{6 \times 10}{150} \right) \text{ N/mm}^2$$
$$= 10(1 + 0.4) = 14 \text{ N/mm}^2 = 14 \text{ MPa}$$

$$\sigma_{\min} = \frac{P}{A} \left(1 - \frac{6e}{b} \right) = \frac{1800 \times 10^3}{18000} \left(1 - \frac{6 \times 10}{150} \right) \text{ N/mm}^2$$
$$= 10(1 - 0.4) = 6 \text{ N/mm}^2 = 6 \text{ MPa}$$

② A rectangular column 200 mm wide and 150 mm thick is carrying a vertical load of 120 kN at an eccentricity of 50 mm in a plane bisecting the thickness. Determine the max^m and min^m intensities of stress in the section.

Given width (b) = 200 mm

Thickness (d) = 150 mm

Load (P) = 120 kN = 120×10^3 N

eccentricity (e) = 50 mm

Area of the column,

$$A = b \times d = 200 \times 150 = 30,000 \text{ mm}^2$$

$$\sigma_{\max} = \frac{P}{A} \left(1 + \frac{6e}{b} \right) = \frac{120 \times 10^3}{30000} \left(1 + \frac{6 \times 50}{200} \right) \text{ N/mm}^2$$
$$= 4(1 + 1.5) = 10 \text{ N/mm}^2 = 10 \text{ MPa}$$

$$\sigma_{\min} = \frac{P}{A} \left(1 - \frac{6e}{b}\right) = \frac{120 \times 10^3}{30,000} \left(1 - \frac{6 \times 50}{200}\right) \text{ N/mm}^2$$

$$= 4(1 - 1.5) = 4(-0.5) = -2 \text{ N/mm}^2$$

$$= 2 \text{ N/mm}^2 = 2 \text{ MPa (tension)}$$

(3) In a tension specimen 13 mm in dia. the line of pull is parallel to the axis of the specimen but is displaced from it. Determine the distance of the line of pull from the axis, when the maximum stress is 15% greater than the mean stress on a section normal to the axis.

Given Diameter (d) = 13 mm

$$\sigma_{\max} = 1.15 \sigma_{\text{mean}}$$

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4} (13)^2$$

$$= 132.7 \text{ mm}^2$$

$$\text{Section modulus} = Z = \frac{\pi}{32} (d)^3 = \frac{\pi}{32} (13)^3$$

$$= 215.7 \text{ mm}^3$$

$$m = P \cdot e$$

$$\sigma_{\text{mean}} = \frac{P}{A} = \frac{P}{132.7} \text{ N/mm}^2$$

$$\sigma_{\max} = \sigma_{\text{mean}} + \frac{m}{Z} = \frac{P}{132.7} + \frac{P \cdot e}{215.7}$$

$$\frac{P}{132.7} + \frac{P \cdot e}{215.7} = \frac{P}{132.7} \times 1.15$$

$$\frac{1}{132.7} + \frac{e}{215.7} = \frac{1.15}{132.70}$$

$$e = \left(\frac{1.15}{132.70} - \frac{1}{132.7} \right) \times 215.7$$

$$= 0.25 \text{ mm Ans.}$$

(4) A hollow rectangular masonry pier is 1.2m x 0.8m wide and 150mm thick. A vertical load of 2MN is transmitted in the vertical plane bisecting 1.2m side and at an eccentricity of 100mm from the geometric axis of the section. Calculate the maximum and minimum stress intensities in the section.

Given :- outer width = 1.2m = 1.2×10^3 mm
 Load (P) = 2MN = 2×10^6 N
 outer thickness = 0.8m = 0.8×10^3 mm
 Thickness = 150mm
 eccentricity = 100mm

Area of the pier,

$$A = (BD - bd)$$

$$= [(1.2 \times 10^3) \times (0.8 \times 10^3)] - [(0.9 \times 10^3) \times (0.5 \times 10^3)]$$

$$= (0.96 \times 10^6) - (0.45 \times 10^6) = 0.51 \times 10^6 \text{ mm}^2$$

Section modulus,

$$Z = \frac{1}{6} [BD^2 - bd^2] = \frac{1}{6} [(1.2 \times 10^3) \times (0.8 \times 10^3)^2 - (0.9 \times 10^3) \times (0.5 \times 10^3)^2] \text{ mm}^3$$

$$= \frac{1}{6} [(768 \times 10^6) - (225 \times 10^6)]$$

$$= 90.5 \times 10^6 \text{ mm}^3$$

$$M = P \cdot e = (2 \times 10^6) \times 100 = 200 \times 10^6 \text{ N-mm}$$

$$\sigma_{\max} = \frac{P}{A} + \frac{M}{Z} = \frac{2 \times 10^6}{0.51 \times 10^6} + \frac{200 \times 10^6}{90.5 \times 10^6} \text{ N/mm}^2$$

$$= 3.92 + 2.21 = 6.13 \text{ N/mm}^2$$

$$= 6.13 \text{ MPa}$$

$$\sigma_{\min} = \frac{P}{A} - \frac{M}{Z} = \frac{2 \times 10^6}{0.51 \times 10^6} - \frac{200 \times 10^6}{90.5 \times 10^6} \text{ N/mm}^2$$

$$= 3.92 - 2.21 = 1.71 \text{ N/mm}^2$$

$$= 1.71 \text{ MPa}$$

- (5) A hollow circular column having external and internal diameters of 300 mm and 250 mm respectively carries a vertical load of 100 kN at the outer edge of the column. Calculate the max and min intensities of stress in the section.

Given :- External diameter (D) = 300 mm
Internal diameter (d) = 250 mm
and load (P) = 100 kN = 100×10^3 N

$$A = \frac{\pi}{4} (D^2 - d^2) = \frac{\pi}{4} [(300)^2 - (250)^2] \text{ mm}^2 \\ = 21.6 \times 10^3 \text{ mm}^2$$

Section modulus

$$Z = \frac{\pi}{32} \times \frac{D^4 - d^4}{D} = \frac{\pi}{32} \times \frac{(300)^4 - (250)^4}{300} \\ = 1372 \times 10^3 \text{ mm}^3$$

$$e = 150 \text{ mm}$$

$$M = P \cdot e = (100 \times 10^3) \times 150 = 15 \times 10^6 \text{ N-mm}$$

$$\sigma_{\max} = \frac{P}{A} + \frac{M}{Z} = \frac{100 \times 10^3}{21.6 \times 10^3} + \frac{15 \times 10^6}{1372 \times 10^3} \text{ N/mm}^2 \\ = 4.63 + 10.93 = 15.56 \text{ N/mm}^2 \\ = 15.56 \text{ MPa}$$

$$\sigma_{\min} = \frac{P}{A} - \frac{M}{Z} = \frac{100 \times 10^3}{21.6 \times 10^3} - \frac{15 \times 10^6}{1372 \times 10^3} \text{ N/mm}^2 \\ = 4.63 - 10.93 = -6.3 \text{ N/mm}^2 \\ = 6.3 \text{ MPa (tension)}$$

Euler's theory

- ① A solid round bar 3 m long and 5 cm in diameter is used as strut with both ends hinged. Determine the crippling load, Take $E = 2.0 \times 10^5 \frac{\text{N}}{\text{mm}^2}$

Given :- $L = 3\text{m} = 3000\text{mm}$

$d = 5\text{cm} = 50\text{mm}$

Young's modulus $E = 2 \times 10^5 \frac{\text{N}}{\text{mm}^2}$

Moment of inertia $I = \frac{\pi}{64} d^4 \text{mm}^4$

$$I = \frac{\pi}{64} \times 5^4$$

$$I = 30.68 \text{cm}^4$$

$$I = 30.68 \times 10^4 \text{mm}^4$$

$P =$ crippling load

$$P_{\text{Euler}} = \frac{\pi^2 EI}{L^2}$$

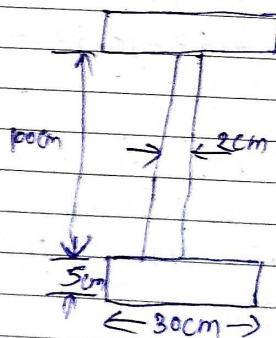
$$= \frac{\pi^2 \times 2 \times 10^5 \times 30.68 \times 10^4}{3000^2}$$

$$= 67288\text{N}$$

$$= 67.288\text{kN}$$

Column with both ends fixed

2. An I-section column of length 14.15m has both of its ends fixed. Use Euler's formula to calculate the safe load that the column can withstand, a factor of safety of 4. $E = 210000 \frac{\text{N}}{\text{mm}^2}$



Given data

$$l = 14.15 \text{ m}$$

$$E = 210 \text{ GN/m}^2$$

$$F.O.S. \text{ of safety} = 4$$

$$l_e = \frac{L}{2} = \frac{14.15}{2} = 7.07 \text{ m}$$

$$I_{xx} = \frac{1}{12} (30 \times 110^3 - 28 \times 100^3)$$

$$= 994166.67 \text{ cm}^4$$

$$= 994.16 \times 10^3 \text{ cm}^4$$

$$I_{yy} = 2 \times \left(\frac{5 \times 30^3}{12} \right) + \left(\frac{100 \times 2^3}{12} \right)$$

$$= 22567 \text{ cm}^4$$

$$= 2.25 \times 10^{-4} \text{ m}^4$$

$$P_{Euler} = \frac{\pi^2 EI}{l_e^2} = \frac{\pi^2 \times 210 \times 10^9 \times 2.25 \times 10^{-4}}{7.07^2 \times 10^{-3}}$$
$$= 9330 \text{ kN}$$

$$\text{Safe load} = \frac{P_{Euler}}{F.O.S.} = \frac{9330}{4} = 2332.5 \text{ kN}$$

Column with one end fixed and other end free

- ③ A mild steel column 5m long and 50mm dia. which is fixed at one end and free at the other end. Determine the Euler's crippling load. Take $E = 2 \times 10^5 \frac{\text{N}}{\text{mm}^2}$

Sol $l = 5 \text{ m}$

$$d = 50 \text{ mm}$$

$$l_e = 2L = 2 \times 5 = 10 \text{ m} = 10 \times 1000 = 10,000 \text{ mm}$$

$$E = 2 \times 10^5 \frac{\text{N}}{\text{mm}^2}$$

$$M.I = I_{xx} = I_{yy} = \frac{\pi d^4}{64}$$

$$\Rightarrow I = \frac{\pi (50)^4}{64} = 306.8 \times 10^3 \text{ mm}^4$$

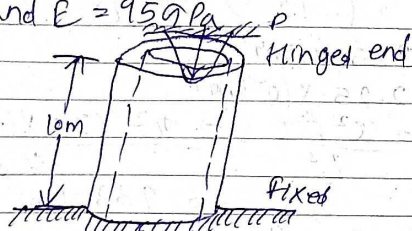
Euler's crippling load $P = \frac{\pi^2 EI}{L^2}$

$$= \frac{\pi^2 \times 2 \times 10^5 \times 306.8 \times 10^3}{(10,000)^2}$$

$$P = 6055.9 \text{ N}$$

$$P = 6.0559 \text{ kN}$$

4. Column with one end fixed and other end hinged.
Calculate the safe compressive load on a hollow cast iron column of 150mm external diameter, 100mm internal diameter and 10m length. Use Euler's formula with a factor of safety of 5 and $E = 95 \text{ GPa}$.



$$D = 150 \text{ mm} = 0.15 \text{ m}$$

$$d = 100 \text{ mm} = 0.1 \text{ m}$$

$$L = 10 \text{ m} = 10 \text{ m}$$

$$FOS = 5$$

$$E = 95 \text{ GPa} = 95 \text{ GN/m}^2 = 95 \times 10^9 \text{ N/m}^2$$

Safe load $= P = ?$

$$\text{Sol} \quad L_e = \frac{L}{\sqrt{2}} = \frac{10}{\sqrt{2}} = 7.07 \text{ m}$$

$$P_{\text{Euler}} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 \times 95 \times 10^9 \times \left(\frac{\pi}{64}\right) (0.15^4 - 0.1^4)}{(7.07)^2}$$

$$= 374 \times 10^3 \text{ N} = 374 \text{ kN}$$

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$$\text{Safe load} = \frac{P_{\text{ult}}}{\text{FOS}} = \frac{374}{5} = 74.8 \text{ kN}$$

Previous Year Paper 2011

④ Define Stress :-

Stress :- It is defined as the internal resistance or force exerted by a body to oppose the applied external force/load. But the internal force per unit cross-sectional area is called the intensity of stress.

$$\text{Stress } (\sigma) = \frac{\text{force}}{\text{Area}} \text{ N/m}^2$$

Strain :- The ratio of change of dimension to the original dimension of the body is called as strain.

$$\text{Strain } (\epsilon) = \frac{\text{change in length } (\Delta l)}{\text{original length } (l)}$$

② Stress has unit but strain has no unit.

(2) State the Hook's law.

Ans: Hook's law :- It states that, when a body is loaded within elastic limit, the stress is proportional to strain.

$$E = \frac{\sigma}{\epsilon} = \frac{\text{stress}}{\text{strain}}$$

(3) Write down the expression for Strain energy.
Expression for Strain Energy - Strain energy = work done by the load.

$$U = \frac{\sigma^2 V}{2E} \quad \text{where } V = \text{Volume}$$

$E = \text{modulus of elasticity}$
 $\sigma = \text{deformation of the bar due to load.}$

(4) Define Hoop Stress?

It is the stress induced in the hollow cylinder or hollow sphere which acts tangentially to the circumference of the cylinder and sphere in direction due to the action of internal fluid pressure. $p_c = \frac{p d}{2t}$ where p = internal fluid pressure.

d = Internal diameter of the cylinder.

(5) What do you understand by Principal Stresses?

t = Thickness of the cylinder wall.

Ans:- Principal planes are defined as those planes in a strained body which do not contain any shear stresses. Such planes are subjected to only direct stress.

The magnitude of direct or normal stresses as the case may be acting normal to the principal planes are known as principal stresses.

(6) Write down the significance of Mohr's circle.

Ans:- Mohr's stress circle illustrates various stresses on an oblique plane of a strained material in graphical format.

(i) The Mohr's circle is plotted by taking normal stress as abscissa and the shear stress is in ordinate.

(ii) The radius of the Mohr's stress circle indicates the maximum shear stress.

(7) State the different types of Beams?

Ans:- Beams are generally classified as:-

(i) statically determinate beam.

(ii) statically indeterminate beam.

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 Statically determinate beams are:-

- (i) Simply supported beam,
- (ii) Cantilever beam,
- (iii) overhanging beam,
- (iv) continuous beam
- (v) fixed beam.

(8) What is Pure Bending

⇒ A beam section can be subjected to either axial force, bending force, shear force, torsion etc. When a beam is subjected to only bending in such a manner that the other actions are absent then that type of bending of the beam is called bending.

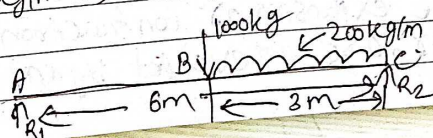
(9) Explain Crippling load

It is the minimum amount of load at which a column or strut will start buckling this load is known as crippling load.

(10) Define Shaft.

A shaft is a rotating machine element. It is used to transmit power from one place to another. The power is delivered to the shaft by some tangential force and the resultant torque set up within the shaft permits the power to be transferred to various machines linked up to the shaft.

(11) State the assumptions made in theory of bending. Find the reactions of simply supported beam when a point load of 1000 kg and a uniform distributed load of 200 kg/m is acting on it.



Sol Given :- $l = 2m = 2000mm$

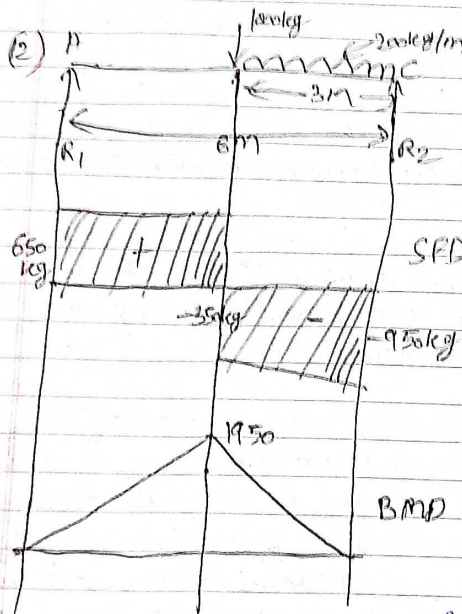
$$D = 150mm^2$$

$$P = 15kN$$

$$E = 200GPa$$

$$= 200kN/mm^2$$

$$\sigma = \frac{P}{A} = \frac{15 \times 10^3}{150 \times 100} = 1mm$$



Span length = 6m

$$P = 1000kg$$

UDL, $P = 200kg/m$

$$\sum M_A = 0$$

$$R_2 \times 6 = (1000 \times 3) + 1000 \times 3$$

$$= 2700 + 3000$$

$$= 5700kg$$

$$R_2 = 950kg$$

$$R_1 = 1000 - (200 \times 3) - 950$$

$$= 650kg$$

BMD

$$S.F. D = R_1 = 650kg = S.F._A$$

$$S.F._B = 650 - 1000 = -350kg$$

$$S.F._C = -350 - 200 \times 3 = -950kg$$

$$B.D.M = B.M_A = B.M_C = 0$$

$$B.M_B = 650 \times 3$$

$$= 1950kg-m$$

(14)

Given

(15)

(13) Explain Temp. Stress and derive its expressions

Ans If the temp. of a body raises or falls, it will undergo expansion or contraction correspondingly, if free expansion or contraction is allowed, it However, are prevented by any means the stress

Given

are produced in the body. These stresses are called temp. or thermal stress and corresponding strain are called temp. or thermal strain.

$$\Rightarrow \frac{P}{E} = \alpha T (\because E = \frac{P}{\epsilon}) \Rightarrow P = \alpha T E$$

Compressive load $\Rightarrow P = p \times A$

$$\Rightarrow P = \alpha T E A$$

- (14) A circular bar is subject an axial Pull of 120 kN. If the maximum intensity of shear stress on any oblique plane is not to exceed 55 MN/m². Find the diameter of the bar.

Given $P = 120 \text{ kN} = \text{Axial Pull}$

$$\tau_{\max} = 55 \text{ MN/m}^2 = 55 \times 10^3 \text{ kN/m}^2 = \text{maximum shear stress}$$

~~Find~~ $d = ?$

Cross sectional area of the bar:-

$$A = \left(\frac{\pi}{4} \right) \times d^2$$

$$\tau_{\max} = \frac{P}{2A}$$

$$\Rightarrow 55 \times 10^3 = \frac{120}{2 \times \frac{\pi}{4} d^2}$$

$$\Rightarrow d = \left(\frac{120 \times 4}{2 \times \pi \times 55 \times 10^3} \right)^{\frac{1}{2}}$$

$$= 0.0372 \text{ m} = 37.2 \text{ mm.}$$

- (15) Find the generalized equation for shear force and bending moment of a simply supported beam with UDL

Given $l = \text{Span length of beam}$

$w = \text{Intensity of loading}$

$$R_A = R_B = \frac{wl}{2}$$

SF at any section - x at a distance x from A.

$$F_x = R_A - wx = \frac{wl}{2} - wx$$

B.m. at any section at a distance x from A.

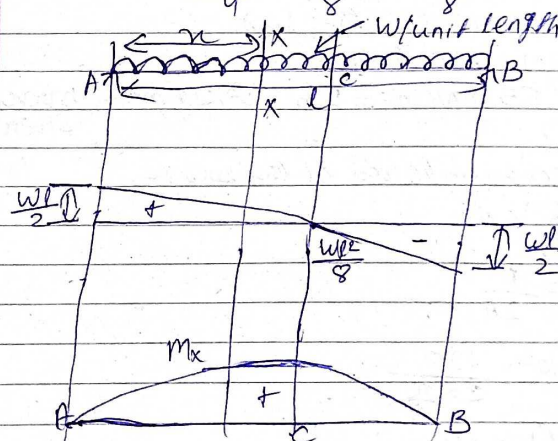
$$m_x = R_A x - \frac{wx^2}{2}$$

$$= \frac{wl}{2} x - \frac{wx^2}{2}$$

$$(Bm)_A = (Bm)_B = 0$$

$$(Bm)_C = \frac{wl}{2} \left(\frac{l}{2}\right) - \frac{w}{2} \left(\frac{l}{2}\right)^2$$

$$= \frac{wl^2}{4} - \frac{wl^2}{8} = \frac{wl^2}{8}$$



(16) A Steel rod of 22mm in dia and 1.5 meters long is sub. to an axial pull of 35kN, find

(i) The intensity of stress (ii) the strain and elongation, Take $E = 2 \times 10^5 \text{ N/mm}^2$

Ans. Given

$$d = 22 \text{ mm}$$

$$l = 1.5 \text{ m} = 1500 \text{ mm}$$

$$P = 35 \text{ kN} = 35000 \text{ N}$$

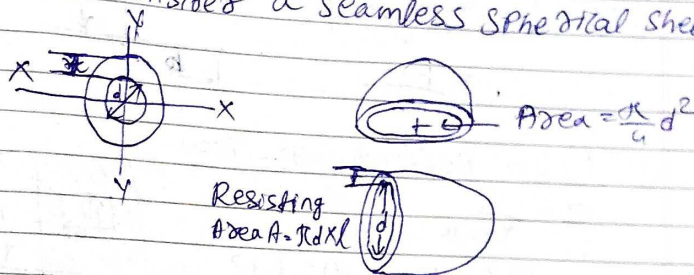
$$E = 2 \times 10^5 \text{ N/mm}^2$$

$$(i) \sigma = \frac{P}{A} = \frac{35 \times 10^3}{\frac{\pi}{4} (22)^2} = 92 \text{ N/mm}^2$$

$$(ii) \text{Strain} = \frac{\text{Stress}}{E} = \frac{92}{2 \times 10^5} = 46 \times 10^{-3}$$

$$(iii) \delta l = \frac{Pl}{AE} = \frac{35000 \times 1500}{\frac{\pi}{4} (22)^2 \times 2 \times 10^5} = 2.16 \text{ mm}$$

(F) Derive the expression for Hoop Stress and Hoop Strain for thin Spherical Shells.
 Ans Let's consider a seamless spherical shell



Let d = Internal diameter

t = Thickness of the shell

P = Internal fluid pressure

σ_c = Hoop Stress

$\frac{1}{m}$ = Poisson's Ratio

Considering the equilibrium of the spherical shell, Resisting force = Bursting force

$$\Rightarrow \sigma_c \times (\pi d \times t) = P \times \frac{\pi d^2}{4}$$

$$\Rightarrow \sigma_c = \frac{Pd}{4t}$$

$\Rightarrow \eta$ = Efficiency of the joint

$$\Rightarrow \sigma_c = \frac{Pd}{4\eta t}$$

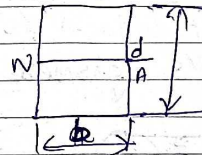
(18) Find out the Expression for section modulus for
 a (i) Rectangular Section (ii) Hollow Rectangular
 Section (iii) Circular Section and (iv) Hollow Circular
 Section.

Ans Section modulus:-

(2) $\frac{\text{moment of inertia about neutral axis}}{\text{Distance of the most extreme fibre from the neutral axis}} = \frac{I}{y_{\max}}$

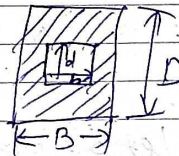
(i) Rectangular Section

$$Z = \frac{I}{y_{\max}} = \frac{bd^3/12}{d/2} = \frac{bd^3}{6}$$



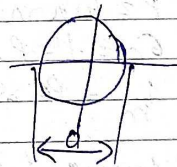
(ii) Hollow Rectangular Section

$$Z = \frac{I}{y_{\max}} = \frac{BD^3 - bd^3}{\frac{D}{2}} = \frac{BD^3 - bd^3}{6D}$$



(iii) Circular Section

$$Z = \frac{I}{y_{\max}} = \frac{\pi d^4/64}{d/2} = \frac{\pi d^3}{32}$$



(iv) Hollow Circular Section

$$Z = \frac{I}{y_{\max}} = \frac{\pi(D^4 - d^4)}{64} \cdot \frac{2}{D} = \frac{\pi(D^4 - d^4)}{32D}$$

- (19) The Principal Stresses at a point in a bar are 150 N/mm^2 (Tensile) and 80 N/mm^2 (compressive). Determine the resultant stress in magnitude and direction on a plane inclined at 60° to the axis of the major principal stress. Also, find the maximum intensity of shear stress in the material at that point.

Ans Tensile stress $\sigma_x = 150 \text{ N/mm}^2$
 Compressive stress $\sigma_y = -80 \text{ N/mm}^2$
 Angle of oblique, $\theta = 60^\circ$
 Normal stress, $\sigma_n = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta$

$$= \frac{150 + (-80)}{2} + \frac{150 - (-80)}{2} \cos 120^\circ$$

$$= 35 + 115 \cos 120^\circ$$

$$= 128.6 \text{ N/mm}^2$$

Shear stress/tangential stress.

$$\tau = \frac{\sigma_x - \sigma_y}{2} \sin 2\theta$$

$$= \frac{150 - (-80)}{2} \sin 120^\circ$$

$$= 115 \sin 120^\circ = 66.77 \text{ N/mm}^2$$

Resultant stress, $\sigma_r = \sqrt{\sigma_n^2 + \tau^2}$

$$= \sqrt{(128.6)^2 + (66.77)^2} = 144.92 \text{ N/mm}^2$$

Maximum shear stress $\tau_{\max} = \frac{\sigma_x - \sigma_y}{2} = \frac{150 - (-80)}{2}$

$$= 115 \text{ N/mm}^2$$

- (20) Derive the formulae for crippling load under various end condition.

Ans :- Condition :- Columns both end hinged of length L and Uniform cross sectional Area A .

Let P be the crippling load at which the Column has just buckled.

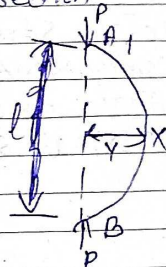
Consider any section XX at a distance x from the end B . Let y be the deflection at the section.

The Bending moment at the section

$$\frac{EI d^2y}{dx^2} = -Py$$

$$\Rightarrow EI \frac{d^2y}{dx^2} + Py = 0$$

$$\Rightarrow \frac{d^2y}{dx^2} + \frac{P}{EI} y = 0$$



The Solution to the above differential equation is

$$y = C_1 \cos \left(x \sqrt{\frac{P}{EI}} \right) + C_2 \sin \left(x \sqrt{\frac{P}{EI}} \right)$$

Where C_1 and C_2 are constant integration

At B the deflection is zero

$$\text{At } x = 0, y = 0$$

$$\text{Hence } C_1 = 0$$

At A also, the deflection is zero

$$\text{At } x = l, y = 0$$

$$0 = C_2 \sin \left(l \sqrt{\frac{P}{EI}} \right)$$

$$\text{Then } C_1 = C_2 = 0$$

Considering the first practical value, we have

$$l \sqrt{\frac{P}{EI}} = \pi$$

$$\Rightarrow P = \frac{\pi^2 EI}{l^2}$$

Case-1 Both end hinged $l_e = l$

$$P = \frac{\pi^2 EI}{l^2}$$

Case-2 = one end fixed other end free, $l_e = 2l$

$$P = \frac{\pi^2 EI}{4l^2}$$

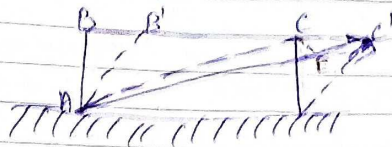
Case-3 one end fixed, other end hinged, $l_e = l\sqrt{2}$

$$P = \frac{2\pi^2 EI}{l^2}$$

Case-4 Both ends fixed $l_e = \frac{l}{2}$

$$P = \frac{4\pi^2 EI}{l^2}$$

Q1) Derive the relationship between the three moduli (Young's, Bulk and Shear).



From fig-1

CE is perpendicular to diagonal AC

Increase in length of diagonal $AC = EC' = CC' \cos 45^\circ$

But angle $CC'E$ is equal to angle $BCD = 45^\circ$

$\therefore$ Strain Along AC = $\frac{EC'}{AC}$

$$\Rightarrow \frac{CC' \cos 45^\circ}{AC} = \frac{CC' \cos 45^\circ}{\frac{DC}{\cos 45^\circ}}$$

$$= \frac{CC' \cos^2 45^\circ}{DC} = \left(\frac{CC'}{DC} \right) \cos^2 45^\circ$$

$$\text{Strain} = E = \frac{1}{2} \phi \quad (i)$$

Linear Strain of Diagonal AC = Strain Produced by tensile force along AC + Strain produced by compressive force along BD

$$e = \frac{T}{E} + \frac{T}{mE}$$

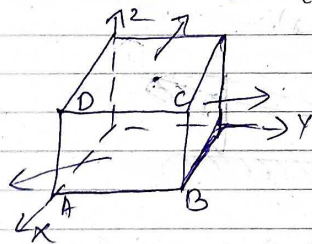
$$e = \frac{T}{E} \left(1 + \frac{1}{m}\right) \dots (ii)$$

From eqn (i) and (ii)

$$\frac{T}{E} \left(1 + \frac{1}{m}\right) = \frac{1}{2} \phi = \frac{1}{2} \frac{T}{C}$$

$$\Rightarrow \frac{T}{E} \left(1 + \frac{1}{m}\right) = \frac{1}{2} \frac{T}{C}$$

$$\Rightarrow E = 2C \left(1 + \frac{1}{m}\right)$$



Volume of cube = l^3

$$\delta V = \delta V = 3l^2 \delta l$$

$$\delta l = 3l^2 \delta l$$

$$\Rightarrow \delta V = 3l^2 \delta l$$

$$\text{Volumetric Strain} = \frac{\delta V}{V} = \frac{3l^2 \delta l}{l^3}$$

$$= \frac{3\delta l}{l}$$

$$\Rightarrow \frac{\delta V}{V} = \frac{3\delta l}{l} \dots (i)$$

Consider the effect of mutual perpendicular stresses on side DC

Strain of DC due to stress on side DC

$$DC \text{ due to } \sigma \text{ along } X = \frac{\sigma}{E}$$

$$" " " " " Y = -\frac{\sigma}{mE}$$

$$" " " " " Z = -\frac{\sigma}{mE}$$

Answer
is

Total strain of D = $\frac{\sigma}{E} = \frac{\sigma}{mE} = \frac{\sigma}{mE}$

$\Delta l = \frac{\sigma}{E} (1 - \frac{2}{m}) \dots (i)$

From eqn (i) and (ii)

$\Delta l = \frac{2\sigma}{E} (1 - \frac{2}{m})$

$E = \frac{2\sigma}{\Delta l} (1 - \frac{2}{m})$

$\Rightarrow E = \frac{2\sigma}{\Delta l} (1 - \frac{2}{m})$

But $\frac{\sigma}{\Delta l}$ is a reciprocal of modulus k

$\Rightarrow E = 2k (1 - \frac{2}{m})$

$\Rightarrow E = 2C (1 + \frac{1}{m})$

$\Rightarrow \frac{E}{2C} = (1 + \frac{1}{m}) \dots (i)$

The relation between E and k is given by

$E = 2k (1 - \frac{2}{m})$

$\frac{E}{2k} = (1 - \frac{2}{m}) \dots (ii)$

From eqn (i) and (ii) $\Rightarrow \frac{E}{2k} = 1 - \frac{2}{m}$

$\frac{E}{2C} = 1 + \frac{1}{m}$

$\frac{E}{2k} + \frac{E}{2C} = 2 - \frac{1}{m}$

$\Rightarrow E = 2k (1 - \frac{2}{m}) = 2C (1 + \frac{1}{m})$

$\Rightarrow 2k - \frac{4k}{m} = 2C + \frac{2C}{m}$

$\Rightarrow 2k - 2C = \frac{4k}{m} + \frac{2C}{m}$

$\Rightarrow 2k - 2C = \frac{1}{m} (4k + 2C)$

$\Rightarrow \frac{1}{m} = \frac{2k - 2C}{4k + 2C} \dots (iv)$

Q3

Q4

Q5

Q6

Q7

Q8

Q9

Q10

Q11

Q12

Q13

Q14

Q15

Q16

Q17

Q18

Q19

Q20

Q21

Q22

Q23

Q24

Q25

Q26

Q27

Q28

$$E = 2C \left(1 + \frac{1}{m}\right)$$

$$E = 2C + \frac{2C}{m}$$

$$\Rightarrow \frac{2C}{m} = 2C - E$$

$$\Rightarrow \frac{1}{m} = \frac{2C - E}{2C} \quad \dots (V)$$

From eqn (IV) and (V)

$$\frac{1}{m} = \frac{3k - 2C}{6k + 2C} = \frac{2C - E}{2C} = \left(1 - \frac{E}{2C}\right)$$

$$\Rightarrow \frac{E}{2C} = 1 - \frac{3k - 2C}{6k + 2C}$$

$$\Rightarrow \frac{E}{2C} = \frac{6k + 2C + 3k - 2C}{6k + 2C} = \frac{9k}{6k + 2C}$$

$$\Rightarrow \frac{E}{2C} = \frac{9k}{6k + 2C}$$

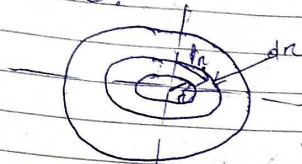
$$\Rightarrow E = \frac{2C \times 9k}{6k + 2C} = \frac{2(9kC)}{2(3k + C)}$$

$$\Rightarrow E = \frac{9kC}{3k + C} \quad \text{Ans}$$

$$\Rightarrow E = \frac{9kC}{3k + C}$$

Torsion eqn

Let us take elementary ring of area on the cross section of the shaft at any radius "r".



Thickness of the ring = dr

Area of the ring = $2\pi r dr$

Shear stress at the radius = t_r

The twisting force on the elementary ring = $t_r \times 2\pi r dr$

Twisting moment due to force $dT = (t_r \times 2\pi r dr) \times r$

$$\int dT = \int_0^R t_r \times 2\pi \times r^2$$

$$= 2\pi \int_0^R t_r \times r^2 \times dr$$

$$= 2\pi \int_0^R \frac{t_R}{R} \times r \times r^2 dr \quad \left[\begin{array}{l} \frac{t_R}{R} = \frac{t_r}{r} \\ t_r = \frac{t_R}{r} \times r \end{array} \right]$$

$$T = 2\pi \frac{t_{max}}{R} \left[\frac{r^4}{4} \right]_0^R$$

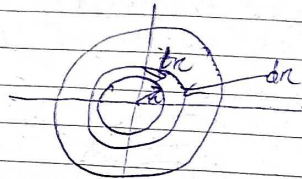
$$T = 2\pi \times \frac{t_{max}}{R} \times \frac{R^4}{4}$$

$$T = \frac{t_{max}}{R} \times \frac{\pi R^4}{2} \quad \left[J = \frac{\pi R^4}{2} \right]$$

$= \frac{\pi D^4}{32}$

$T = \frac{t_{max} \times J}{R}$

Shear stress in an elementary of shaft.



$$\int d\tau = \int_0^R \tau r \cdot 2\pi r \cdot dr$$

$$= \int_0^R \frac{\tau_{\max}}{R} \cdot r \cdot 2\pi r^2 dr$$

$$= \frac{2\pi \tau_{\max}}{R} \int_0^R r^3 dr$$

$$= \frac{2\pi \tau_{\max}}{R} \left[\frac{r^4}{4} \right]_0^R$$

$$= \frac{\pi \tau_{\max} R^3}{2}$$

$$\Rightarrow T = \frac{\tau_{\max} R^3}{2}$$

$$= \frac{2\pi \times \tau_{\max} \times R^4}{R \times 4}$$

$$= \pi \times \frac{\tau_{\max}}{R} \times \frac{D^4}{32}$$

$$= \frac{\tau_{\max}}{R} \times J$$

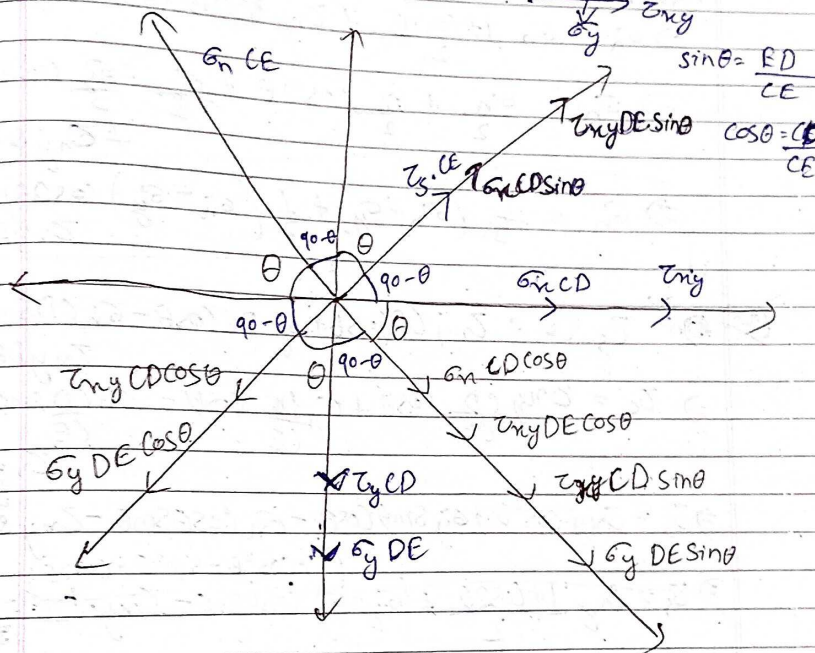
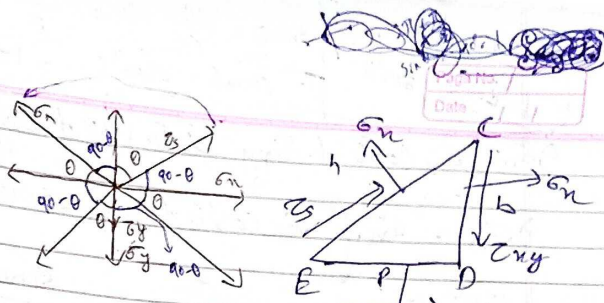
$$J = \frac{\pi D^4}{32}$$

$$T = \int d\tau$$

$$= \frac{\tau_{\max} \times J}{R}$$

$$\Rightarrow \frac{T}{J} = \frac{\tau_{\max}}{R} \Rightarrow \frac{\tau}{r} \Rightarrow \frac{G\theta}{L}$$

$$\Rightarrow \frac{T}{J} = \frac{\tau}{r} = \frac{G\theta}{L}$$



$$\sigma_n \cdot CE = \sigma_n CD \cos \theta + \tau_{ny} DE \cos \theta + \tau_{ny} CD \sin \theta + \sigma_y DE \sin \theta$$

$$\sigma_n = \frac{\sigma_n CD \cos \theta + \sigma_y DE \sin \theta + \tau_{ny} DE \cos \theta + \tau_{ny} CD \sin \theta}{CE}$$

$$= \frac{\sigma_n (CD \cos \theta + DE \sin \theta) + \tau_{ny} (DE \cos \theta + CD \sin \theta)}{CE}$$

$$= \frac{\sigma_n CD \cos \theta}{CE} + \frac{\sigma_y DE \sin \theta}{CE} + \frac{\tau_{ny} DE \cos \theta}{CE} + \frac{\tau_{ny} CD \sin \theta}{CE}$$

$$= \sigma_n \cos^2 \theta + \sigma_y \sin^2 \theta + \tau_{ny} \sin \theta \cos \theta + \tau_{ny} \cos \theta \sin \theta$$

$$\frac{1 + \cos 2\theta}{2} = \cos^2 \theta$$

$$\frac{1 - \cos 2\theta}{2} = \sin^2 \theta$$

$$\sin \theta = \frac{ED}{CE} \quad \cos \theta = \frac{CD}{CE}$$

$$\cos 2\theta = 2\cos^2 \theta - 1 = 1 - 2\sin^2 \theta$$

$$\Rightarrow \sigma_n = \sigma_n \cos^2 \theta + \tau_{xy} \sin \theta \cos \theta + \tau_{xy} \cos \theta \sin \theta + \sigma_y \sin^2 \theta$$

$$\Rightarrow \sigma_n = \sigma_n \cos^2 \theta + \tau_{xy} 2 \sin \theta \cos \theta + \sigma_y \sin^2 \theta$$

$$\Rightarrow \sigma_n = \sigma_n \frac{1 + \cos 2\theta}{2} + \tau_{xy} \sin 2\theta + \sigma_y \frac{1 - \cos 2\theta}{2}$$

$$\Rightarrow \sigma_n = \frac{\sigma_n}{2} + \frac{\sigma_n}{2} \cos 2\theta + \frac{\sigma_y}{2} - \frac{\sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$\Rightarrow \sigma_n = \frac{1}{2} (\sigma_n + \sigma_y) + \frac{1}{2} (\sigma_n - \sigma_y) \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$\Rightarrow \tau_s = \tau_{xy} \cos \theta \sin \theta + \sigma_y \sin \theta \cos \theta - \sigma_n \cos \theta \sin \theta - \tau_{xy} \sin \theta \cos \theta$$

$$\Rightarrow \tau_s = \tau_{xy} \frac{CD}{CE} \cos \theta + \sigma_y \frac{DE}{CE} \cos \theta - \sigma_n \frac{CD}{CE} \sin \theta - \tau_{xy} \frac{DE}{CE} \sin \theta$$

$$\Rightarrow \tau_s = \tau_{xy} \cos^2 \theta + \sigma_y \sin \theta \cos \theta - \sigma_n \cos \theta \sin \theta - \tau_{xy} \sin^2 \theta$$

$$\Rightarrow \tau_s = \tau_{xy} \frac{1 + \cos 2\theta}{2} + (\sigma_y - \sigma_n) \cos \theta \sin \theta - \tau_{xy} \frac{1 - \cos 2\theta}{2}$$

$$\Rightarrow \tau_s = \tau_{xy} (\cos^2 \theta - \sin^2 \theta) + (\sigma_y - \sigma_n) \cos \theta \sin \theta$$

$$\Rightarrow \tau_s = \tau_{xy} \cos 2\theta - \frac{\sigma_n}{2} \sin 2\theta + \frac{\sigma_y}{2} \sin 2\theta$$

$$\Rightarrow \tau_s = -\frac{1}{2} (\sigma_n - \sigma_y) \sin 2\theta + \tau_{xy} \cos 2\theta$$

$$\vec{E} = E_x \hat{i} + E_y \hat{j}$$

$$\frac{dE}{d\theta} = 0$$

$$\Rightarrow \frac{1}{2} (E_x - E_y) \frac{d}{d\theta} \cos 2\theta + E_y \frac{d}{d\theta} \sin 2\theta = 0$$

$$\Rightarrow \frac{1}{2} (E_x - E_y) (-2 \sin 2\theta) + E_y (2 \cos 2\theta) = 0$$

$$\Rightarrow E_x - E_y \sin 2\theta + E_y \cos 2\theta = 0$$

$$\Rightarrow (E_x - E_y) \sin 2\theta = 2 E_y \cos 2\theta$$

$$\Rightarrow \tan 2\theta = \frac{2 E_y}{E_x - E_y}$$

$$\Rightarrow \tan 2\theta = \frac{2 E_y}{E_x - E_y}$$

$$\Rightarrow 2\theta = \tan^{-1} \frac{2 E_y}{E_x - E_y}$$

$$\Rightarrow \theta_p = \frac{1}{2} \tan^{-1} \frac{2 E_y}{E_x - E_y}$$

$$E_1 = (E_x)_2 = \frac{1}{2} (E_x + E_y) + \frac{1}{2} (E_x - E_y) \cos 2\theta + E_y \sin 2\theta$$

$$E_2 = (E_x)_2 = \frac{1}{2} (E_x + E_y) + \frac{1}{2} (E_x - E_y) \cos 2\theta - E_y \sin 2\theta$$

$$\begin{aligned} \cos 2\theta &= \cos(2\theta_p) \\ &= \cos(2\theta_p) \\ &= \cos(2\theta_p) \\ \sin 2\theta &= \sin(2\theta_p) \\ &= \sin(2\theta_p) \\ &= \sin(2\theta_p) \end{aligned}$$

Let (i) and (ii)

$$\vec{E}_1 + \vec{E}_2 = (E_x + E_y) \hat{i}$$

Normal stress of an object plane located at an angle θ $\sigma_n = \frac{1}{2}(\sigma_n + \sigma_y) + \frac{1}{2}(\sigma_n - \sigma_y)\cos 2\theta + \tau_{xy}\sin 2\theta$

Shear Stress

$$\tau_s = -\frac{1}{2}(\sigma_n - \sigma_y)\sin 2\theta + \tau_{xy}\cos 2\theta$$

Location of P.P

$$\tan 2\theta_p = \frac{\tau_{xy}}{\frac{(\sigma_n - \sigma_y)}{2}} = \frac{d\tau_s}{d\theta} = 0$$

$$\Rightarrow -\frac{1}{2}(\sigma_n - \sigma_y)\frac{d}{d\theta}\sin 2\theta + \tau_{xy}\frac{d}{d\theta}\cos 2\theta = 0$$

$$\Rightarrow \frac{1}{2}(\sigma_n - \sigma_y) \cdot 2\cos 2\theta = \tau_{xy} - 2\sin 2\theta$$

$$\Rightarrow \tan 2\theta_s = \frac{1}{2}(\sigma_n - \sigma_y)$$

$$\Rightarrow \tan 2\theta_s = \frac{-\tau_{xy}}{2\tau_{xy}} = -1 = -\tan 2\theta_p$$

$\tau_s \text{ max} \rightarrow \text{max}^m \text{ s plane} \rightarrow \theta_2 = \theta_1 = \theta_3 + 90^\circ$

P.P. $\rightarrow$ zero shear plane / pure normal stress plane

$\rightarrow$ Normal stress $\neq 0$

Plane of pure shear - $\sigma_n = 0$

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_n - \sigma_y}$$

$$p^2 + b^2 = h^2 \Rightarrow h = \sqrt{p^2 + b^2}$$

$$(h) = \sqrt{\left(\frac{\sigma_n - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \text{Radius of mohr's circle.}$$

$$= \frac{1}{2}\sqrt{(\sigma_n - \sigma_y)^2 + 4\tau_{xy}^2}$$

$$\sin 2\theta_p = \frac{2\tau_{xy}}{\sqrt{(\sigma_n - \sigma_y)^2 + 4\tau_{xy}^2}}$$

$$\sin 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

$$\cos 2\theta_p = \frac{\sigma_x - \sigma_y}{\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}}$$

$$\Rightarrow \sigma_n = \frac{1}{2}(\sigma_x + \sigma_y) + \frac{1}{2}(\sigma_x - \sigma_y)\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\Rightarrow \frac{(\sigma_x - \sigma_y)^2}{2} + 2\tau_{xy}^2 = \frac{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}{2\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}} = \pm \frac{\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}}{2}$$

$$\Rightarrow \sigma_{n1} = \sigma_{n2} = \frac{1}{2}[\sigma_x + \sigma_y] \pm \frac{\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}}{2}$$

$$\Rightarrow \sigma_1 - \sigma_2 = \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2} = \text{Diameter of Mohr's circle}$$

$$\sigma_1 = + \sigma_{n1}$$

$$\sigma_2 = - \sigma_1 + \sigma_y = \sigma_{n2} + \sigma_y$$

$$\sigma_1 > \sigma_2 \text{ (Location of } \sigma_1 \text{ plane)}$$

* Assumptions in Theory of Simple Bending

- (i) The beam is initially straight and of uniform cross section.
- (ii) The beam is subjected to pure or simple bending only and no shear force acts on it.
- (iii) The beam material is stressed within elastic limit and hence obeys Hooke's law.
- (iv) The value of Young's modulus (E) of the beam material is same for tension as well as compression.
- (v) The resultant pull or push on a transverse section of the beam is zero.
- (vi) The beam section is symmetrical about the plane of bending.
- (vii) The transverse section of the beam which is plane before bending remains plane after bending.